

Economic Capital Allocation under Liquidity Constraints[§]

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Abstract

Since the capital structure affects the performance of financial institutions confronted to liquidity constraints, the *Economic Capital* is determined by the maximisation of value. Allowing economic decisions to be characterised by a *distorted* probability distribution — so assessing the attitude towards risk as well as information and knowledge — the optimal surplus is expressed as a *Value-at-Risk* — as recommended by the Basel Committee. Thus, demanding more capital than regulatory requirements accounts for different expectations about risks. The optimal surplus is allocated to the lines of business of a conglomerate according to the borne risk and the type of divisional managers. Full allocation is assured and no covariances are required. Further, a mechanism is provided, which allows for the distribution of equity in a decentralised organisation.

1 Introduction

In a seminal paper, Modigliani and Miller (1959) claimed that in perfect markets the capital structure of financial institutions does not matter, for at any time it is possible to raise or release funds if required. Accordingly, the optimal plan — when the objective is maximising value — is to attract as much debt as possible. Since this fact is not observed in practice, Modigliani and Miller gave several explanations in subsequent papers, even questioning the skills of decision makers — as in Miller (1998). However, averse-to-risk customers are sensible to fluctuations and then the performance of intermediaries depends on providing guarantees that assumed liabilities are default-free — see Merton (1997). This situation leads manager's decisions to be determined also by risk aversion — as long as their reputation depends on performance.

Usual practices to protect against default risk are *hedging*, *re-insuring* and *capital cushions*. By *Economic* or *Risk Capital* we mean an amount of money invested in non risky assets that serves as a buffer in order to prevent insolvency. Since a price has to be paid

for raising capital, there is a level of surplus which properly combines the two conflicting objectives: maximisation of shareholder's value and minimisation of default risk. Within a multibusiness environment, the problem of allocation arises due to the gain acquired — through diversification — when merging the activities of the firm. Such benefit should be distributed fairly among the subsidiaries — i.e. according to the risk borne. In this context, many of the allocation principles present in the literature are based on covariances. Full allocation is also considered as a desirable property — for the aggregate surplus maintained by divisions should be equal to the level regarded as appropriate for the conglomerate¹.

In Merton and Perold (1993) a capital allocation principle is developed based on the incremental risk of subsidiaries, which is obtained by subtracting the capital required after suppressing a line of business to the surplus demanded by the whole portfolio. Then the sum of individual surplus might be lower than the capital hired by the conglomerate — the difference is explained by the gains in efficiency due to the knowledge of divisional managers. On this basis, Merton and Perold argue that it is inappropriate to full allocate the capital — for doing so incentives may be distorted. Myers and Read (2001) consider instead the marginal capital requirement, defined as the marginal change in the total surplus in response to a small increment in the equity demanded by a certain line of business. They prove that full allocation is guaranteed by this principle, provided that some conditions on the valuation function of capital are satisfied.

Stoughton and Zechner (1999) propose a model to deal with firms that are not able to continuously raise funds — see also Froot et al (1993) and Jensen (1986). Thus, equity is distributed in order to maximise the *Economic Value Added (EVA)* by the lines of business, and capital allocation is justified as a mechanism that stimulates the exchange of information inside the institution. In the process, the attitude towards risk is considered, which is supposed to depend on the ability to apply and transfer skills — as well as the effort expended to accumulate information. Thus, an optimal mechanism is advanced based on the internal price of capital. Distortions are allowed in the form of under and overinvestment.

In the following, an allocation principle is proposed which, instead of accounting for stochastic dependencies, focuses on agency costs due to discrepancies in the expectations kept by central and divisional managers. Actually, the case of perfect correlation is considered — when no diversification is possible — in this way modelling the situation when the failure in any division may damage the credit quality of the whole conglomerate. *Section 2* is devoted to the determination of the optimal amount of economic capital. The attitude towards risk is determined by a single — functional — parameter, which in imperfect markets accounts for differences in expectations among decision makers. Thus, the demanded surplus depends on the risk profile — or the informational type — of decision makers, as well as on the risk involved. The problem of capital allocation within a multibusiness setting is addressed in *Section 3*. A *centralised* solution is obtained depending on individual exposures. In *Section 4* the *stand alone* allocation is attained by letting subsidiaries to act on their own. Finally, the problem of agency costs is addressed by establishing an *optimal contract*. When the types are not accessible — a situation most probably found in practice — a mechanism can be designed by fixing the cost of raising capital inside the conglomerate. In this way, subsidiaries are forced to reveal their type. *Section 5* concludes.

