APPENDIX: Modeling Cournot Competition in an Electricity Market with Transmission Constraints

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This is the appendix of my article in the Energy Journal. It derives the reaction functions and the Nash equilibrium under proportional and efficient rationing.

1 Proportional rationing

1.1 Reaction function

If a generator switches from strategy GNE to strategy II, his market size increases from $Q_i^{GNE}(Q_j)$ to $\frac{1}{1+Q_j}$, but the price drops from $p(Q_i^{GNE}(Q_j)+Q_j)$ to $p(1)$:

$$\pi_i^{prop}(Q_i^{II}(Q_j), Q_j) = (p(1) - c_i) \cdot \frac{1}{1+Q_j}$$

Generator i switches his strategy when the incentive function changes sign. This happens at $Q_j^{prop,cr}(\theta_i)$, the solution of $g_i^{prop}(Q;\theta_i) = 0$ (See Figure 1):

$$Q_j^{prop,cr}(\theta_i) = \begin{cases} \frac{1}{2}\left(\theta_i + 1 - \sqrt{\theta_i^2 + 6\theta_i - 7}\right) & \text{if } \theta_i > 1 \\ 1 & \text{if } \theta_i \leq 1 \end{cases} \quad (1)$$

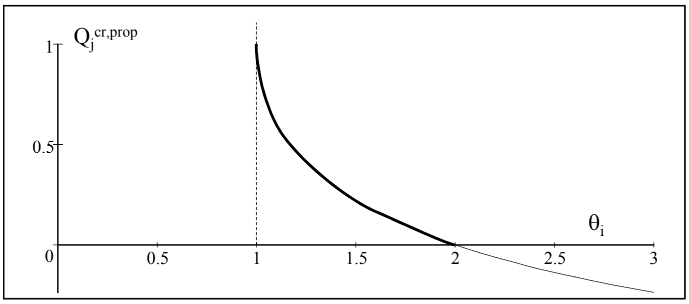

Figure 1: Critical values $Q_j^{cr}(\theta_i)$ as function of the parameter θ_i. For $\theta_i > 2$ the generator always plays aggressively. For $\theta_i < 1$ the generator always plays the standard Cournot reaction function.

The optimal reaction function 'jumps' at the cut off value $Q_j^{prop,cr}$ from the GNE bid to the aggressive bid.

$$Q_i^{prop}(Q_j) = \begin{cases} Q_i^{GNE}(Q_j) & \text{if } Q_j \leq Q_j^{prop,cr}(\theta_i) \\ 1 & \text{if } Q_j > Q_j^{prop,cr}(\theta_i) \end{cases} \tag{2}$$

1.2 Nash Equilibrium

The Nash equilibrium is the intersection of the reaction functions:

$$Q_1^{prop}(Q_2) = Q_1 \tag{3}$$

$$Q_2^{prop}(Q_1) = Q_2 \tag{4}$$

In equilibrium the total bid can be larger or smaller than the available transmission capacity.

If the total equilibrium bid is smaller than the available transmission capacity, the equilibrium conditions (3 and 4) can be rewritten as :

$$Q_1^{GNE}(Q_2) = Q_1 \tag{5}$$

$$Q_2^{GNE}(Q_1) = Q_2 \tag{6}$$

$$Q_2 \leq Q_2^{prop,cr}(\theta_1) \tag{7}$$

$$Q_1 \leq Q_1^{prop,cr}(\theta_2) \tag{8}$$

The couple $(Q_{1,eq}^{GNE}, Q_{2,eq}^{GNE})$ is the solution of the first two equations (5 and 6). So it only remains to be checked if it satisfies equations 7 and 8.

If the total equilibrium bid is larger than transmission capacity, the following conditions should be satisfied:

$$1 = Q_1 \tag{9}$$

$$1 = Q_2 \tag{10}$$

$$Q_1 > Q_2^{prop,cr}(\theta_1) \tag{11}$$

$$Q_2 > Q_1^{prop,cr}(\theta_2) \tag{12}$$

The couple $(1,1)$ is an equilibrium if $1 > Q_2^{prop,cr}(\theta_1)$ and $1 > Q_2^{prop,cr}(\theta_1)$.

2 Efficient rationing

2.1 Reaction function of high cost firm

The *high cost generator* (1)has the same profit with efficient and with *aon* rationing:

$$\pi_1^{eff}(Q_1, Q_2) = \begin{cases} \pi_1^c(Q_1, Q_2) & \text{if } Q_1 \leq 1 - Q_2 \\ 0 & \text{otherwise} \end{cases} \tag{13}$$

His reaction function is thus $Q_1^{GNE}(Q_2)$

$$Q_1^{eff}(Q_2) = Q_1^{GNE}(Q_2) \tag{14}$$

2.2 Reaction function of the low cost firm

The *low cost generator* (2) has a similar profit with efficient rationing as with proportional rationing. He obtains the Cournot profit in the first region (small Q_2), and in the second region ($Q_2 > 1 - Q_1$) his profit increases with his bid.

$$\pi_2^{eff}(Q_2, Q_1) = \begin{cases} \pi_1^c(Q_1, Q_2) & \text{if } Q_2 \leq 1 - Q_1 \\ (\theta_2 - \max\{\theta_1, 1\})\, Q_2 & \text{otherwise} \end{cases} \tag{15}$$

The optimal action in the first region is $Q_2^{GNE}(Q_1)$. In the second region bidding aggressively is optimal ($Q_2 = 1$).

