Optimal Capital Allocation confronting Bankruptcy and Agency Costs[§]

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Abstract

The value of a firm that cannot modify continuously its hedging strategy is supposed to depend on the level of risk capital. Specifically, the cost of bankruptcy is characterised as the expectation with respect to a distorted probability distribution, in this way assessing the type of investors. The optimal capital is allocated to the lines of business of a conglomerate according to the borne risk and the type of divisional managers. Full-allocation is assured and no covariance is required. Further, a mechanism is provided, which allows for the distribution of equity in a decentralised structure. Since no conditions are imposed on the distribution functions describing risks, the raising and allocation principles are suitable both to financial and insurance applications.

Key words: Capital Structure; Risk Capital; Capital Allocation; Bankruptcy Costs; Distorted Probability Distributions.

1 Introduction

Financial intermediaries attract funds by establishing contractual liabilities with customers, a critical concern being how much debt to hire and how much capital to supply. According to the Modigliani and Miller (1959) proposition, the capital structure does not matter, for at any time it is possible to raise or release funds if required. Hence the value of the firm does not depend on the amount of debt and the optimal plan — when the objective is maximising value — is to attract as much liabilities as possible, especially when the presence of taxes increases the cost of holding capital. Since this behaviour is not observed in practice, Modigliani and Miller gave several explanations in subsequent papers, even questioning the skills of decision-makers (Miller, 1998).

However, as long as intermediaries cannot suppress risk, the value of the firm is affected by the risk policy. Moreover, the aggregate portfolio of a company is not observed by outsiders

— who have to expend effort to gather information and properly assess its credit quality — and so performance depends on providing guarantees that assumed liabilities are default-free (Merton, 1997). In this context, natural incentives for managers to maximise value will be confronted with the necessity of diminishing the risk of financial distress. This situation leads managers decisions to be determined by risk aversion — for their reputation is affected by the performance of the firm.

Usual practices to protect against default risk are hedging, re-insuring and risk capital cushions. *Risk-capital* — also known as *economic capital* or *equity* — is an amount of money invested in non-risky assets that serves as a buffer in order to prevent insolvency. Typically, it is provided by shareholders or by a third part guarantor — a role that can be assumed by another financial institution as well as by a governmental division. In any case, since a cost has to be paid for holding capital, a criterion is needed which combines the two conflicting objectives: maximisation of shareholder's value and minimisation of default risk.

Within a multi-business environment, equity must be distributed among divisions in a *fair* way — according to the risk borne. The procedure is justified on the gain obtained when putting risks together — covariances among risks play a crucial role (see Albrecht, 2004; Hallerbach, 2003). Additionally, full-allocation is required, as long as the total capital is expected to fit the conglomerate's needs. In Merton and Perold (1993), a principle based on the incremental risk is advanced — which is defined as the additional surplus required to holding each line of business. The total amount assigned to subsidiaries under this prescription is less than the capital hired by the conglomerate. The difference is explained by the gains in efficiency due to the combination of risks. Myers and Read (2001) consider instead the marginal change of total equity in response to a small increment on the size of the divisional portfolio. Full-allocation is guaranteed provided that some conditions on the valuation function of capital are satisfied.

Stoughton and Zechner (2004) propose a model where firms are not able to continuously raise funds without cost. Thus, equity is distributed so that divisions maximise the *Economic-Value-Added* (EVA) and capital allocation is justified as part of a general mechanism that stimulates the exchange of information among divisional and top managers inside the institution. Distortions can be present in the form of under or overinvestment — which are supposed to depend on the ability to apply and transfer skills. Thus, an optimal mechanism is defined and the internal price of capital appears as a tool for implementation.

In the following, I propose a model which follows the basics of the Stoughton and Zechner's at the time that explicitly deals with the cost of financial distress. *Section 2* is devoted to the determination of the optimal amount of capital. An expression is obtained depending on the risk involved, which allows to work with any distribution function — including those presenting heavy tails. Hence, the principles that will soon be introduced are suitable both to financial and insurance applications. Moreover, in imperfect markets, the type of decision makers accounts for differences in expectations affecting the levels of raised capital. In *Section 3* the optimal capital allocation is determined according to the information accessed by central managers alone. The problem of *agency-costs* is addressed in *Section 4* by establishing an optimal contract depending on the *internal cost* of capital and including the information managed by divisional managers. When the types are not accessible — a situation most probably found in practice — divisions may be induced to reveal their type. *Section 5* concludes.