¹See Albrecht (2004), Hallerbach (2003) and Saita (2004).

2 Economic Capital as the Optimal Level of Surplus

Consider a financial institution holding assets and liabilities for total market values of A and L respectively. The net — random — loss suffered each period is then given by $X = L - A$. Merton (1977) defines the fair price of insuring liabilities — at any time before the maturity date — as the present value of the liability claim less the value of a put option on assets with strike price equal to the value of liabilities². In the same way, it follows that shareholders are the owners of a call option on the portfolio of assets whose exercise price is the value of liabilities. From the *Put-Call Parity Theorem*, the following relation must hold:

$$A = C(A, L) + Le^{-r_0T} - P(A, L).$$

Thus, though both the market value of assets and equity are functions of leverage, by the *Put-Call Parity* their sum is independent of it. Hence, the market value of the firm — i.e. the market value of the portfolio of assets A — is independent of the capital structure, as stated in the Modigliani and Miller proposition — see Miller (1998). However, this reasoning holds true in perfect markets, i.e. when no restrictions are to be found when borrowing and lending. Moreover, the hedged portfolio remains non risky only a short period of time ahead, assuming that during a short period of time market conditions remain unchanged. Thus, continuous rebalancing is needed. Under these conditions, the conglomerate will be indifferent between hedging and reinsurance. But decision makers confronted to liquidity constraints might be interested in replacing — or complementing — their hedging strategy.

By now, assume that central managers know the distribution function of losses F_X and that funds may be hired at the interest rate r_k , with $r_k \geq r_0$, where r_0 denotes the risk free interest rate. Decisions are affected by the *net cost of capital* $\eta_k = r_k - r_0$. Moreover, notice the firm simultaneously acts in two markets. So whenever a loss occurs cash is demanded to avoid default, while in the case a gain is obtained, the surplus can be used to buy more assets or to pay liabilities. Assuming that investors keep different expectations about risk — as long as they own different information, knowledge, social contacts and capabilities — and denoting respectively by φ and β the types for lending and borrowing, *corporate EVA* is given by:

$$\text{EVA} = E_\varphi [(X + k)_-] - E_\beta [(X - k)_+] - \eta_k k.$$

The term $E_\varphi [(X + k)_-]$ denotes the value of the firm when the portfolio is solvent (i.e. when $X < -k$), which is diminished by raising the level of surplus. On the other hand, the term $E_\beta [(X - k)_+]$ represents the *cost of bankruptcy* — or more properly, the cost of *assuming* bankruptcy. Demanding more capital leads to a reduction of the burden of default. Thus, financial intermediaries are able to create value to shareholders as long as the cost of insuring the aggregate exposure — which can be related to the credit quality, as perceived by lenders — plus the cost of raising capital is less than expected gains. Notice how crucial is the role played by the differences in expectations and the symmetry of risks.

²Whenever $A \geq L$ the firm can afford the debt, but when $A < L$ the guarantor suffers a loss equal to $L - A$. Consequently, the guarantor's claim equals $\min(A - L, 0)$ which is identical to that of a put option — where the promised payment L corresponds to the exercise price and the value of assets corresponds to the common stock's price. See also Cummins and Sommer (1996).

Under homogeneous expectations and symmetric risks, keeping a surplus produces a total loss and so no capital should be hired — the value of the firm in this case is zero, which is a reasonable claim in a competitive setting. In this way, the result of the Modigliani and Miller proposition is obtained — see Stiglitz (1972).

