Is Last Minute Bidding Bad?

Tao-yi Joseph Wang*

Department of Economics, UCLA

March 14, 2003

Abstract

Last minute bidding on internet auctions is not compatible with standard private value (PV) settings. However, it can be rationalized by adding (at least) another identical auction. In the sequential ebay auction model, it can be proved that the last minute bidding equilibrium, in which bidders only bid at the last minute, is the unique symmetric perfect Bayesian equilibrium in (weakly) undominated strategies. Moreover, bidders would sometimes shade their last minute bids as in a first price auction. Nevertheless, revenue equivalence results show that last minute bidding does not hurt the seller, in contrast to the non-transmission model where last minute bidding is not efficient and lowers sellers' revenue. Thirdly, a bidder's maximum willingness to pay for the current stage depends on others' types. This introduces a "common value" component to the private value environment. Empirically, online event ticket auction data shows some support of the sequential model, while laboratory experiments reject the alternative non-transmission model.

*Preliminary. Comments welcomed. Please do not quote without permission.

1 Introduction

During the past ten years, taking advantage of costless flows of information on the internet, online auction houses such as ebay help millions of buyers and sellers meet and trade with each other. One interesting phenomenon of ebay auctions that has not been fully understood is "sniping," or last minute bidding, meaning that most buyers submit their bids very late during these online auctions.[1] As Bajari and Hortacsu (2000) note, "more than 50% of final bids are submitted after 90% of the auction duration has passed."

Intuitively, if there were overlapping auctions, it would be natural to see people bid on the auctions that end first, and then bid on the next auction if they did not win the first one. This would result in bids coming late after all previous auctions are gone. However, sniping refers not to "late" bidding, but attempts to bid almost exactly on the last second. In fact, attempting to bid at the last second is a common practice on all kinds of items. There are even various specialized softwares online that place last minute bids for you.

The standard auction theory with private value (PV) has little to say about last minute bidding. Since auction houses like ebay have proxy bidding systems, buyers could set a maximum bid and let the ebay robot bid for them when overbid by others. Hence, setting a maximum bid of one's valuation as soon as he or she sees the auction would be a dominant strategy.[2] Nevertheless, last minute bidding is commonly observed in all categories of items, common value and private value alike.

Moreover, most bidders rationalize last minute bidding saying it can "avoid triggering a price war between each other." On the contrary, theory tells us that the timing of the maximum bids does not matter as long as everyone bids their valuation, and hence, such common sense rationale should not hold. Interesting enough, the common sense rationale assumes the possibility of a price war if bidders bid early, hinting that bidders would "revise" their valuation and bid aggressively if bidding early, and would not do so if they bid on the last minute. Is this simply a misperception, or is there something

[1] See Roth and Ockenfels (2002) for a full description of this phenomenon.

[2] This is why Bajari and Hortacsu (2000) think that last minute bidding can only be explained by common value models.

that is not captured in the standard model?

On the other hand, it would not be surprising to see similar or identical items sold *repeatedly* for auction houses like ebay where thousands of items are auctioned off daily. If there are two identical auctions conducted in a row, bidders would not want to bid up to their true valuation in the first auction if they expects to win the second one at a lower price. Hence, their maximum willingness to bid for the current period would not equal their true valuation. In fact, it would be the (conditional) expectation of the highest loser's valuation, which is correlated with other bidders' valuation. This creates a "common value component" in this private value setting, and brings back all of the information issues we might face in a common value environment.

The idea of similar auctions conducted simultaneously or repeatedly is not new. Milgrom and Weber (1982b) consider three types of simultaneous auctions and four repeated ones.[3] Under the affiliated private value(APV) setting, they proved that the winning price path of repeated first price auctions with price revelation follow a martingale.[4] Weber (1983) characterized the equilibria of various repeated auctions under the independent private value (IPV) assumption and proved the revenue equivalence theorem behind these repeated auctions. Ashenfelter (1989) describes repeated wine auctions which produced decreasing prices within seconds apart. McAfee and Vincent (1993) build on Ashenfelter's idea and show how increasing risk aversion can create this so called, "decreasing price anomaly." However, most literature on empirical auctions abstract away repeated auctions and stick to the single auction model. Although repeatedness may not be important in traditional auctions, it should play an important role in internet auctions since an easy search tool could locate hundreds or even thousands of similar auction conducted repeatedly.

In this paper we follow the repeated auctions literature and construct a repeated auc-

[3]A set of "repeated auctions" is a sequence of identical single-unit auctions conducted repeatedly, and is what Milgrom and Weber called "sequential auctions." Since the term "sequential auctions" might refer to other types of auction designs, we use the term "repeated auctions" instead.

[4]Unfortunately, their conjectures of other sequential auctions could not be proved. See their own discussion in their recently published version of Milgrom and Weber(1982b) in *The Economic Theory of Auctions*, edited by P. Klemperer.

tion model to explain last minute bidding in ebay auctions. In repeated ebay auctions with affiliated private value(APV), bidders only bid at the last minute of each auction and shade their bids in the first auction. However, since we have a private value setting, we prove that expected revenue is the same for each auction, which equals to the expected valuation of the highest bidder not winning an item. Thus, last minute bidding does not hurt the seller, and the ending rules of ebay does induce efficiency.

Roth and Ockenfels (2000) propose a different explanation for last minute bidding. They model the ending period of ebay auctions as second price auctions with a positive probability p that a last minute bid is not successfully transmitted. With the probability p as common knowledge, there exists an equilibrium where all bidders jump in at the last minute bidding their valuations and hoping that only their own bids are successfully transmitted and all other bids are lost. Since there is a positive probability of losing some bids, sellers' expected revenue, and hence, ebay's service charges, are lowered. Moreover, overall efficiency is not always obtained since there is a positive probability that the highest bid is not transmitted. In this case, the policy implication would be to change the ending rule by allowing a "waiting period" to avoid last minute bids from getting lost.[5]

We use empirical data and laboratory experiments to test between different explanation of last minute bidding. The empirical data supports implications of the sequential model, while lab experiments conducted by Ariely, Ockenfels, and Roth (2002) actually rejects the non-transmission model. Hence, it seems that the sequential model explains last minute bidding better, and according to it, last minute bidding hurts neither efficiency nor revenue.

The rest of the paper is consisted as following. Section 2 states the finitely repeated auction model for internet auctions, and explains last minute bidding. Section 3 describes the empirical data, section 4 shows the results of the laboratory experiments,

[5]Note that this "collusive bidding" equilibrium supported by the non-transmission of the last minute bid might not be the unique equilibrium and would exist only for a certain range of p. Also, it would be odd for the auction house to have no incentive to reduce p, the probability of non-transmission, since it lowers its expected revenue. One possible explanation is the auction house wants to keep the bidders from leaving the auction house. See Ariely, Ockenfels, and Roth (2002) for some reasons they suggest.

and section 5 concludes.

2 The Repeated ebay Auction Model

2.1 Basic Settings

Assumption 1. *There are N bidders, M sellers where $N > M$. There is no discounting and all players are risk neutral.*

Assumption 2. *Each seller auctions off one item. The sellers value the items at $v_0 = 0$.*

Assumption 3. *Bidders only want one item and have valuations v_i, which are called their types. Types v_i are private and drawn from the symmetric*[6] *joint distribution function $F(V) = F(v_1, v_2, \cdots, v_N)$ and pdf $f(V) = f(v_1, v_2, \cdots, v_N)$, with support $[\underline{v}, \overline{v}]$. Types are* affiliated.[7] *The order statistics of $\{v_1, v_2, \cdots, v_N\}$ are $\{v_{(1)}, v_{(2)}, \cdots, v_{(N)}\}$ where*

$$v_{(1)} \geq v_{(2)} \geq \cdots \geq v_{(N)}.$$

Assumption 4. *The same single-unit auction is conducted* **repeatedly** *according to the auction rules described below. In other words, the entire auction game consists of M stage, and each stage is a single-unit auction.*

To simplify things, we focus on symmetric perfect Bayesian equilibrium. Also, we do not consider (weakly) dominated strategies, and assume that there is no waiting cost. To sum up, we (implicitly) have the following assumption:

Assumption 0. *We consider only symmetric perfect Bayesian equilibrium in monotonic, undominated strategies, and the bidders are always available. i.e. There is no*

[6] i.e. for all $1 \leq i, j \leq N$, $F(v_1, \cdots, v_i, \cdots, v_j, \cdots, v_N) = F(v_1, \cdots, v_j, \cdots, v_i, \cdots, v_N)$.