2 Optimal Amount of Risk-Capital

Consider a financial conglomerate holding assets and liabilities for total market values of A and L respectively — which are assumed to be *uncertain* quantities. The net *loss* the company suffers each period is then given by $X = L - A$. Following Merton (1977), the price of insuring liabilities — at any time before the maturity date — is given by the riskless present value of the liability claim minus the value of a *put* option on the assets of the firm with *strike-price* equal to the value of liabilities (see also Merton, 1997, and Cummins and Sommer, 1996). In the same way, shareholders are the owners of a *call* option on the portfolio of assets with *strike-price* equal to the value of liabilities. From the *Put-Call Parity Theorem*, though both the market value of debt and equity are functions of the firm's leverage, their sum does not depend on it and so the market value of the firm is independent of its capital structure, as stated in the Modigliani-Miller proposition (Miller, 1998).

However, such analysis holds true under *perfect markets* conditions alone. Moreover, the hedged portfolio remains non-risky only a short period of time ahead — assuming that during a short period of time market conditions remain unchanged — and so continuous rebalancing is required. Some firms that are not able to modify their hedging strategy so frequently will seek protection against default risk through reinsurance and risk-capital cushions.

By now, assume that central managers know the distribution function of losses F_X. Let the *cost of bankruptcy* or, more precisely, the *cost of assuming bankruptcy* — as perceived by decision-makers — be quantified by the term $E_\theta\left[(X-k)_+\right]$, where k denotes the preferred level of surplus and the parameter θ accounts for the *aversion-to-risk* profile of the decision-maker, as well as all collected information, skills, educational background, training and social contacts. Actually, even agents who behave as risk-neutrals if facing the *average* understanding of the market might under or overestimates risk. So differences in the parameter are expected to be found in *imperfect* markets, where information is not fairly distributed (Stiglitz 1967, 1972).

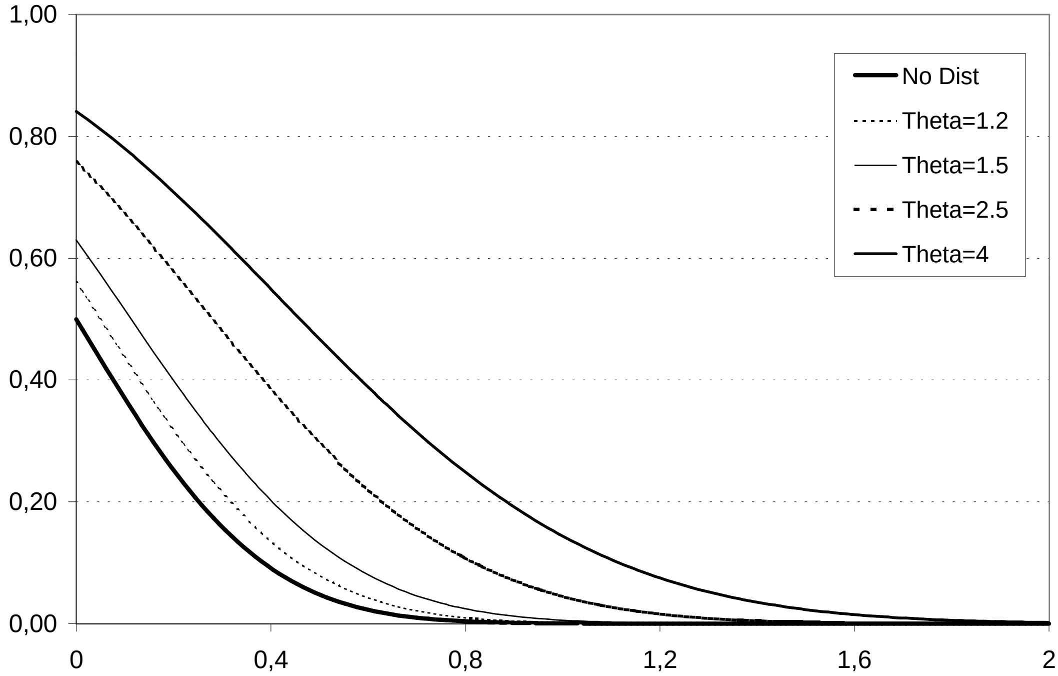
Suppose the company may raise funds at the interest rate r_k — with $r_k > r_0$ in general, where r_0 denotes the *risk-free interest rate*. Hence, if θ and λ respectively denote the types of the institution and the insurer, *corporate EVA* is given by (note that a *negative* value of the variable X represents a *benefit* in the model):

$$EVA = E_\theta\left[(X+k)_-\right] - E_\lambda\left[(X-k)_+\right] - r_k \cdot k$$