The *Wang's risk principle* allows for a characterisation of the mathematical expectation with respect to a *distorted* probability distribution, which is obtained by applying a *distortion* transformation — i.e. a continuous, strictly increasing function, defined on the unit interval $\varphi : [0, 1] \rightarrow [0, 1]$ such that $\varphi(0) = 0$ and $\varphi(1) = 1$ — to the *decumulative distribution function* $S_X(x) = 1 - F_X(x) = \mathbb{P}[X > x]$ in the following way³:

$$\mathbb{E}_\varphi[X] = \int x dF_{\varphi,X}(x) = \int [1 - F_{\varphi,X}(x)] dx = \int \varphi(S_X(x)) dx.$$

The traditional expectation operator is obtained when the *neutral distortion* — equal to the identity operator $\varphi(x) = x, \forall x$ — is introduced. Further, Wang and Young (1998) state the properties:

$$\begin{aligned} \varphi \text{ concave} &\Rightarrow \varphi(y) \geq y \quad \forall y \in [0, 1] \Rightarrow \mathbb{E}_\varphi[X] \geq \mathbb{E}[X] \\ \varphi \text{ convex} &\Rightarrow \varphi(y) \leq y \quad \forall y \in [0, 1] \Rightarrow \mathbb{E}_\varphi[X] \leq \mathbb{E}[X]. \end{aligned}$$

Therefore, *concave* distortion functions characterise the decisions of *averse-to-risk* investors — who overestimates risks — and *convex* distortions the behaviour of *risk lovers* — who underestimates risks. Moreover, applying a *Taylor series* around zero leads to:

$$\mathbb{E}_\varphi[(X + k)_-] \approx \mathbb{E}_\varphi[X_-] + \left[\frac{\partial \mathbb{E}_\varphi[(X + k)_-]}{\partial k} (k = 0) \right] \cdot k.$$

Let us accordingly define:

$$r_{\varphi,X} := - \frac{\partial \mathbb{E}_\varphi[(X + k)_-]}{\partial k} (k = 0) = F_{\varphi,X}(0).$$

The coefficient $r_{\varphi,X}$ corresponds to a *premium for solvency* — specifically, it expresses the marginal reduction of the insured return when hiring an additional unit of equity. When the risk accumulates more probability in gains — remember the variable X represents an aggregated loss — a higher premium has to be paid. On this basis, the level of *Economic Capital* is determined in order to maximise corporate EVA⁴:

³The distorted probability principle is extended to real-valued random variables as — see Wang et al (1997): $\mathbb{E}_\varphi[X] = \int_{-\infty}^0 [\varphi(S_X(t)) - 1] dt + \int_0^\infty \varphi(S_X(t)) dt$. Hence, after performing a change of variables, we can write: $\mathbb{E}_\varphi[X] + \mathbb{E}_\varphi[X_-] = \mathbb{E}_\varphi[X_+]$. The right-hand-side of the equation shows the price of a portfolio containing an insured version of the asset, while the left-hand-side shows the price of a fund containing the asset and a guarantee to pay the loss incurred by X . Both portfolios have the same value at the end of period, and hence both should be assigned the same market price. Therefore the condition is consistent with the no-arbitrage principle.

⁴A raising principle is presented in this fashion by Dhaene et al (2003) though they propose to minimise the total capital cost. See also Goovaerts et al (2004), Laeven and Goovaerts (2004) and Froot et al (1993).

$$\mathbf{Max}_k \mathbb{E}_\varphi [X_-] - \mathbb{E}_\beta [(X - k)_+] - (r_{\varphi,X} + \eta_k)k.$$

Applying *Lagrange optimisation* yields the first order condition:

$$-\frac{\partial}{\partial k} \mathbb{E}_\beta [(X - k)_+] - (r_{\varphi,X} + \eta_k) = S_{\beta,X}(k^*) - (r_{\varphi,X} + \eta_k) = 0.$$

Hence, the firm attracts debt until the marginal benefit equals the total cost of capital and the optimal level of surplus is given by:

$$k^* = F_{\beta,X}^{-1}(1 - r_{\varphi,X} - \eta_k) = S_{\beta,X}^{-1}(r_{\varphi,X} + \eta_k) = S_X^{-1}(\beta^{-1}(r_{\varphi,X} + \eta_k)).$$

