Physica A 389 (2010) 1215–1222

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Uncertainty about fundamentals and herding behavior in the FOREX market

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ARTICLE INFO

Article history:
Received 22 June 2009
Received in revised form 8 September 2009
Available online 18 November 2009

Keywords:
Herding behavior
Belief bias and uncertainty
Fundamental exchange rate
Econophysics

ABSTRACT

It is traditionally assumed in finance models that the fundamental value of assets is known with certainty. Although this is an appealing simplifying assumption it is by no means based on empirical evidence. A simple heterogeneous agent model of the exchange rate is presented. In the model, traders do not observe the true underlying fundamental exchange rate and as a consequence they base their trades on beliefs about this variable. Despite the fact that only fundamentalist traders operate in the market, the model belongs to the heterogeneous agent literature, as traders have different beliefs about the fundamental rate.

1. Introduction

The traditional rational expectations efficient market hypothesis model typically assumes a representative agent that has the capacity to fully understand the world. As a consequence, this agent is able to compute the fundamental value of assets at every point in time by correctly using all available information in the market. Clearly this is an appealing simplifying assumption; it is however by no means based on empirical evidence. At the opposite side of the finance literature spectrum, the behavioral approach has been widely used and it has offered an interesting alternative to rational expectations. Nevertheless, in this branch of the literature it is also typically assumed that the fundamental value of assets is known with certainty. Most commonly, models of finance with interacting agents consider two groups of traders: chartist and fundamentalists (Brock and Hommes [1,2], De Grauwe and Grimaldi [3–5], Frankel and Froot [6,7], Lux and Marchesi [9]). The fundamentalist traders buy or sell the asset depending on the magnitude of the misalignment of the current asset price with respect to its fundamental value which, by assumption, is perfectly known at every time. This is also an appealing simplifying assumption, as the analysis is clearly simplified, but it is again not based on empirical evidence.

Take for instance the foreign exchange market (FOREX). For quite a few years, economists have been actively debating whether the value of the Chinese Renminbi is in line with its fundamentals. The answer is far from settled. Goldstein and Lardy [10] for instance have estimated that the Renminbi should appreciate by about 20% to 35% against the US dollar.[1] Interesting enough, based on two different estimates using the same methodology Wang [11] concludes that the

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[1] The Renminbi has experienced a large appreciation since this reference was published. One of the authors has recently claimed however that the Renminbi still needs a further appreciation.

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doi:10.1016/j.physa.2009.11.012

undervaluation level was in one case 'minor' and in the other case the Renminbi should even further depreciate to be in line with its fundamentals. The objective of this paper is not to emit a judgment in favor of one or the other view. There can be little doubt however, that there is a great degree of uncertainty when it comes up to estimate the long-run equilibrium value of the Renminbi and any other currency or asset in general.

In this paper a heterogeneous agent model of the FOREX market is presented. The model extends the setup proposed by Alfarano et al. [12] by excluding all chartist trading rules and thus including only fundamentalist ones. Few papers in the literature of heterogeneous agents model the uncertainty about fundamentals in a systematic way, e.g., De Grauwe and Rovira Kaltwasser [13], Diks and Dindo [14], Manzan and Westerhoff [15], Naimzadaa and Ricchiuti [16] and Westerhoff [17]. Traders in the model presented here do not observe the true underlying fundamental and as a consequence they take positions in the market based on beliefs about it. Since there are only fundamentalists in the current setup, heterogeneity stems only from the difference in beliefs that agents have about the fundamental rate and not from the type of trading strategy that they use to trade.

2. Theoretical exchange rate model

2.1. Population dynamics as a stochastic process

Assume a fixed population of N traders. Traders in the market only behave as fundamentalists implying that they expect any deviation of the observed market exchange rate from its fundamental value to be corrected in the future. Motivated by the empirical evidence presented in the previous section, we assume that traders do not observe the true fundamental rate and therefore they take positions in the market based on beliefs about it. The population of traders therefore will be divided into n optimists and $N-n$ pessimists, where the label optimist–pessimist refers to traders that systematically overestimate or underestimate the fundamental rate respectively. Accordingly, the opinion index $y = 2n/N - 1$ is defined. Clearly, if $y = 1$ there will only be optimists in the market whereas if $y = -1$ there will only be pessimists in the market.

The probability $p(n, t)$ determines the state of the socioeconomic configuration $(n, N-n)$. Assuming that the Markov property applies, we can define $p_{t+\tau,t}(n+k|n)$ as the transition probability from state n to state $n+k$ after the time interval τ.

Following Alfarano et al. [12] the corresponding transition rates per unit time between adjacent states are defined as:

$$\begin{aligned}\pi(n+1|n) &= (N-n)(u+v\,n)\\ \pi(n-1|n) &= n(u+v(N-n))\end{aligned} \tag{1}$$

where u is a parameter measuring the probability of a trader changing from opinion autonomously and v is a herding parameter measuring the influence of others' opinion on each trader.[2] In the current setup both parameters remain constant over time. While this assumption allows to keep the model parsimonious, it should be noted that in principle they can be time varying. See De Grauwe and Rovira Kaltwasser [13] and Westerhoff [18] for an alternative treatment.