Consequently, in markets where agents keep different expectations about risks, financial companies are able to create value to shareholders as long as the cost of insuring their exposure plus the cost of raising capital is less than expected gains. Notice how crucial is the role played by the differences in expectations and the symmetry of risks. When expectations are homogeneous and risks are symmetric, no capital should be raised — the value of the firm is zero in this case, which is a reasonable claim in a competitive setting — and the Modigliani-Miller proposition is obtained (see Stiglitz, 1972).

An explicit expression can be found for the optimal level of capital by introducing a measure for the cost of bankruptcy. I propose to consider the *distorted-risk-measure*, which allows for the following representation (see Wang, 1995, and Wang et al., 1997):

Figure 1: Distorted Decumulative Distributions



$$E_\theta[X] = \int x \cdot dF_{\theta,X}(x) = \int [1 - F_{\theta,X}(x)] \cdot dx = \int S_{\theta,X}(x) \cdot dx = \int S_X(x)^{\frac{1}{\theta}} \cdot dx$$

where $S_X(x) = P\{X > x\} = 1 - F_X(x)$ denote the *decumulative* or *survival* distribution function. Values of the parameter greater than one overestimates the price of risk (in this way representing the behaviour of averse-to-risk decision-makers) while values less than one sub estimates the price (risk-lovers) and the value one represents no distortion (risk-neutral investors). Consider, for example, a Gaussian risk with mean zero and volatility 30%. Small differences in the parameter may induce big differences in the perception of risk. Thus, in *Figure 1*, the survival probability is depicted, starting with the case when no distortion is applied (to the left down side) and raising the distortion parameter to the right upper side. Values of $\theta = 1.2$, 1.5, 2.5 and 4.0 may produce price discrepancies in the order of 30%, 70%, 190% and 340% respectively.

As long as decision-makers regard the *insured return* $E_\theta\left[(X+k)_-\right]$ as linearly dependent on k, the marginal benefit of the guarantee may be added to the cost of capital r_k, in such a way that maximising value is equivalent to minimising bankruptcy costs $E_\theta\left[(X-k)_+\right] + r_k \cdot k$ (as in Dhaene et al., 2003, and Laeven and Goovaerts, 2004). Applying *Lagrange optimisation* yields the firm raises capital until the marginal benefit equal the cost of the guarantee and the optimal level is given by:

$$k^* = F_{\theta,X}^{-1}(1 - r_k) = F_X^{-1}\left(1 - r_k^\theta\right)$$

Therefore, the optimal amount of capital is expressed as a *Value-at-Risk* under a transformed probability measure, a criterion that coincides with the capital adequacy requirement established by the Basel Capital Accord (see Basel Committee on Banking Supervision, 1996, 2004). However, this time the confidence level depends on the cost of raising capital, which means that even risk-neutral investors internalise the price of equity. When this cost is high, less equity is demanded and the contrary occurs when capital is cheap. The cases $r_k \geq 1$ and $r_k \leq 0$ respectively lead to preferring the minimum and the maximum levels of capital.

Table 1: Capital Requirements for Gaussian Risks

Mean	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Volatility	5%	5%	10%	10%	20%	20%	35%	35%
Net Cost of Capital	0,50%	5,00%	5,00%	10,00%	1,00%	5,00%	5,00%	35,65%
Confidence Int	99,50%	95,00%	95,00%	90,00%	99,00%	95,00%	95,00%	64,35%
Capital Requirement / EUR	**0,1288**	0,0822	0,1645	**0,1282**	0,4653	0,3290	0,5757	**0,1287**
Distortion	0,25	1,77	0,77	9,50	1,50	2,30	0,34	3,10
Distorted Conf Int	73,41%	99,50%	90,04%	100,00%	99,90%	99,90%	36,11%	95,91%
Distorted Cap Requ / EUR	0,0313	**0,1289**	**0,1284**	0,6183	0,6181	0,6170	**0,1244**	0,6092
% Capital Distortion	-76%	57%	-22%	382%	33%	88%	-78%	373%