The term $(r_{\varphi,X} + \eta_k)$ accounts for the *total cost* of holding an additional unit of capital. When this index is high — i.e. when a high premium is asked for solvency or a high cost is confronted when attracting liabilities — less equity is provided. The contrary occurs when the total cost is low — i.e. when the premium for solvency or the price of capital is low. Whenever $(r_{\varphi,X} + \eta_k) \geq 1$ and $(r_{\varphi,X} + \eta_k) \leq 0$, the minimum and the maximum level of cash are preferred respectively. There is an additional motivation to demand as much surplus as possible in the later case, for the deterioration in the credit quality of the firm might raise the net cost η_k . Moreover, averse-to-risk investors, for whom the distortion function is concave, so that $\varphi^{-1}(\eta) < \eta$, underestimate the price of equity.

The optimal amount of capital — or the *Economic Capital* — is thus expressed as a *Value-at-Risk* under a transformed probability measure. This criterion coincides with the capital requirement established by the Basel Capital Accord⁵. Accordingly, the *Regulatory Capital* is obtained by applying the *neutral distortion* and introducing a *level of confidence* α — in this way implicitly determining the premium for solvency as well as the cost of capital by letting $\alpha = r_{\varphi,X} + \eta_k$. Typically, $\alpha = 5\%$ or $\alpha = 1\%$. Since the same confidence level is asked for every company, the most efficient — which are asked a higher premium for they hold better investments — are forced to keep more surplus than the optimal level. This loss in efficiency makes sense from the perspective of the regulator, as long as the social losses produced because of the simultaneous default of many firms in the industry might be huge — by affecting the economic activity and the aggregate demand. But on the other hand, the minimum level required for the intermediaries that perform badly might be underestimated.

3 Optimal Allocation of Economic Capital among Lines of Business

In order to hold the viewpoint of central managers — or a regulatory authority — confronting a multibusiness environment, let us suppose that X denotes the aggregate loss of a financial conglomerate consisting of $n \in N$ subsidiaries — or lines of business — such that X equals the sum of individual risks:

$$X = X_1 + \dots + X_n.$$

⁵See Basel (1996) and Basel (2004).

Marginal distributions (F_1, \dots, F_n) are assumed to be known and since a failure in any division may damage the reputation of the whole conglomerate, the *comonotonic dependence structure* is considered⁶. When capital decisions are centralised, the cost of the guarantee can be diminished by merging the individual losses⁷, for in this way funds can be assigned only to insolvent divisions — and no idle surplus is maintained. Accordingly, let us establish an allocation principle based on the minimisation of the sum of exposures — the value of the firm is already maximised by choosing the level k^* as the total surplus kept by the conglomerate:

$$\begin{aligned} \mathbf{Min}_{k_i} \quad & \mathbb{E}_\varphi \left[\sum_{i=1}^n (X_i - k_i)_+ \right] \\ \text{subject to} \quad & \sum_{i=1}^n k_i = k^*. \end{aligned}$$

Therefore, a diversification effect exists, but it depends on liquidity constraints — and not on covariances. The only condition imposed is full allocation — as long as capital decisions on business units are taken by central managers, no other concerns are needed. For the *Lagrange multiplier* γ the first order conditions are the following:

$$\begin{aligned} \frac{\partial}{\partial k_i} \mathbb{E}_\varphi \left[\sum_{i=1}^n (X_i - k_i)_+ \right] + \gamma &= -S_{\varphi, X_i}(k_i^*) + \gamma = 0 \quad \forall i = 1, \dots, n \\ \sum_{i=1}^n k_i^* &= k^*. \end{aligned}$$

Let us denote by F_{X^c} the probability distribution of the *comonotonic sum* $X^c = X_1^c + \dots + X_n^c$, where (X_1^c, \dots, X_n^c) represents the *comonotonic random vector* with same marginal distributions as (X_1, \dots, X_n) . Since the inverse distribution of the comonotonic sum is given by the sum of the inverse marginal distributions — see Dhaene et al (2002) — we get that γ is determined such that:

$$F_{\varphi, X^c}^{-1}(1 - \gamma) = \sum_{i=1}^n F_{\varphi, X_i}^{-1}(1 - \gamma) = \sum_{i=1}^n k_i^* = k^*.$$