[7] See Milgrom and Weber (1982a) for the formal definition of affiliation and its properties. In particular, for vectors z and z', let $z \vee z'$ be the component-wise maximum, and $z \wedge z'$ be the component-wise minimum. Then, random variables $\mathbf{X}$ with joint pdf $f(\mathbf{X})$ is **affiliated** if, for all z and z',

$$f(z \vee z') f(z \wedge z') \geq f(z) f(z').$$

waiting cost.[8]

One might ask why we consider repeated or sequential auctions since in practice, ebay auctions do *overlap.* However, overlapping auctions are almost the same as repeated auctions except each that cross-bidding is allowed.

Assuming that the bidders are always available, bidding on the next auction before the previous one has ended is weakly dominated, for you can always wait and bid afterwards. Hence, we can delete these weakly dominated strategies and model internet auctions as repeated auctions without overlap.

2.2 The Continuous-time English Auction

In **standard English auctions**, prices ascend while bidders press on a button indicating their participation. Bidders decide what price to drop out and let go of the button at that price. Drop-outs are observed immediately, and the auction ends when all but one bidder has dropped out. The remaining bidder wins the item and pays the **current price**, which equals to the price when its last rival dropped out. In this setting, dropping out at one's valuation is a weakly dominant strategy.

The ebay auctions are designed to resemble repeated continuous-time English auctions with a fixed ending time, and differ from standard English auctions in several important ways. First of all, they are repeatedly conducted, as stated above. Moreover, we only observe bids in the open ascending part of the auction instead of "drop-outs."[9]

The third aspect ebay auctions differ from standard English auctions is that ebay has a *proxy bidding system* where you can submit your reservation price and let the

[8]This might not be true in internet auctions, but is always assumed in the auction literature. One way to approximate this is to have an agent or even a robot bid for you. Besides, the critical thing needed for our results is that the waiting cost for being available *exactly* at the last minute is lower than the gains from "last minute revisions."

[9]This information issue matters in a repeated setting since we learn bidders' valuations by observing the price they drop out. In particular, Milgrom and Weber (1982b) showed that a M stage repeated standard English auction where drop-outs are observed would yield exactly the same prices in each single-unit standard English auction since we observe the valuation of the $(M+1)$th highest bidder when that bidder drops out. However, such an equilibrium does not exist if "drop-outs" are not observed.

proxy system bid for you. The proxy bidding system will only bid up to the amount which is necessary. In particular, it would only overbid the standing bid (or the hidden reservation price) by at most the minimum increment s. When your bid is the standing bid and another bidder overbids you, the proxy bidding system will revise your bid to counter it unless the other bidder's bid exceeds your reservation price.

The last but most important aspect ebay auctions differ from stand English auctions is that it has a *fixed ending rule.* Therefore, the proxy bidding system at the fixed "last minute" works more like a *second price* auction. Similar to Roth an Ockenfels (2000)[10], we specifically model the ebay auction as a sequence of repeated continuous-time English auctions ending with a sealed-bid second price auction.

To see the impact of each aspect of discrepancy, we add these feature one at a time, starting from the continuous time setting.

We define the **Continuous-time English Auction** as the modified version of the standard English auction with the following rules:

Definition. (Continuous-time English Auction)

1. *Starting Period: Bidding starts from the reserve price* $r = v_0$ *at time* $t = 0$.

2. *Submitting Bids: Bidders can bid any time* $t \in [0, T]$. *If no bid is submitted in* $[0, T]$, *the auction ends without a sale. Bidder* i*'s bid at time* t *is recorded as* $b(i, t)$. *This bidding record is available immediately after time* t. *Hence, the bidding record at time* $\tilde{t}$ *is* $h_{\tilde{t}} = \left\{b(i, t)\right\}_{t \in [0, \tilde{t})}$

3. *Bidding Sequence: Bids are ordered according to its submission time* t. *If two or more bids enter at the same instance* t, *they are randomly ordered. i.e. the bidding sequence is* $b_1 = b(i_1, t_1)$, $b_2 = b(i_2, t_2)$, $\cdots$, $b_n = b(i_k, t_k)$, $\cdots$, *where* $t_k \leq t_{k+1}$.

4. *Ascending Bids: Latter bids have to be higher than their predecessors by a minimum increment* s. *In other words, if the bidding sequence is* b_1, b_2, $\cdots$, b_k, $\cdots$

[10]However, they do not consider repeated auctions, but assume a exogenous probability p of non-transmission for the last-minute bids instead.

, then $b_{k+1} \geq b_k + s$ for all $k \in \mathbf{N}$.[11] *We denote the last and highest bid as the* **standing bid** *or* **current price***.*

5. *Ending Rule: The auction ends if no further bids came in during the "going-going-gone" period $\left(t_n, t_n + \overline{T}\right]$ after the last bid was submitted at t_n. The high bidder wins the item and pays the standing bid S_{t_n}. This is called the* **going-going-gone ending rule***.*[12] *If $t_n + \overline{T} > T$, then the auction is* **automatically extended** *to time $T' = (t_n + \overline{T})$.*[13] *The bidding history of this ended auction is the finite*[14] *bidding sequence $h = \left\{b_k(i_k, t_k)\right\}_{k=1}^{n}$ where $b_k(\cdot)$ is bidder i_k's bid at time t_k.*

When there is only one auction conducted, i.e. $M = 1$, we may characterize the equilibrium of the Continuous-time English auction by the following lemma:

Lemma 1. *Assume that Assumption 0-4 hold and $M = 1$. Under the Continuous-time English Auction rules, for any $s < v_{(1)} - v_{(2)}$, an equilibrium is characterized by a finite bidding sequence $\{b_1, \cdots, b_n\}$ with the bidder with valuation $v_{(1)}$ bidding the last bid $b_n \in \left((v_{(2)} - s), (v_{(2)} + s)\right]$, and all other bidders cease to bid during the* going-going-gone *period $\left(t_n, t_n + \overline{T}\right)$. Hence, the outcome is efficient, and as $s \to 0$, the winning price converge to the winning price in the standard English auction.*

Proof. First note that for bidder i, bidding $b_k > v_i$ is strictly dominated by bidding $b_k = v_i$ since the former adds a positive probability of winning and getting a negative payoff.

Furthermore, subgame perfection requires that after bidder i bid b_k at time t_k, bidder $j \neq i$ with valuation v_j has to submit a **counter bid** $b_{k+1} \geq b_k + s$ at time $(t_k + \overline{T})$, as long as $b_k + s \leq v_j$, if there were no other bids submitted during t_k and $(t_k + \overline{T})$, or, equivalently, bidding history satisfies $h_{t_k} = h_{t_k + \overline{T}}$. Hence, as the standing bid

[11] If two bids are submitted at the same instance t, the (randomly selected) latter bid would be rejected and not recorded if it does not satisfy this minimum increment requirement. However, this is fine since the bidder can always resubmit another bid at $t' > t$. Only if there is a fixed last minute to submit bids would this be an issue.

[12] This corresponds to an auctioneer soliciting bids several times before announcing that the item is sold.

[13] Actually, automatic extension is not necessary if T is large enough, just as in practice the auctioneer usually has enough time to solicit for new bids. What is crucial is the "going-going-gone" ending rule.

[14] Since we have a minimum increment s, the sequence is finite.

increases, lesser bidders would be required to submit a counter bid since the condition to counter bid, $b_n + s \leq v_j$, is more likely to fail. Regardless of the bidding path, as the standing bid increases, counter bids would cease to exist when the only bidder that might satisfy the condition $b_n + s \leq v_i$ is the the one who submitted the last bid b_n itself. In that case, no one will submit a counter bid during the going-going-gone period $\left(t_n, t_n + \overline{T}\right]$, and the auction ends.