In *Table 1*, the *optimal* capital requirements are depicted for Gaussian risks considering different volatilities, costs of capital and distortions. As expected, more capital is required for the same cost and higher volatility (compare the figures for volatilities equal to 5%, 10%, 20% and 35%, and capital cost equal to 5%) and less capital is required when its cost is higher. Moreover, looking at any column it can be noticed that distortions greater than 1.00 raise the level of optimal capital, while distortions between 0.00 and 1.00 reduce it. The same optimal capital may come from different combinations of costs and volatilities. This is the case in *Table 1* for volatilities 5%, 10% and 35% and costs equal to 0.5%, 5%, 10% and 35.65%. Also the same level is achieved when the cost of capital equals 5% and volatilities are equal to 5%, 10% and 35%, by applying the distortions 1.77, 0.77 and 0.34 respectively.

3 Optimal Capital Allocation

When a company faces a multi-business environment, a failure in any division may cause damage to the reputation of the whole conglomerate. Accordingly, central managers are interested in distributing provisions in such a way the default of subsidiaries is avoided. This situation reinforces the necessity of allocating capital in a fair way — according to the borne risk. An *optimal allocation-principle* can then be defined, in such a way that the sum of divisional bankruptcy costs $E_\theta\left[\sum_{i=1}^{n}(X_i - k_i)_+\right]$ is minimised (see Dhaene et al., 2003, and Laeven and Goovaerts, 2004). Full-allocation is additionally required: $\sum_{i=1}^{n} k_i = k^*$ — as long as capital decisions on business units are taken by central managers no other concern is needed.

Let us denote by F_{X^c} the probability distribution of the *comonotonic sum* $X^c = X_1^c + \ldots + X_n^c$, where $(X_1^c, \ldots, X_n^c)$ represents the *comonotonic random vector* with same marginal distributions as $(X_1, \ldots, X_n)$. *Comonotonicity* characterises an extreme case of dependence — opposite to *independence* (Dhaene et al., 2000a, 2000b). Thus, the optimal risk-capitals allocated to the business units are given by:

$$k_i^* = F_{\theta,X_i}^{-1}\left(F_{\theta,X^c}\left(k^*\right)\right) = F_{\theta,X_i}^{-1}\left(1 - r_k\right) \quad \forall\, i = 1, \ldots, n$$

Such principle is regarded as the *centralised* allocation, for it is established according to the type of central managers alone. No dependence structure is required, but by introducing the comonotonic random vector the *lest favourable* situation — when no diversification is possible — is considered.

4 Capital Allocation as an Optimal Decentralised Mechanism

Full-allocation suffices when the firm acts in a centralised fashion. But divisional managers access better information about investment opportunities and hence it is desirable to allow them to choose on their own. The question is how to achieve the optimal allocation in a decentralised way. Let us consider subsidiaries as separate units that maximise value. Therefore, following the reasoning of *Section 2*, the *stand-alone* risk-capital is defined by:

$$k_i(r) = F_{\theta_i,X_i}^{-1}\left(1 - r\right) \quad \forall\, i = 1, \ldots, n$$

Notice that subsidiaries consider their own subjective probabilities when deciding the level of risk capital. It is possible to prove that $k_i(r) \leq k_i(r_k)\ \forall\, r \geq r_k$ and the opposite occurs when $r < r_k$. Hence subsidiaries incorporate the cost of capital to decision-making. By means of it, (risk-neutral) central managers may then distort divisional manager's decisions to make them to act according to the interest of the conglomerate. Thus, they may overcharge the cost of capital to averse-to-risk subsidiaries in order to encourage them to raise less capital. A return over the market rate r_k should be assigned in this situation. By contrast, when divisional managers are risk-lovers, they must be assigned a return below the market rate — and so their access to equity must be subsidised. Finally, for risk-neutral decision-makers, the market return should be assigned.