Thus, the optimal risk capitals allocated to the business units are given by:

$$k_i^* = F_{\varphi, X_i}^{-1}(F_{\varphi, X^c}(k^*)) \quad \forall i = 1, \dots, n.$$

These levels of equity determine the *centralised solution* — for both the raising and the allocation principles have been established according to the risk attitude and knowledge of central managers.

⁶Comonotonicity characterises an extreme case of dependence, when no benefit can be obtained from diversification — see Dhaene et al (2002).

⁷Mathematically, this result is sustained by the fact that the distorted probability principle preserves the first stochastic order, defined by $X \leq Y \Leftrightarrow S_X(t) \leq S_Y(t), \forall t$. Therefore, $\mathbb{E}_\varphi [(X - k)_+] \leq \mathbb{E}_\varphi [\sum_{i=1}^n (X_i - k_i)_+]$ when $\sum_{i=1}^n k_i = k$. See Goovaerts et al (2004) and Laeven and Goovaerts (2004).

4 Optimal Decentralised Mechanism

Full allocation suffices for centralised organisations. But divisions are run by managers who access better information about investment opportunities, a situation that leads shareholders to incur in agency costs — see Jensen (1986). So let us consider subsidiaries as separate units that maximise value but do not assume the reduction of the insured return — and hence do not internalise the premium for solvency in decision making. By putting the burden of bankruptcy on their shoulders, central managers attain a gain due to the diversification of the liquidity constraint — as stated in *Section 3*. Accordingly, as long as subsidiaries hire capital from central management at the net internal cost η , *divisional EVA* is defined in the following way:

$$\text{EVA} = \mathbb{E}_{\varphi_i} [(X_i)_-] - \mathbb{E}_{\varphi_i} [(X_i - k_i)_+] - \eta k.$$

Therefore, divisions maximise value by minimising the total loss $\mathbb{E}_{\varphi_i} [(X_i - k_i)_+] + \eta k$. After the first order condition, the *stand alone risk capital* is determined by:

$$k_i(\eta) = F_{\varphi_i, X_i}^{-1}(1 - \eta) \quad \forall i = 1, \dots, n.$$

By means of the net cost η , the capital decisions of subsidiaries may be distorted — forcing them to internalise bankruptcy according to the interest of the conglomerate. So in order to encourage averse-to-risk managers to raise less capital, its cost might be overcharged. A return over the market rate r_k should be assigned in this situation — such that $\eta > \eta_k$. On the contrary, risk lovers might be subsidised — so that $\eta < \eta_k$ — for giving them incentives to hire more capital. The optimal levels of economic capital and internal cost are simultaneously determined by introducing the following allocation principle — see Diamond and Verrecchia (1982):

$$\begin{aligned} & \mathbf{Max}_{k, \eta} \quad \mathbb{E}_{\varphi} [X_-] - \mathbb{E}_{\varphi} [(X - k)_+] - (r_{\varphi, X} + \eta_k) \cdot k \\ & \text{subject to} \quad k_i = k_i(\eta) \quad \text{and} \quad \sum_{i=1}^n k_i = k. \end{aligned}$$

Applying *Lagrange optimisation* leads the solution to be characterised by:

$$S_{\varphi, X}(k^*) = r_{\varphi, X} + \eta_k \quad \text{and} \quad \sum_{i=1}^n k_i^* = k^*.$$