Since there is a minimum increment s, and the bidding sequence $\{b_k\}$ is strictly increasing, there exist a finite $\overline{n}$ such that $b_{\overline{n}} > v_{(2)} - s$, submitted by either the highest bidder (with valuation $v_{(1)}$) or the second highest bidder (with valuation $v_{(1)}$). If it was the highest bidder submitting $b_{\overline{n}}$, the second highest bidder would not counter bid since $b_{\overline{n}} + s > v_{(2)}$.

However, as long as $s < v_{(1)} - v_{(2)}$, the highest bidder can always counter any possible bid b_k of the second highest bidder, which is at most $v_{(2)}$. Hence or otherwise, the bidding sequence is finite, and will end when the highest bidder bids b_n in $\left((v_{(2)} - s), (v_{(2)} + s)\right]$.

This yields an efficient outcome, and as $s \to 0$, the winning price b_n converges to $v_{(2)}$, the winning price in the standard English auction. □

Note that when $M \geq 2$, the equivalence result between the standard English auction and the continuous-time English auction may not hold. Although there are equilibria of the repeated continuous-time English auction that correspond to the repeated standard English auction, there are possibly other equilibria that yield different outcomes. This is because the continuous-time English auction does *not* require all other bidders to counter bid, which is equivalent to not dropping out in the standard English auction, but just at least *one* counter bid. Hence, two bidders might engage in a bidding war against each other which is unnecessary since one of them could stop counter bidding, wait, and bid on subsequent auctions. However, this creates incentives for high valuation bidders to jump bid, eliminating the chance for multiple opponents to counter bids revealing their relatively high valuation, and hence, deter other bidders from counter bidding.[15]

[15]Such jump-bid equilibria rationalizes the imposed game structures in the jump-bid literature. However, further discussion is beyond the scope of this paper.

2.3 The Proxy Bidding System

Now we add the proxy bidding system, which was designed as a bidding tool, but play an important role in ebay auctions:

Definition. (Continuous-time English Auction with Proxy Bidding)

1. *Starting Period: Bidding starts from the reserve price* $r = v_0$ *at time* $t = 0$.

2. *Submitting Proxy Bids: Bidders can submit a "proxy bid" any time* $t \in [0, T]$. *If no bid is submitted in* $[0, T]$, *the auction ends without a sale. Bidder* i*'s* **proxy bid** *at time* t *is recorded as* $b(i, t)$. *However, this proxy bid is not revealed immediately. The (hidden) bidding record of proxy bids at time* $\tilde{t}$ *is* $h_{\tilde{t}} = \left\{ b(i, t) \right\}_{t \in [0, \tilde{t})}$

3. *High Bidder and its Standing Bid: For any time* $t \in [0, T]$, *the* **high bidder** *is the bidder who submitted the highest proxy bid*

$$\max_{\tilde{t} \in [0,t)} b(i, \tilde{t}).$$

The **standing bid** *or "current price"* S_t *at any time* $t \in [0, T]$ *is, if there are two or more bidders who submitted their proxy bid(s),*

$$\min \left\{ (b_{k'} + s), b_{k_t} \right\} \qquad \textit{where} \qquad b_{k'} = \max_{j \neq i, \tilde{t} \in [0,t)} b(\tilde{t}),$$

the highest proxy bid that is submitted by a different bidder, or, if there is only one active bidder, the reserve price r. *If two bidders submitted the same highest proxy bid, the bidder who submitted their proxy bid(s) earlier becomes the high bidder, and the standing bid equals to the tied highest proxy bid. The standing bid* S_t *is common knowledge immediately after time* t.[16]

4. *Ascending Proxy Bids: Latter proxy bids have to be higher than the bidder's last submitted proxy bid*[17] *and higher than the standing bid by a minimum increment*

[16] Note that once a proxy bid is overbid, it is automatically revealed through change in the standing bid. Hence, most proxy bids are revealed with a delay, except the winning proxy bid, which is never revealed—not even after the end of the auction.

[17] In other words, a bidder cannot lower its still hidden proxy bid. The only way to lower an existing proxy bid, is to retract it and submit a new bid. However, bid retractions are viewed as bad signals about a bidder and are very rare.

s. *In other words, if* $b(i, \hat{t})$ *is the last proxy bid submitted by bidder* i*, then its new proxy bid* $b(i, t)$ *must satisfy*

$$b(i,t) > b(i,\hat{t}), \qquad \textit{and} \qquad b(i,t) > S_t + s.$$

5. *Ending Rule: The auction ends if no further bids came in during the "going-going-gone" period* $\left(t, t+\overline{T}\right]$ *after a last bid was submitted at* t*. The high bidder wins the item and pays the standing bid* S_t*. This is called the* **going-going-gone ending rule***, or the* **Amazon rule***. If* $t+\overline{T} > T$*, then the auction is* **automatically extended** *to time* $(t + \overline{T})$*.*

6. *Bidding History: The bidding history of an ended auction is*

$$h_{\tilde{T}} = \left\{ b(i,t) \right\}_{t\in[0,\tilde{T})} \setminus \left\{ \max_{t\in[0,\tilde{T})} b(i,t) \right\}$$

   where $\tilde{T}$ *is the extended ending period.*[18]

There are two effects of the proxy bidding system. On the one hand, it eliminates the possibility of jump-bidding, and hence, eliminates the possible jump-bid equilibria.[19] On the other hand, the proxy bidding system significantly simplifies the bidding process since it automatically makes counter bids for you immediately when you are overbid by another bidder. In fact, for $M = 1$, we can characterize the unique equilibrium with the following proposition:

Proposition 1. *Assume that Assumption 0-4 hold and* $M = 1$*. Under the rules of the Continuous-time English Auction with proxy bidding, the equilibrium is characterized by bidders submitting proxy bids* $b(i, 0) = v_i$ *at the beginning of the auction, and cease to bid after the initial bids are submitted. Hence, the unique equilibrium of the continuous-time English auction is the same as the equilibrium in the standard English auction and is efficient.*

Proof. It suffice to prove that for bidder i, submitting a proxy bid of $b(i, t) = v_i$ at $t = 0$ is a strictly dominant strategy.

[18] The winning proxy bid is not revealed.

[19] However, this is beyond the scope of this paper.

Suppose bidder i's strategy leads to the submission of several proxy bids, $B(1, t_1) < B(2, t_2) < \cdots < B(n, t_n)$, and cease to bid after t_n. The final proxy bid B_n must be in $(v_i - s, v_i]$. Otherwise, adding a proxy bid $B(n+1, t_{n+1}) = v_i$ at time $t_{n+1} > t_n$ would strictly dominate the proposed strategy. Due to the automatic extension rule, there exists t_{n+1} between t_n and the ending period of the auction.

Due to the proxy bidding system, submitting a proxy bid $b(i, 0) = v_i$ at time $t = 0$ would yield the same outcome if no bidder j has valuation $v_j \in (v_i - s, v_i)$, or if $B(n, t_n) = v_i$. However, by submitting a proxy bid equal to one's reservation price one strictly gains if there exists a bidder j who has valuation $v_j \in (v_i - s, v_i)$, and hence, submits a proxy bid $b_j = v_j$.

Since we assume continuous types with full support, there is a positive probability of having such an opponent. Therefore, submitting a proxy bid of $b_i = v_i$ at time $t = 0$ strictly dominates the proposed strategy.

Considering submitting the proxy bid of $b(i, t') = v_i$ at another time $t' \neq 0$, we should note that the rule of submission requires that the proxy bid to exceed the standing bid by at least a minimum increment s. However, there is a positive probability that there exists a bidder j who has valuation $v_j \in (v_i - s, v_i)$. If this bidder submits a proxy bid $b(j, 0) = v_j$ earlier than bidder i, bidder i would lose its chance to submit a bid and win the item. Thus, due to the tie breaking rule, bidders race to bid early, and end up all bidding at time $t = 0$.[20] □

Note that in this equilibrium, all bidders submit a proxy bid equal to their valuations at the time 0 and the automatic extension rule has no bite.