We now compare the profit of both actions. If the generator plays aggressively, he increases his market size from $Q^{GNE}(Q_1)$ to 1, but his price drops from $p(Q^{GNE}(Q_1) + Q_1)$ to $\min(c_1, p(1))$.

$$\pi_2^{eff}(Q_2^{II}(Q_1), Q_1) = (\min(c_1, p(1)) - c_2) \cdot 1$$

The incentive function $g_2^{eff}(Q_1; \theta_1, \theta_2) = \pi_2^{eff}(Q_2^{GNE}(Q_1), Q_1) - \pi_2^{eff}(Q_2^{II}(Q_1), Q_1)$ is a decreasing function in Q_1 and has one root $Q_1^{eff,cr}(\theta_1, \theta_2)$. The reaction function of the low cost generator is

$$Q_2^{eff}(Q_1) = \begin{cases} Q_2^{GNE}(Q_1) & \text{if } Q_1 \leq Q_1^{eff,cr}(\theta_1, \theta_2) \\ 1 & \text{if } Q_1 > Q_1^{eff,cr}(\theta_1, \theta_2) \end{cases} \tag{16}$$

The critical value $Q_1^{eff,cr}(\theta_1, \theta_2)$ has a different specification in four regions A, B, C and D :

$$Q_1^{eff,cr} = \begin{cases} 1 & \text{if } \vec{\theta} \in A \\ \theta_2 - 2\sqrt{\theta_2 - 1} & \text{if } \vec{\theta} \in B \\ \theta_2 - 2\sqrt{\theta_2 - \theta_1} & \text{if } \vec{\theta} \in C \\ \frac{\theta_1 - 1}{\theta_2 - 1} & \text{if } \vec{\theta} \in D \end{cases} \tag{17}$$

where $A = \{\theta |\ \theta_1 < 1 \text{ and } \theta_2 < 1\}$, $B = \{\theta | \theta_1 < 1 \text{ and } \theta_2 > 1\}$, $C = \left\{\theta | \theta_1 > 1 \text{ and } \theta_1 < \theta_2 - (\theta_2 - 1)^2\right\}$ and $D = \left\{\theta | \theta_1 > 1 \text{and } \theta_1 > \theta_2 - (\theta_2 - 1)^2\right\}$. These regions are represented in Figure 2.

2.3 Nash Equilibrium

The equilibrium is the intersection of the reaction functions. This equilibrium can be calculated quite easily. The total equilibrium bid has to be smaller than the transmission capacity, as otherwise the high cost generator would decrease his bid. The equilibrium conditions are thus:

$$Q_2^{GNE}(Q_1) = Q_2 \tag{18}$$

$$Q_1^{GNE}(Q_2) = Q_1 \tag{19}$$

$$Q_1 \leq Q^{eff,cr} \tag{20}$$

The couple $(Q_{1,eq}^{GNE}, Q_{2,eq}^{GNE})$ is the solution of the first two equations (18 and 19) so we only have to check the third condition. For a large range of $\vec{\theta}$ this condition is satisfied. Hence, efficient and all-or-nothing rationing have similar equilibria.

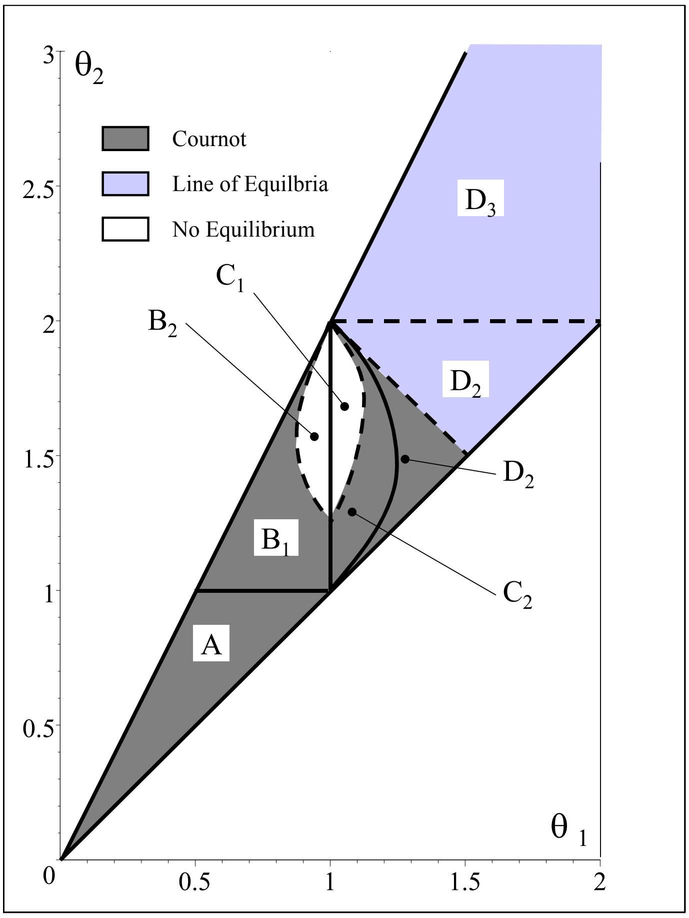

Figure 2: The parameter space $\vec{\theta}$ of the game with efficient rationing. The different regions represent the type of reaction function of the low cost firm.