The optimal contract is defined such that the value of the firm (that is, corporate EVA) is maximised, at the time that subsidiaries maximise the value of their respective divisions (see Diamond and Verrecchia, 1982):

$$\begin{array}{rc} & Min_{r,k}\ E_\theta\left[(X-k)_+\right] + r_k \cdot k \\ subject\ to & \\ & k_i = k_i(r)\ \ \&\ \ \sum_{i=1}^{n} k_i = k \end{array}$$

Therefore, if $F_{\theta_1,\ldots,\theta_n,X^c} = \left(\sum_{i=1}^{n} F_{\theta_i,X_i}^{-1}\right)^{-1}$ denotes the distribution function of the comonotonic sum when marginal distributions are $(F_{\theta_1,X_1}, \ldots, F_{\theta_n,X_n})$, the optimal levels of aggregated risk-capital and internal cost of capital are respectively given by:

$$k^* = F_{\theta,X^c}^{-1}\left(1 - r_k\right) \quad \& \quad r^* = 1 - F_{\theta_1,\ldots,\theta_n,X^c}\left(k^*\right)$$

This mechanism leads to a *decentralised* allocation, as long as subsidiaries act as independent units. Further, the information of divisional managers is incorporated to decision-making assuring that corporate EVA is maximised.

Table 2: Optimal Capital Allocation for Gaussian Risks (Millions EUR)								
Aggregated Loss	**$ 100,00**	**MM EUR**		Dec-Maker's Distortion	1,00			
Optimal Conglomerate's Capital	**$ 15,51**	**MM EUR**		Comonotonic Portfolio's Volatility	9,43%			
Optimal Internal Cost of Capital	**1,3688%**			External Cost of Capital	5,00%			
Lines of Business	Distortion	Volatility	Contribution	Centralised Allocation	Stand-Alone Allocation	DIF	Decent Allocation	DIF
1	0,35	12,00%	35%	6,91	1,61	5,30	3,21	3,70
2	0,85	25,00%	15%	6,17	5,31	0,86	7,28	-1,12
3	0,90	5,00%	8%	0,66	0,60	0,06	0,81	-0,16
4	**1,00**	5,00%	8%	**0,66**	0,66	0,00	**0,88**	**-0,22**
5	1,15	1,00%	7%	0,12	0,13	-0,01	0,17	-0,06
6	1,75	0,50%	2%	0,02	0,03	-0,01	0,03	-0,02
7	2,30	2,00%	15%	0,49	0,93	-0,43	1,16	-0,67
8	3,50	3,00%	10%	0,49	1,21	-0,72	1,50	-1,00
Total Allocation (MM EUR)			100%	**15,51**	**10,47**	**5,04**	**15,05**	**0,4607**

In *Table 2*, the optimal capital allocation is presented for a conglomerate confronting Gaussian risks and facing an aggregated loss of 100 millions of euros — the volatilities of the lines of business are related to individual exposures. Then the comonotonic sum is a Gaussian variable as well, whose volatility (expressed in terms of the total loss) is given by $\sigma = \sum_{i=1}^{n} \alpha_i \cdot \sigma_i$, where the coefficients $\alpha_1, \ldots, \alpha_n$ denote the per cent contributions to risk (Dhaene et al., 2002a). Business units with high and low volatility are supposed to be managed by risk-lover and adverse-to-risk individuals respectively. Since the composition of the portfolio favours more volatile investments, the total capital supplied by the stand alone allocation — obtained when subsidiaries maximise value and hire equity at the interest rate of the conglomerate — is less than the optimal amount when implementing the centralised solution. Such difference is explained by the gains in efficiency attained because of the knowledge and skills of divisional managers — agency-costs . When applying the decentralised mechanism, the same amount of capital as under the centralised allocation is collected — by distorting the decisions of divisional managers.

When the types of divisions are not observed by central managers — a case most probably found in practice — they can look for the distortions which are compatible with the levels of capital already chosen. Then the process of capital allocation can be performed in two stages. First subsidiaries are asked to suggest an amount of equity for their units. After the estimations of distortions have been realised, the decentralised allocation may be implemented, in this way assuring the same optimal equity as under the centralised solution is obtained at the time that divisional managers reveal their type. Hence the allocation principle provides a basis to measure the disagreement between central management and business units.

5 Concluding Remarks

According to the Modigliani and Miller (1959) proposition, the capital structure of a financial institution does not affect its value for it is always possible to raise or release funds in the market. However, this is not a suitable assumption for imperfect markets. Therefore (after Merton and Perold, 1997, and Stiglitz, 1967, 1972) I claim the capital structure matters for averse-to-risk customers who are sensible to the possibility of bankruptcy of the firm. Accordingly, the decisions of managers — whose reputation depends on performance — are also affected by aversion-to-risk.