2.4 The Fixed Ending Rule

Finally, we add the well-known fixed ending rule and complete our description of the ebay auctions.

Assumption 5. (ebay Auction) *The ebay auction is a continuous-time English auction with proxy bidding but has a fixed ending rule, defined by:*

[20] In practice, this corresponds to bidders submitting their bid when they first see the auction.

1. *Starting Period: Bidding starts from the reserve price* $r = v_0$ *at time* $t = 0$.

2. *Submitting Proxy Bids: Bidders can submit a "proxy bid" any time* $t \in [0, T]$. *If no bid is submitted in* $[0, T]$*, the auction ends without a sale. Bidder* i*'s* **proxy bid** *at time* t *is recorded as* $b(i, t)$. *However, this proxy bid is not revealed immediately. The (hidden) bidding record of proxy bids at time* $\tilde{t}$ *is* $h_{\tilde{t}} = \left\{b(i, t)\right\}_{t \in [0, \tilde{t})}$

3. *High Bidder and its Standing Bid: For any time* $t \in [0, T]$*, the* **high bidder** *is the bidder who submitted the highest proxy bid*

$$\max_{\tilde{t} \in [0,t)} b(i, \tilde{t}).$$

The **standing bid** *or "current price"* S_t *at any time* $t \in [0, T]$ *is, if there are two or more bidders who submitted their proxy bid(s),*

$$\min\left\{(b_{k'} + s), b_{k_t}\right\} \qquad \textit{where} \qquad b_{k'} = \max_{j \neq i, \tilde{t} \in [0,t)} b(\tilde{t}),$$

the highest proxy bid that is submitted by a different bidder, or, if there is only one active bidder, the reserve price r. *If two bidders submitted the same highest proxy bid, the bidder who submitted their proxy bid(s) earlier becomes the high bidder, and the standing bid equals to the tied highest proxy bid. The standing bid* S_t *is common knowledge immediately after time* t.

4. *Ascending Proxy Bids: Latter proxy bids have to be higher than the bidder's last submitted proxy bid and higher than the standing bid by a minimum increment* s. *In other words, if* $b(i, \hat{t})$ *is the last proxy bid submitted by bidder* i*, then its new proxy bid* $b(i, t)$ *must satisfy*

$$b(i, t) > b(i, \hat{t}), \qquad \textit{and} \qquad b(i, t) > S_t + s.$$

5. *Ending Rule: The auction ends at time* T. *The high bidder wins the item pays the standing bid* S_T. *This is called the* **Fixed Ending Rule***.[21]

6. *Bidding History: The bidding history of an ended auction is*

$$h_T = \left\{b(i, t)\right\}_{t \in [0,T]} \setminus \left\{\max_{t \in [0,T]} b(i, t)\right\}$$

[21] Note that due to the proxy bidding system, bidders may submit the reservation price at time $t = T$. This makes the last minute T as if a sealed bid second price auction.

However, bids submitted at the same time t are randomly ordered and later bids might be dropped. Such cases are important for time T since there is no time to re-submit another bid. For simplicity, we assume that the last minute bids submitted at time T are ordered in the way that they are all revealed.[22]

If $M = 1$, the unique equilibrium is still the same as the Continuous-time English Auction since the ending rule has no bite. In other words, we have the following proposition:

Proposition 2. *Assume that Assumption 0-4 hold and $M = 1$. Under the rules of the ebay auction, the unique equilibrium is characterized by bidders submitting proxy bids $b(i, 0) = v_i$ at the beginning of the auction, and cease to bid after the initial bids are submitted. Hence, the unique equilibrium of the (single-unit) ebay auction is the same as the equilibrium in the standard English auction and is efficient.*

The proof is identical to that in the previous section.

Note that when there is only one auction ($M = 1$), all bidders submit proxy bids equal to their valuation *at time* 0, not at the last minute (time T) observed in practice. This is why "sniping," or last minute bidding, is not explained by (and even contradicts) standard auction literature.

To explain last minute bidding, Roth and Ockenfels (2000) add an exogenous probability p that the last minute bid submitted at time T is not successfully transmitted due to network congestion or disconnection. They construct an equilibrium in which bidder only bid at the last minute hoping for the small probability of gaining a lot when others'

[22]Note that information revelation is actually determined by the the order of the last minute bids. Later bids which are lower then the standing bid are rejected by the ascending bid rule, and hence, are not revealed. If we assume the order of the last minute bids are random, the revelation is also random. Alternatively, we might think of endogenously determining the order. If last minute bids are ordered from the smallest to the largest, all last minute bids (except the highest one) are revealed. If last minute bids are ordered from the largest to the smallest, then only the second highest last minute bid, which determines the final standing bid, is revealed. Other cases are in between.
Information revelation would influence the results. Nevertheless, when $M = 2$, all information structures result in the same equilibrium.

bids are lost. Such an equilibrium is not unique, as the early bidding equilibrium shown here still exists, and relies heavily on appropriate p's.[23]

We take an alternative approach, and add repetition to the model.

2.5 Repeated ebay Auctions and their Bidding History

The rules of repeated ebay auctions are identical to that of the single-unit auction, except that we repeat the same single-unit auction M times, each for one item. We denote each single-unit auction as a "stage."

However, the history of repeated auction becomes a little more complicated. We may define the bid history of the repeated ebay auction as:

Assumption 6. (Information Structure for ebay Auctions)

a. Revelation of Last minute bids:

We assume that last minute bids are ordered from the smallest to the largest. Hence, no last minute bid would be rejected due to the ascending rule.[24]

b. Bidding history of Ended Auctions:

At any stage m, the bidding history of the ended auction at stage $m' < m$ is

$$\tilde{h}_{m',T} = \left\{ b_{m'}(i,t) \right\}_{t\in[0,T]} \setminus \left\{ \max_{t\in[0,T]} b_{m'}(i,t) \right\}$$

c. Interim Information for the current stage:

At stage m, time t, all bids, except the highest proxy bid, submitted between 0 and t are known from the change in the standing bid S_t. Hence, the bidding record of stage m, time $t \in [0,T]$ is denoted by

$$\tilde{h}_{m,t} = \left\{ b_m(i,\tilde{t}) \right\}_{\tilde{t}\in[0,t)} \setminus \left\{ \max_{\tilde{t}\in[0,t)} b_m(i,\tilde{t}) \right\}$$

where $b_m(i,\tilde{t})$ is bidder i's proxy bid at time $\tilde{t}$ (of stage m).

d. Bidding History: The entire bidding history up to stage m, time t is defined by

$$h_{m,t} = \left\{ \tilde{h}_{1,T}, \cdots, \tilde{h}_{m-1,T}, \tilde{h}_{m,t} \right\}$$

[23] See Roth and Ockenfels (2000) for the bounds of p required to sustain their equilibrium.

[24] If smaller bids are put behind larger ones, they would be rejected by the "ascending bid rule" and not recorded in the bid history.

As for notation, we define $\beta_m(v_i, h_{m,t})$ as the bid function for bidder i, according to their valuation v_i and the bidding history for stage m, time t is $h_{m,t}$. Since bidders can submit multiple bids in each stage, the (symmetric) bidding function is a sequence of bids, $\beta_m(v_i, h_{m,t}) = \Big\{\beta_{m,t}(v_i, h_{m,t})\Big\}_{t\in[0,T]}$ where $h_{m,t}$ refers to the history up to time t of stage m.

Note that the ebay auction has the sealed-bid second price auction as a special cases when $T = 0$. We first consider this special cases as a benchmark.

2.6 The Repeated Sealed-bid Second Price Auctions (T=0)

When $T = 0$, the ebay auction reduces to a sequence of sealed-bid second price auctions. The equilibrium is as follows:

Theorem 1. (APV, Any Information Structure, 2 Stages)
Assume that $M = 2$, Assumption 1–6 hold, and $T = 0$. Then, no matter how many last minute bids are revealed after the auctions (arbitrary on Assumption 6a), a symmetric equilibrium of the repeated sealed-bid second price auction is determined by bidding $\beta_1(v_i) = E\Big[v_{(3)}\Big|v_{(1)} > v_i > v_{(3)}\Big]$ in stage 1, and $\beta_2(v_i) = v_i$ in stage 2. Moreover, Expected revenue is the same in both stages, which equals to $E\Big[v_{(3)}\Big]$.