Hence, the same optimal surplus of *Section 2* is obtained for the conglomerate, while the internal cost of capital is determined such that full allocation is assured:

$$\sum_{i=1}^n F_{\varphi_i, X_i}^{-1}(1 - \eta^*) = k^*.$$

Therefore, if $F_{\varphi_1, \dots, \varphi_n, X^c} = \left(\sum_{i=1}^n F_{\varphi_i, X_i}^{-1} \right)^{-1}$ denotes the distribution function of the comonotonic sum when marginal distributions are given by $(F_{\varphi_1, X_1}, \dots, F_{\varphi_n, X_n})$, then the optimal level of the net internal cost of capital is given by:

$$\eta^* = 1 - F_{\varphi_1, \dots, \varphi_n, X^c}(k^*).$$

In this way, a *decentralised* allocation is determined — the same benefit as under the *centralised* prescription is obtained and so no efficiency is lost. When the types of subsidiaries are not observable, central managers may calibrate their estimations by comparing the preferred amounts of equity with the optimal levels k_i^* . Therefore, by letting divisional managers to act independently they are forced to reveal their type. We can then say the proposed mechanism provides a basis to measure the disagreement between central management and business units.

5 Conclusions

According to the Modigliani and Miller (1959) proposition, the capital structure of a financial institution does not affect its value for it is always possible to raise or release funds in the market. However, this is not a suitable assumption for imperfect markets. Actually, after Merton (1997), the level of surplus matters for averse-to-risk lenders who are sensible to the possibility of bankruptcy of the borrower. Accordingly, the decisions of managers, whose reputation depends on performance, are also affected by risk aversion. In this context, the value of the firm depends on its capital structure, and the *Economic Capital* is defined such that the *Economic Value Added (EVA)* is maximised.

The *Wang's principle* — see Wang et al (1997) — allows expressing the *cost of bankruptcy* as an expectation with respect to a *distorted probability distribution*. The — functional — distortion type simultaneously accounts for risk attitude and knowledge, and investors are supposed to maintain different expectations — an approach already adopted by Stiglitz (1972). The optimal level of surplus is then a function of the total cost of equity — defined as the premium for solvency plus the net capital cost — as well as the risk involved, and since no restrictions are imposed on the distribution functions of returns, the model is suitable both to financial and insurance applications. Thus, decision makers internalise the price of equity, though it is underestimated by risk averse investors — who apply a concave transformation to the probability distribution and consequently demand more capital.

Specifically, the *Economic Capital* is expressed as a *Value-at-Risk* under a distorted probability measure, at the time the *Regulatory Capital* is obtained by applying no distortion and fixing a confidence level α — which in this way plays the same role as the total equity cost and hence in the model both coefficients are given the same meaning. Capital decisions over the minimum regulatory requirement are then explained by risk aversion — for payments are overestimated in this case. However, risk lover investors may overestimate exposures as well, as long as the type also accounts for information and knowledge. In this context, the excess of surplus induces a gain in efficiency, and not the opposite.

A *centralised* allocation of equity is determined by maximising *corporate EVA* and minimising *bankruptcy costs* according to the expectations of central managers. For a decentralised organisation, an *optimal mechanism* is proposed, whose instrument is the internal cost of capital. The same level of surplus is maintained by the conglomerate under both principles. When central managers do not know the types of subsidiaries, the estimations may be calibrated a posteriori — by looking for the functional types which are consistent

with the preferred levels of equity. Thus, the mechanism promotes transparency within the institution. Moreover, the burden of arithmetic operations may be reduced if the distortion function is parametrically determined, such that a single real number accounts for the informational type⁸.

Finally, the mechanism can be useful for regulatory purposes by determining the types which are consistent with the levels of risk capital observed in the industry. Institutions demanding the minimum capital requirement might be expecting a higher performance from their investments than suggested by *average* knowledge. Moreover, though it is not possible to know when companies are underestimating their risk — as long as some information is private — rational decision makers reveal their type — for they maximise value.