A *raising principle* based on the maximisation of value is presented in this paper, which explicitly considers the *cost of bankruptcy*. Following the Stiglitz's (1972) approach, investors are supposed to keep different expectations about risks — which are determined by their type. The optimal level of surplus is then a function of the price of capital and the risk involved and since no restrictions are imposed on the probability distributions, the model is suitable both to financial and insurance applications.

An optimal centralised *allocation principle* is obtained by maximising the value of the firm. For a decentralised structure, a mechanism is proposed, whose instrument is the internal cost of capital. To avoid incurring in agency-costs, the decentralised allocation can be performed, in such a way that the optimal allocation is obtained and subsidiaries are forced to reveal their type.

References

[1] Albrecht, P. (2004). "Risk-based capital allocation", *Encyclopedia of Actuarial Science*: 1459-1466. John Wiley & sons.

[2] Basel Committee on Banking Supervision (1996). "Amendment to the capital accord to incorporate market risks", http://www.bis.org

[3] Basel Committee on Banking Supervision (2004). "International convergence of capital measurement and capital standards. A revised framework", http://www.bis.org

[4] Cummins, J.D. and Sommer, D.W. (1996). "Capital and risk property-liability insurance markets", *Journal of Banking and Finance*, 20:1069-1092.

[5] Dhaene, J., Goovaerts, M. and Kaas, R. (2003). "Economic Capital Allocation Derived from Risk Measures", *North American Actuarial Journal*, 7 (2): 44-59.

[6] Dhaene, J., Denuit, M., Goovaerts, M., Kaas, R. and Vyncke, D. (2002a). "The Concept of Comonotonicity in Actuarial Science and Finance: Theory", *Insurance: Mathematics & Economics*, 31 (1): 3-33.

[7] Dhaene, J., Denuit, M., Goovaerts, M., Kaas, R. and Vyncke, D. (2002b). "The Concept of Comonotonicity in Actuarial Science and Finance: Applications", *Insurance: Mathematics & Economics*, 31 (2): 133-161.

[8] Diamond, D.W. and Verrecchia, R.E. (1982). "Optimal managerial contracts and equilibrium security prices", *The Journal of Finance*, 37 (2): 275-287.

[9] Hallerbach, W.G. (2003). "Capital allocation, portfolio enhancement and performance measurement: a unified approach", http://www.gloriamundi.org

[10] Laeven, R.J.A. and Goovaerts, M.J. (2004). "An optimization approach to the dynamic allocation of economic capital", http://www.gloriamundi.org

[11] Merton, R.C. (1977). "An analytic derivation of the cost of deposit insurance and loan guarantees", *Journal of Banking and Finance*, 1:3-11.

[12] Merton, R.C. (1997). "A model of contract guarantees for credit-sensitive, opaque financial intermediaries", *European Finance Review*, 1:1-13.

[13] Merton, R.C. and Perold, A.F. (1993). "Theory of risk capital in financial firms", *Journal of Applied Corporate Finance*, 5:16-32.

[14] Miller, M.H. (1998). "The Modigliani-Miller propositions after thirty years", *Journal of Economic Perspectives*, 2 (4): 99-120.

[15] Modigliani, F. and Miller, M.H. (1959). "The cost of capital, corporation finance and the theory of investment", *American Economic Review*, 49 (4): 655-69.

[16] Myers, S.C. and Read Jr., J.A. (2001). "Capital Allocation for Insurance Companies", *Journal of Risk and Insurance* 68 (4): 545-580.

[17] Stiglitz, J. (1967). "A re-examination of the Modigliani-Miller Theorem", *Cowles Foundation Dicussion Paper* No 242.

[18] Stiglitz, J. (1972). "Some aspects of the pure theory of corporate finance: bankruptcies and take-overs", *Bell Journal of Economics and Management Science*, 3 (2): 458-482.

[19] Stoughton, N.M. and Zechner, J. (1999). "Optimal capital allocation using RAROC and EVA", http://www.gloriamundi.org

[20] Wang, S. (1995). "Insurance pricing and increased limits ratemaking by proportional hazards transforms", *Insurance: Mathematics & Economics*, 17: 43-54.

[21] Wang, S., Young, V. and Panjer, H. (1997). "Axiomatic characterization of insurance prices", *Insurance: Mathematics & Economics*, 21: 173-183.