Proof. In the last auction, or stage M, we have the standard case in which the remaining bidders bid (up to) their valuation $\beta_M = v_i$ and the item is sold at price $v_{(M+1)}$ since there are $(N - M + 1)$ bidders left.

This can be shown by solving the maximization problem of bidder (of type) i choosing her bid b_i, given other bidders' (identical) strategies b^*_{-i}. Let $b_{(2)}$ be the second highest bid, and let the history be $h_m = h_{m,0}$, then the maximization problem is

$$\max_{b_i} u(b_i, b_{-i}) = Pr(b_i > b_{(2)}) \cdot v_i - E\Big[b_{(2)}\Big|b_i > b_{(2)}, h_M\Big]$$

Suppose the (monotonic) equilibrium bid function is $\beta^*_M(v)$, we may transform the maximization problem from choosing bid b_i into choosing to "pretend to be type x" in which $\beta^*_M(x) = b_i$. Given other bidders' strategy $\beta^*_M(v_{-i})$, the maximization problem becomes:

$$\max_{x} u(x, v_{-i}) \quad = \quad Pr(x > v_{(2)}) \cdot v_i - E\Big[\beta^*_M(v_{(2)})\Big|x > v_{(2)}, h_M\Big]$$

$$= \quad F_{-i}(x|v_i, h_M) \cdot v_i - \int_{\underline{v}}^{x} \beta_M^*(\overline{v}_{-i}) f_{-i}(\overline{v}_{-i}|v_i, h_M) d\overline{v}_{-i}$$

where $\overline{v}_{-i}$ is the highest type among the bidders other than bidder i, $F_{-i}(\overline{v}_{-i}|v_i, h_M)$ is its conditional distribution function given v_i, h_M, and $f_{-i}(\overline{v}_{-i}|v_i)$ is its conditional pdf given v_i, h_M. Then, the first order condition is

$$\frac{\partial u}{\partial x} = f_{-i}(x|v_i, h_M) \cdot v_i - \beta_M^*(x) f_{-i}(x|v_i, h_M) = 0$$

Therefore, $\beta_M^*(x) = v_i$ maximizes bidder i's expected payoff.

Knowing this, in the next-to-last stage, or stage $(M-1)$, the remaining $(N-M+2)$ bidders can calculate the expected winning price of the last stage, and that would be the maximum amount bidder i is willing to pay at this stage. If the expected winning price in this stage is more than that of the next stage, they would rather wait and bid in the next stage. This can be shown by comparing the payoff of bidder i at stage $(M-1)$ bidding $\beta_{M-1}(v_i)$ and $\beta_M(v_i) = v_i$ (bid now according to the proposed strategy) with bidding $\beta_{M-1} = 0$ and $\beta_M(v_i) = v_i$ (wait and bid later in stage M), when all other bidders $j \neq i$ follow the proposed strategy bidding $\beta_{M-1}(v_i)$ and $\beta_M(v_i) = v_i$.

Let $y_{(1)} > y_{(2)} > \cdots > y_{(N-1)}$ be the order statistics of the valuations of the $(N-1)$ bidders $j \neq i$, then

$$\begin{aligned}
U(\text{bid now}) &= Pr(v_i > y_{(1)}) \cdot v_i - E_{h_M}\left\{E\Big[\beta_{M-1}(y_{(1)})\Big|v_i > y_{(1)}, h_M\Big]\right\} \\
&\quad + Pr(y_{(1)} > v_i > y_{(2)}) \cdot v_i - E_{h_M}\left\{E\Big[\beta_M(y_{(2)})\Big|y_{(1)} > v_i > y_{(2)}, h_M\Big]\right\} \\
&= Pr(v_i > y_{(2)}) \cdot v_i - E\Big[\beta_{M-1}(y_{(1)})\Big|v_i > y_{(1)}, h_{M-1}\Big] \\
&\quad - E\Big[y_{(2)}\Big|y_{(1)} > v_i > y_{(2)}, h_{M-1}\Big], \\
U(\text{bid later}) &= Pr(v_i > y_{(2)}) \cdot v_i - E_{h_M}\left\{E\Big[\beta_M(y_{(2)})\Big|v_i > y_{(2)}, h_M\Big]\right\} \\
&= Pr(v_i > y_{(2)}) \cdot v_i - E\Big[y_{(2)}\Big|v_i > y_{(2)}, h_{M-1}\Big],
\end{aligned}$$

and the equilibrium requires

$$\begin{aligned}
0 &\leq U(\text{bid now}) - U(\text{bid later}) \\
&= E\Big[y_{(2)}\Big|v_i > y_{(1)}, h_{M-1}\Big] - E\Big[\beta_{M-1}(y_{(1)})\Big|v_i > y_{(1)}, h_{M-1}\Big].
\end{aligned}$$

Following the same method as above, instead of choosing to bid b_i, we let bidder i choose to "pretend to be" type x where $\beta_{M-1}(x) = b_i$ according to the monotonic bid function. We maximize bidder i's payoff constrained to the equilibrium requirement shown above:

$$\begin{aligned} \max_x \quad & Pr(x > y_{(1)}) \cdot v_i - E\Big[\beta_{M-1}(y_{(1)})\Big| x > y_{(1)}, h_{M-1}\Big] \\ s.t. \quad & E\Big[\beta_{M-1}(y_{(1)})\Big| x > y_{(1)}, h_{M-1}\Big] \leq E\Big[y_{(2)}\Big| x > y_{(1)}, h_{M-1}\Big] \end{aligned}$$

Suppose the constraint does not bind. Since the unconstrained maximization is the same as that of stage M, the solution is $\beta_{M-1}(v_i) = v_i$. However, plugging in the solution, we find that the constraint is violated since

$$E\Big[y_{(1)}\Big| v_i > y_{(1)}, h_{M-1}\Big] \geq E\Big[y_{(2)}\Big| v_i > y_{(1)}, h_{M-1}\Big]$$

Therefore, we know that the constraint binds, and hence,

$$\beta_{M-1}(y_{(1)}) = E\Big[y_{(2)}\Big| y(1) \text{loses in stage } (M-1)\text{, but wins in stage } M, h_{M-1}\Big]$$

or,

$$\beta_{M-1}(v_i) = E\Big[v_{(3)}\Big| v_{(M-1)} > v_i > v_{(M+1)}, h_{M-1}\Big]$$

Thus, the unique symmetric perfect Bayesian Nash equilibrium for this stage is to bid

$$\beta_{M-1}(v_i) = E\Big[v_{(M+1)}\Big| v_{(M-1)} > v_i > v_{(M+1)}, h_{M-1}\Big].$$

The item is won by the bidder with the highest valuation among the $(N - M + 2)$ bidders, which has valuation $v_{(M-1)}$ and pays the second highest bid (submitted by the bidder with valuation $v_{(M)}$):

$$\beta_{M-1}(v_{(M)}) = E\Big[v_{(M+1)}\Big| v_{(M-1)} > v_{(M)} > v_{(M+1)}, h_{M-1}\Big].$$

Note that the expected revenue of stage $(M-1)$ is

$$\begin{aligned} E\Big\{\beta_{M-1}(v_{(M)})\Big\} &= E_{v_{(M)}}\Big\{E\Big[v_{(M+1)}\Big| v_{(M-1)} > v_{(M)} > v_{(M+1)}, h_{M-1}\Big]\Big\} \\ &= E\Big[v_{(M+1)}\Big| h_{M-1}\Big]. \end{aligned}$$

For $M = 2$, we are done since $h_{M-1} = \emptyset$, and there is revenue equivalence:[25]

$$E\{\beta_1(v_{(2)})\} = E_{v_{(2)}}\Big\{E\Big[v_{(3)}\Big| v_{(1)} > v_{(2)} > v_{(3)}\Big]\Big\} = E\Big[v_{(3)}\Big].$$

[25] First, stage 1 and stage 2 yield the same expected revenue. Furthermore, it equals to the expected revenue of a multi-unit sealed-bid third price auction.