References

- [1] Albrecht, P. (2004). “Risk-based capital allocation”, *Encyclopedia of Actuarial Science*: 1459-1466. John Wiley & sons.
- [2] Basel Committee on Banking Supervision (1996). “Amendment to the capital accord to incorporate market risks”, <http://www.bis.org>
- [3] Basel Committee on Banking Supervision (2004). “International convergence of capital measurement and capital standards. A revised framework”, <http://www.bis.org>
- [4] Cummins, J.D. and Sommer, D.W. (1996). “Capital and risk property-liability insurance markets”, *Journal of Banking and Finance*, 20:1069-1092.
- [5] Dhaene, J., Goovaerts, M. and Kaas, R. (2003). “Economic Capital Allocation Derived from Risk Measures”, *North American Actuarial Journal*, 7 (2): 44-59.
- [6] Dhaene, J., Denuit, M., Goovaerts, M., Kaas, R. and Vyncke, D. (2002). “The Concept of Comonotonicity in Actuarial Science and Finance: Theory”, *Insurance: Mathematics & Economics*, 31 (1): 3-33.
- [7] Diamond, D.W. and Verrecchia, R.E. (1982). “Optimal managerial contracts and equilibrium security prices”, *The Journal of Finance*, 37 (2): 275-287.
- [8] Froot, K.A., Scharfstein, D.S. and Stein, J.C. (1993). “Risk management: coordinating corporate investment and financing policies”, *The Journal of Finance*, 48 (5): 1629-1658.
- [9] Goovaerts, M.J., Van den Borre, E. and Laeven, R.J.A. (2004). “Managing economic and virtual economic capital within financial conglomerates”, <http://econ.kuleuven.be/tew/academic/actuawet/research.htm>
- [10] Hallerbach, W.G. (2003). “Capital allocation, portfolio enhancement and performance measurement: a unified approach”, <http://www.gloriamundi.org>

⁸In Mierzejewski (2006) the model is presented in these terms. See also Wang (1995).

- [11] Jensen, M.C. (1986). “Agency Costs of Free Cash Flow, Corporate Finance, and Takeovers”, *The American Economic Review*, 76 (2):323-329.
- [12] Laeven, R.J.A. and Goovaerts, M.J. (2004). “An optimization approach to the dynamic allocation of economic capital”, <http://www.gloriamundi.org>
- [13] Merton, R.C. (1977). “An analytic derivation of the cost of deposit insurance and loan guarantees”, *Journal of Banking and Finance*, 1:3-11.
- [14] Merton, R.C. (1997). “A model of contract guarantees for credit-sensitive, opaque financial intermediaries”, *European Finance Review*, 1:1-13.
- [15] Merton, R.C. and Perold, A.F. (1993). “Theory of risk capital in financial firms”, *Journal of Applied Corporate Finance*, 5:16-32.
- [16] Mierzejewski, F. (2006). “Optimal Capital Allocation confronting Bankruptcy and Agency Costs”, *Bank- en Financieuzen / Revue bancaire et financiere*, 2006/2 (March): 72:77.
- [17] Miller, M.H. (1998). “The Modigliani-Miller propositions after thirty years”, *Journal of Economic Perspectives*, 2 (4): 99-120.
- [18] Modigliani, F. and Miller, M.H. (1959). “The cost of capital, corporation finance and the theory of investment”, *American Economic Review*, 49 (4): 655-69.
- [19] Myers, S.C. and Read Jr., J.A. (2001). “Capital Allocation for Insurance Companies”, *Journal of Risk and Insurance* 68 (4): 545-580.
- [20] Saita, F. (2004). “Risk capital aggregation: the risk managers perspective”, <http://www.gloriamundi.org>
- [21] Stiglitz, J. (1972). “Some aspects of the pure theory of corporate finance: bankruptcies and take-overs”, *Bell Journal of Economics and Management Science*, 3 (2): 458-482.
- [22] Stoughton, N.M. and Zechner, J. (1999). “Optimal capital allocation using RAROC and EVA”, <http://www.gloriamundi.org>
- [23] Wang, S. (1995). “Insurance pricing and increased limits ratemaking by proportional hazards transforms”, *Insurance: Mathematics & Economics*, 17: 43-54.
- [24] Wang, S. and Young, V. (1998). “Ordering risks: Expected utility theory versus Yaaris dual theory of risk”, *Insurance: Mathematics & Economics*, 22: 145-161.
- [25] Wang, S., Young, V. and Panjer, H. (1997). “Axiomatic characterization of insurance prices”, *Insurance: Mathematics & Economics*, 21: 173-183.