In fact, when $M = 2$, the information structure plays no role since the last auction is the standard case, and hence, everybody bids their valuation. $\square$

For $M \geq 3$ with independent private value(IPV), and partial or no revelation, Weber (1983) has

Lemma 2. (IPV, Partial Revelation, M stages) (Weber 1983)
Assume that Assumption 1–6 hold, v_i are independent, and $T = 0$. Then, if at most the winning price (second highest last minute bid) is revealed after the auctions, a symmetric equilibrium of the IPV repeated sealed-bid second price auction is determined by $b_m(v_i)$ at stage m according to one's type v_i

$$b_m(v_i) = E\left[v_{(M+1)}\middle|v_i = v_{(m+1)}\right]$$

independent of history $h_{m,t}$.

Note that the assumption that few bids are revealed is crucial. If more bids are revealed and the bid function is strictly monotonic, the inverse function exists, and after the first stage, the types of those whose bids are revealed are public information.

This is a problem since bidders could pretend to have $v_i = \underline{v}$ at stage m and gain in the intermediate stages. Hence, there are no equilibrium with strictly monotonic bid functions for $M \geq 3$.[26]

2.7 Equilibrium of the Repeated Ebay Auction

Now we consider the general case where T is positive and $M = 2$, in which Assumption 1 through 6 gives us repeated ebay auctions. First, there exists a last minute bidding equilibrium where everyone jumps in and bids at the last minute.

[26] Note that this was not possible when $M = 2$ since there is no "intermediate" stage to gain from. To solve this problem, a possible alternation would be to have no information revealed. If the auctioneer does not reveal any information except that "A item is sold," Milgrom and Weber (1982b) suggest that $b_m(v_i) = E\left[v_{(M+1)}\middle|v_i\right]$ forms a symmetric equilibrium strategy of the sequential sealed-bid second price auction. However, as they note later in the "foreword" twenty years later, the proof for the affiliated private value(APV) case did not go through. Besides, (nearly) full information revelation is usually the norm rather than the exception in internet auctions. Amazon reveals all of the bids as soon as they are placed. Ebay reveals the proxy bids immediately after the auction ends.

2.7.1 Existence of Last Minute Bidding Equilibrium

Theorem 2. *For the repeated ebay auctions, under assumption 1–6, there exists a (symmetric)* **last minute bidding equilibrium** *where all bidders wait and jump in at the last minute, $t = T$, bidding the* repeated sealed-bid second price auction equilibrium *characterized in the previous section, $\beta_m(v_i, T) = E\Big[v_{(M+1)}\Big|v_{(M-1)} > v_i > v_{(M+1)}, h_{m-1,0}\Big]$, according the players' ex ante information in the beginning of this stage $h_{m-1,0}$.*[27]

Proof. Suppose all other opponents bid according to the last minute bidding strategy. Then, since nobody else bids, there is no information to update, and hence, playing the *repeated sealed-bid second price auction equilibrium, $\beta_m(v_i) = E\Big[v_{(M+1)}\Big|v_{(M-1)} > v_i > v_{(M+1)}, h_{m-1,0}\Big]$*, according to your ex ante information is your best response at the last minute $t = T$ of each stage m. □

2.7.2 Weak Dominance with Last Minute Bidding

Now we attempt to argue for the uniqueness of this last minute bidding equilibrium. First, I would like to eliminate bidding strategies that are weakly dominated. Recall that focusing on symmetric equilibrium rules out asymmetric equilibria where one bidder bids very high and others bid zero. These equilibria rely on weakly dominated strategies. Also, to model ebay as repeated auctions instead of overlapping auctions, we have eliminated weakly dominated strategies where people bid on later auctions while earlier ones are still available.

At the last minute T of stage m, we define $\beta_{m,T}(v_i, h_{m,T}) = E\Big[v_{(M+1)}\Big|v_{(M-1)} > v_i > v_{(M+1)}, h_{m,T}\Big]$ as the **last minute bidding strategy** such that bidders bid their expected valuation of the $(M+1)$ highest bidder conditioning on winning and on current bidding history.

Then a bidding strategy $\beta_m(v_i, h_m) = \{\beta_{m,t}(v_i, h_{m,t})\}_{t\in[0,T)}$ is weakly dominated by the bidding strategy $\beta_m(v_i, h_m) = \Big\{\beta_{m,t}(v_i, h_{m,t})\Big\}_{t\in[0,T)} \cup \Big\{\beta_{m,T}(v_i, h_{m,T})\Big\}$ where $\beta_{m,T}(v_i)$ is the last minute bidding strategy since it would only increase your

[27] Since there is no information update, the interim information $h_{m-1,T}$ is the same as the ex ante information $h_{m-1,0}$.

probability of winning the last minute second price auction. In other words, we have proved the following lemma:

Lemma 3. *A bidding strategy without a last minute bidding feature is weakly dominated by the bidding strategy which is almost identical except adding a last minute bidding feature.*

2.7.3 Uniqueness of the Last Minute Bidding Equilibrium

Another thing to note is that revealing information cannot revise the expected prices lower in these ebay open auctions.[28]

Lemma 4. *In each stage of the sequential ebay auction, if there is new information revealed after the bidders bid up to* $\underline{v}$*, it would revise the conditional expectation upward or unchanged. i.e.*

$$E\left[v_{(M+1)}\middle|v_{(M-1)} > v_i > v_{(M+1)}, h_{m,t}\right]$$

weakly increase as $h_{m,t}$ *evolves.*

Proof. For the lowest type $v_i = \underline{v}$, there is no information issue since she knows that she can only win if all other bidders are also the lowest type, and hence, $v_{(M+1)} = \underline{v}$ for sure were she to win. Hence, we consider the higher types.

For any $v \in [\underline{v}, \overline{v}]$, if bidders observed that $(M+1)$ different bidders have bid above v, they would immediately realize that $v_{(M+1)} \in [v, \overline{v}]$, instead of $[\underline{v}, \overline{v}]$ since bidding above ones valuation is weakly dominated. Hence, they would update their beliefs about the distribution of types for these $(M+1)$ bidders according to Bayes' rule. Such an update clearly can only (weakly) increase $E\left[v_{(M+1)}\middle|v_{(M-1)} > v_i > v_{(M+1)}, h_{m,t}\right]$.

Since the auction rule (Assumption 5) specifies that later proxy bids must be higher than previous ones, the bidding history of the current stage $\tilde{h}_{m,t} = \left\{b_m(i,\tilde{t})\right\}_{\tilde{t}\in[0,t)} \setminus$

[28] Note that Milgrom and Weber (1982a) provided a similar result for the seller in the general affilated value (AV) settings:

Theorem 16 1. *In the first price auctions, a policy of publicly revealing the seller's information cannot lower, and may raise, the expected price.*

$\Big\{ \max_{\tilde{t}\in[0,t)} b_m(i,\tilde{t})\Big\}$ satisfies $b_m(i,\tilde{t}) > b_m(i,\hat{t})$ as long as $\tilde{t} > \hat{t}$. However, that means that as $\tilde{h}_{m,t}$ evolves, or as another bid $b(i,\tilde{t})$ is placed, for all $v < b(i,\tilde{t})$, one more bidder has bid above v. Thus, the updating can only be upward, as there are $(M+1)$ bidders bidding above v, or unchanged. □

We can now state our uniqueness theorem:

Theorem 3. *The last minute bidding equilibrium is the unique symmetric perfect Bayesian equilibrium in undominated and monotonic strategies for repeated ebay auctions. In other words, people simply play the last minute bidding equilibrium strategy where everybody jumps in at the last minute of each stage, and hence, bid according to their ex ante information, obtaining the repeated sealed-bid second price auction outcome.*

Proof. Suppose there is another symmetric perfect Bayesian equilibrium in undominated strategies where all bidders play $\beta'_m(v_i,h_m)$ containing bidding before the last minute. By weak dominance of last minute bidding, we know that people *must* revise their bids at the last minute, and hence, do update their information according to Bayes' rule, using the last minute history $h_{m,T}$.

For player i, consider deviating to solely using the *last minute bidding strategy:* wait and bid

$$\beta_m\Big(v_i,h_{m,T}\Big) = E\Big[v_{(M+1)}\Big|v_{(M-1)} > v_i > v_{(M+1)},h_{m,T}\Big]$$

only at the last minute according to the then last minute history $\widehat{h_{m,T}}$. By Lemma 3, we know that others' last minute bids are now

$$\widehat{\beta_m} = E\Big[v_{(M+1)}\Big|v_{(M-1)} > v_i > v_{(M+1)},\widehat{h_{m,T}}\Big] \le E\Big[v_{(M+1)}\Big|v_{(M-1)} > v_i > v_{(M+1)},h_{m,T}\Big] = \beta'_m.$$

since player i has "retracted" all of her bids that were in $h_{m,T}$ except the last minute one. Therefore, expected payoffs for player i goes up or stays constant when she deviates to the last minute bidding strategy. However, that means that $\beta'_m(v_i,h_m)$ is a (weakly) dominated strategy, and thus, not an equilibrium strategy since we only consider symmetric perfect Bayesian equilibrium in undominated strategies.

Intuitively, since revealing information would increase the conditional expectation, and hence, increase your opponents' bids, you would rather withhold information until the last minute. □

According to the theorem, last minute bidding can be rationalized even in the private value case, and theoretically, I have shown that with repeated auctions, bidders' reservation prices in the first period are actually correlated across bidders since the conditional expectations of $v_{(M+1)}$, which are their ex ante reservation prices in the early stages, depends on not only one's type but others' types as well.[29]

Therefore, the debate on whether real world auctions are of private values(PV) or common values(CV) would be difficult to resolved empirically when we have repeated auctions. Unfortunately, this is typically the case since similar items are auctioned off over and over across time, and in fact, we need to observe similar auctions performed repeatedly to identify our empirical estimates.

Moreover, though in theory we should find no winner's curse in private value auctions, there might still be winner's curse in repeated private value auctions.[30]

2.8 Revenue Equivalence and Welfare

First of all, the symmetric equilibrium we consider assumes monotonic strategies. Hence, the winners of stage $1, 2, \cdots, M$ have valuations $v_{(1)}, v_{(2)}, v_{(3)}, \cdots, v_{(M)}$, respectively. Therefore, the outcome is efficient.

Moreover, Weber (1983) proved that general revenue equivalence theorem for the IPV case:

Theorem 4. (Revenue Equivalence; IPV; Weber) *Suppose v_i are independent, Assumption 1–4 hold. For any auction rule that assumes that in equilibrium, the M highest types win the auction for sure, and the lowest type $\underline{v}$ expects to get 0. Then, the seller's (total) expected revenue is $M \cdot E\Big[v_{(M+1)}\Big]$.*

In particular, the repeated ebay auction yields the same expected revenue for each stage, and sellers are indifferent between selling in different stages.

Although revenue equivalence generally breaks down in the APV case,[31] with the

[29] A possible way is to test if the revenue equivalence result below holds.

[30] To avoid the winner's curse, we perform all of the expectations stated above conditional on $v_i > v_{(M+1)}$. However, it is not clear if people do take this into account in the real world.

[31] Milgrom and Weber (1982a) ranks different auction rules according to their expected revenue in the gen-

two stage case, we still have revenue equivalence, as shown in Theorem 1. Here, we restate it as

Theorem 5. (Revenue Equivalence; $M = 2$, APV) *Suppose $M = 2$ and Assumption 1–6 hold. Then, the expected revenue of each stage is $E\left[v_{(M+1)}\right]$, and the seller's total expected revenue is $2 \cdot E\left[v_{(M+1)}\right]$.*

Note that in both cases, we have the efficient outcome, and revenue equivalence. Therefore, under the repeated ebay auction model, last minute bidding is not "bad" in the sense of both efficiency and revenue.

3 Empirical Data

There are some testable implication of the repeated auction model. First of all, with repeated auctions, we should see many last minute bidding in early stages, but few in the last stage; without repeatedness, there would be much less last minute bidding. Moreover, off the equilibrium path, bidders revise their stage valuations and bid more aggressively after seeing others bid. This coincides with the "common sense" that last minute bidding helps avoid bidding wars.

What is more important is that, except for the last stage, bidders do *not* bid their private valuation even in the last minute. Instead, they bid the conditional expectation of the next winning price. Hence, we should see very different behavior comparing the last auction and the earlier ones. In particular, we should see that the variance of bids in the last auction is higher than the earlier ones since people are bidding their own private valuation in the last auction, which is scattered out on $[\underline{v}, \overline{v}]$, but merely bidding their expectation of $v_{(M+1)}$ in the earlier ones, which is more concentrated near $E\left[v_{(M+1)}\right]$.

Finally, even if bidder's true valuations are independent, their stage valuations are not. For example, even if the last auctions satisfy an empirical IPV test, the next-to-last auctions might still fail the same IPV test.

eral setting. They argue that (standard) English auctions are widely used due to its higher expected revenue, compared to the sealed-bid first price or second price auctions.

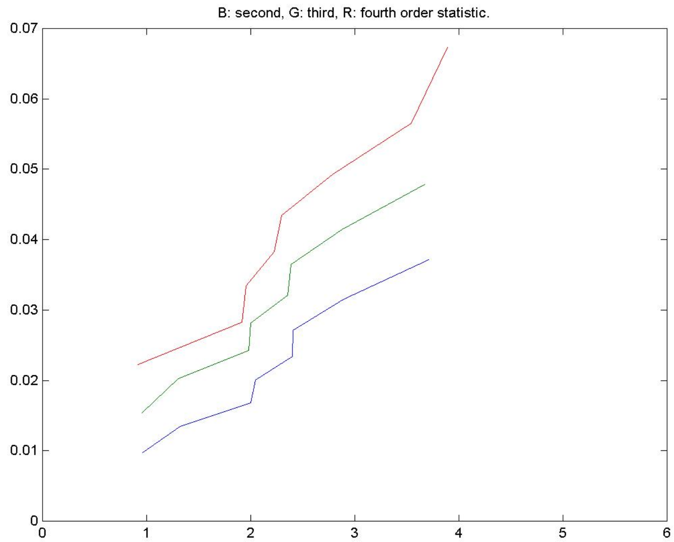

Figure 1: Test of IPV for the Last Auctions: Estimating the Distribution using Different Order Statistics.

To test the last implication, we gather some ebay data to estimate the repeated auction model. One preliminary analysis is done in Wang and Caves (2002) where they use nonparametric methods to analyze the Lakers' ticket auctions. They find somewhat difference between the last and "next-to-last" auction.

Under the IPV assumption, we should be able to estimate the bidding distribution nonparametrically, using any set order statistic. Hence, as Athey and Haile (2001) argue, comparing the distribution estimated using different order statistics can be a test for the IPV assumption.

Since the repeated ebay auction model predicts that even if the IPV assumption holds in the last auctions, it would not hold in the next to last auctions, we test this implication using the IPV test of Athey and Haile (2001). The comparison is drawn from figure one and two below. If the IPV assumption holds, the three colored lines should overlap. It seems that the IPV assumption fails even more in the next-to-last auctions.

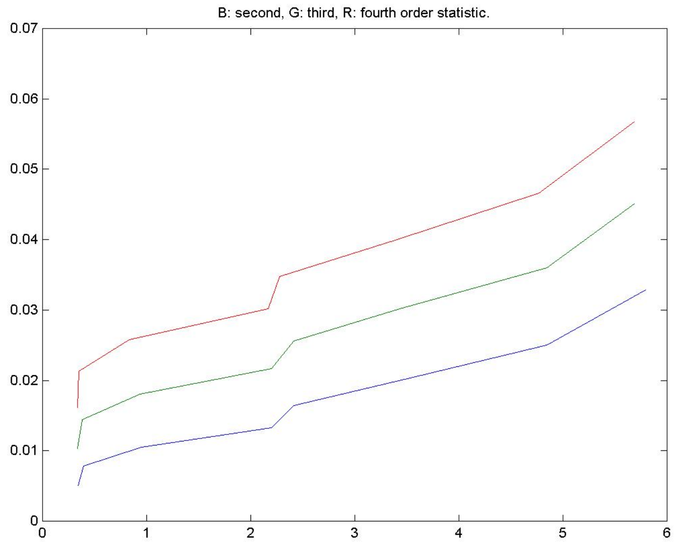

Figure 2: Test of IPV for the Next-to-last Auctions: Estimating the Distribution using Different Order Statistics.

4 Lab Experiments

To test between Roth and Ockenfels' non-transmission model and the repeated auction model with field data might be difficult. This is because to test the validity of the RO model, we should try to set up the rules so that the probability of not transmitting the last bid is zero, so we are back to the ebay auction model (without repetition) where we cannot support last minute bidding as shown in Proposition 2. However, with field data it is hard to verify people's belief about the probability of bid transmission to fail. Furthermore, if the test fails, we cannot verify whether it is because the private value assumption failed, or the none-transmission belief story is false.

An alternative lab experiment with independent private values (IPV) bidders can be conducted by the following. There are more than enough bidders participating in the auction and each have private valuations $\{v_i\}$ with distribution $f(\cdot)$. Consider a game with two stages. One item is auctioned in each stage with the ebay auction rules. However, there is, as Roth and Ockenfels suggests, a probability p that the last minute bid is *not* transmitted.

By perturbing the probability of having the last minute bid transmitted,$1-p$, we may observe the last minute bidding activity, and also, whether or not people bid their valuation in each round. If we still see, in stage 1, people bidding at the last minute, and *not* bidding their valuation v_i's at the last minute, we may conclude that repeatedness, or the existence of a next auction, is indeed the fundamental reason of last minute bidding. Furthermore, as the revenue equivalence result shows, last minute bidding does not hurt expected revenue in the private value case. i.e. Last minute bidding is not *bad* or *collusive*, as Roth and Ockenfels claim.

In fact, Ariely, Ockenfels and Roth (2002) performs a similar experiment containing three auction rules: *Amazon*, *ebay.8*, and *ebay1*, where $.8$ and 1 represents the probability $1-p$. The *ebay.8* and *ebay1* rules are almost identical to the proposed experiment above, except that they both have only one auction, and model the English auction with finite periods of simultaneously bidding.[32] Interesting enough, they observe last minute bidding in both *ebay.8* and *ebay1*, though theoretically the non-transmission model cannot sustain a last minute bidding equilibrium when $p \to 0$. Therefore, their own experiment results are against their non-transmission model.[33]

Also, considering the last 9 rounds, the average efficiency significantly drops from 98% to 88% when $1-p$ drops from 1 to 0.8, while average revenue significantly drops from $\$6.73$ to $\$6.62$. Hence, for the non-transmission model, having $p > 0$ is indeed "bad" for both welfare and revenue.

Given the lab results are already against the non-transmission model, we have little need to run further lab experiments any more. However, we may attempt to run the experiment directly on ebay. We can create an independent private value (IPV) environment by conducting worthless auctions and inviting bidders on ebay to participate by compensating them randomly drawn values.[34]

[32]The *Amazon* rule has the auction start again if anyone bids at the last minute. As expected, there was little, if any, last minute bidding under the *Amazon* rule.

[33]Ariely, Ockenfels and Roth (2002) explains last minute bidding in *ebay1* with the commitment of participants to stay to the end of the experiment. However, this is not satisfactory, as one can explain the *Amazon* result by appealing to the boredom of participants for "starting over again" in the *Amazon* rule.

[34]A similar procedure is done by Garratt, Walker, and Wooders (2002) to conduct a sealed-bid second price auctions with experienced bidders.

As for the probability p, since ebay allows the seller to end an auction early, either accepting or rejecting any of the bids, the seller can explicitly state that he or she would end the auction ten minutes early, leaving enough time for every bidder's last bid to come in. Then, the seller would end the auction early and honor the high bid. If anyone submits more than one bid during the grace period, the seller can legally cancel his or her bid since he is ending the auction anyway, and honor the then high bid.[35]

5 Conclusion

In this paper, we rationalize last minute bidding by adding (at least) another identical auction. In the repeated ebay auction model, we proved that the last minute bidding equilibrium, in which bidders only bid at the last minute, is the unique symmetric perfect Bayesian equilibrium in (weakly) undominated and monotonic strategies. Moreover, bidders do *not* always bid their valuations. However, the auctions are still efficient since high valuation bidders bid higher and win, and sellers are indifferent because of the revenue equivalence result.

Under the repeated ebay auction model, a bidder's maximum willingness to pay for this stage depends on on others' types. This introduces a "common value" component to the private value environment. Hence, it is difficult to distinguish private value and common value. Such implications are tested using field data and laboratory experiments. In repeated auctions of event tickets, it is shown that the IPV test fails more severe in the next-to-last auctions than the last auction, providing somewhat evidence supporting the repeated ebay model.

In a lab experiment, Ariely, Ockenfels and Roth (2002) show that with non-transmission of the last minute bid, there is both efficiency and revenue loss. However, their result also rejects the non-transmission model of Roth and Ockenfels (2000) since they ob-

[35] A personal communication with ebay's customer service confirmed that the method suggested here is not against ebay's policy. However, if the bidders do not understand this kind of construction, or misinterpret it as a bad signal, such a field experiment might not generate the real environment needed. We may avoid this problem by recruiting laboratory subjects, explain the settings, and ask them to participate in auctions we set up on ebay.

serve last minute bidding even without non-transmission. Thus, the repeated auction model explains last minute bidding better, and implies that last minute bidding is not "bad" (to both efficiency and revenue).

References

[1] Ariely, Dan, Axel Ockenfels, and Alvin E. Roth (2002), "An Experimental Analysis of Ending Rules in Internet Auctions," *mimeo*, Harvard University.

[2] Ashenfelter, Orley (1989) "How Auctions Work for Wine and Art," *Journal of Economic Perspectives* 3 (3), pp.23–36.

[3] Athey, Susan and Philip A. Haile (2002) "Identification of Standard Auction Models," *Econometrica*, 70 (6), pp. 2107–2140.

[4] Bajari, Patrick and Ali Hortacsu (2000) "Winner's Curse, Reserve Prices and Endogenous Entry: Empircal Insights from ebay," *Rand Journal of Economics*, forthcoming.

[5] Garratt, Rod, Mark Walker, and John Wooders (2002) "Experienced Bidders in Online Second-Price Auctions," *mimeo*, University of California, Santa Barbara.

[6] Izmalkov, Sergei (2002) "English Auctions with Reentry," *mimeo*, MIT.

[7] McAfee, R. Preston and Daniel Vincent (1993) "The Declining Pirce Anomaly," *Journal of Economic Theory*, 60 (1), pp.191–212.

[8] Milgrom, Paul R. and Robert J. Weber (1982a) "A theory of auctions and competitive bidding," *Econometrica* 50 (5), pp.1089–1122.

[9] Milgrom, Paul R. and Robert J. Weber (1982b) "A theory of auctions and competitive bidding: Part 2," *mimeo*, Northwestern University. Published in *The Economic Theory of Auctions*, P. Klemperer (ed.), Edward Elgar Publishing, 2000.

[10] Roth, Alvin E. and Axel Ockenfels (2000) "Last-Minute Bidding and the Rules for Ending Second-Price Auctions: Theory and Evidence from a Natural Experiment on the Internet," *mimeo*, Harvard University.

[11] Roth, Alvin E. and Axel Ockenfels (2002) "Last-Minute Bidding and the Rules for Ending Second-Price Auctions: Evidence from eBay and Amazon Auctions on the Internet," *American Economic Review*, 92 (4), pp. 1093–1103.

[12] Wang, Tao-yi J. and Kevin Caves (2002) "The 'Common Value' in Sequential IPV Auctions—Evendence from Online Event Ticket Auctions," *mimeo*, University of California, Los Angeles.

[13] Weber, Robert J. (1983) "Multiple-Object Auctions," published in *Auctions, Bidding, and Contracting: Uses and Theory*, R. Engelbercht-Wiggans, et al, (ed.), NYU Press, 1983.