2.2. The financial market model

We now turn to the description of the law of motion of the exchange rate. An optimistic trader will expect the exchange rate to increase whenever the market exchange rate $e_t < e_{opt}$, where e_{opt} is the optimistic belief about the fundamental. The traders' excess demand function is assumed to be log-linear. In particular, every time that agents make a transaction, they buy or sell U_f units of the currency in the foreign exchange market. Since there are n optimistic traders exerting an upward 'pulling' force to make the exchange rate reach e_{opt} we can write the following expression representing the optimists' excess demand:

$$ED_{opt,t} = U_f \log\left(\frac{e_{opt}}{e_t}\right) n_t. \tag{2}$$

By analogy the excess demand function for pessimist traders can be described as:

$$ED_{pes,t} = U_f \log\left(\frac{e_{pes}}{e_t}\right)(N-n_t). \tag{3}$$

The total excess demand function is simply the sum of the excess demand functions of the two groups:

$$ED = ED_{opt} + ED_{pes}. \tag{4}$$

[2] This definition of the transition rates goes back to Kirman [8]. As in Alfarano et al. [12] though, the transition rates are defined as a function of the absolute number of traders rather than their proportion in the total population. That eliminates the n-dependence present in Kirman's work. For further details see Alfarano et al. [12].

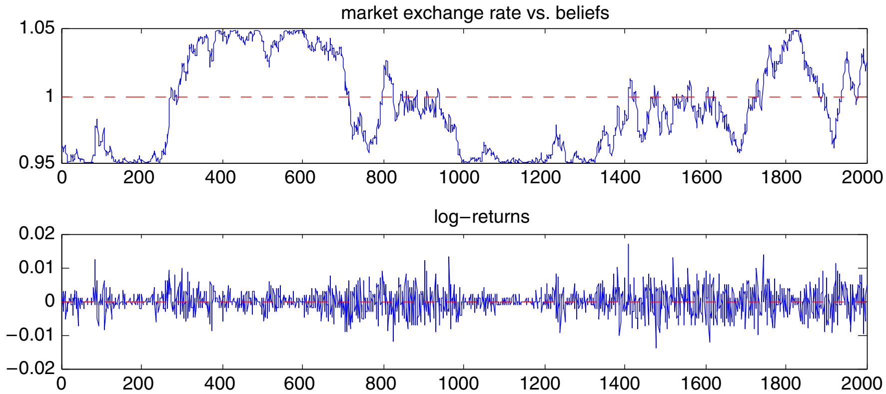


Fig. 1. Upper panel: market log-exchange rate e (continuous line) and unobserved fundamental exchange rate (dashed line fixed at 1). Lower panel: log-returns of the exchange rate. The sample size is $1 \times$ e1000000. Only a subsample of 2000 periods (chosen at random) are shown. Other parameter values: $u = 0.01$, $v = 0.2$, $N = 100$ and $a = 0.05$.

For simplicity we assume that $e_{opt} = e^* + a$ and $e_{pes} = e^* - a$, where $a > 0$ is the belief bias and e^* is the true unobserved fundamental. If we further assume that a market maker collects all the individual orders from the traders and adjusts the price continuously, we have that the continuous change in the exchange rate is

$$\dot{e} = \delta \left(ED_{opt} + ED_{pes} \right) \tag{5}$$

where $\dot{e} = \mathrm{d}e/e\,\mathrm{d}t$ and δ measures the speed of adjustment. In terms of the sentiment index y:

$$\dot{e} = \delta u_f \left[\log \left(\frac{e_{opt}}{e_t} \right) (1 + y_t) + \log \left(\frac{e_{pes}}{e_t} \right) (1 - y_t) \right] \tag{6}$$

with $u_f = \frac{N}{2} U_f$.

Setting $\dot{e} = 0$ allows us to compute the Walrasian market clearing price:

$$e_{eq,t} = \exp \left(e^* + a y_t \right) \tag{7}$$

From Eq. (7) it can be seen that the equilibrium exchange rate is directly related to the sentiment index y. The boundaries of the equilibrium log-price are given by $\log(e_{eq}) = e_{opt}$ if $y \to 1$ and by $\log(e_{eq}) = e_{pes}$ if $y \to -1$. Additionally, when the number of optimists coincides with the number of pessimists, i.e. when the sentiment index y equals 0, then $\log(e_{eq})$ is equal to the value of the true unobserved fundamental rate e^*.

3. Simulation results

Fig. 1 shows a typical realization of the simulated (log)exchange rate equation (7) (upper panel) and the corresponding (log)returns of it (lower panel). Throughout the simulations the true unobserved fundamental exchange rate e^* and the belief bias a as well as the parameters u and v of the stochastic contagion process have been left unchanged.[3] Several features immediately emerge from the figure. Firstly, the market exchange rate series is permanently disconnected from the true (unobserved) fundamental and it is clearly governed by periods of optimism and pessimism. The 'disconnect puzzle' is a vastly reported stylized facts in economics (Obstfeld and Rogoff [20]). In the model presented here the disconnection is not really a puzzle, as the market exchange rate closely follows the sentiment index y (see Eq. (7)).

Furthermore, by simple eye inspection at the returns series the presence of excess volatility and clusters of volatility is immediately recognized. The 'excess volatility puzzle' is also a well documented empirical regularity in financial economics. Looking once again at Eq. (7) it is straightforward to see where the excess volatility in the model comes from. Finally, the clusters of volatility are associated with periods in which the socioeconomic configuration is such that there is no optimistic or pessimistic consensus, that is, when uncertainty is the highest.

Fig. 2 plots the series of sample autocorrelation of the log-returns, absolute returns and squared returns of the simulated data (panel a) and three empirical exchange rate series at daily frequency: Dollar–Pound (panel b), Deutsche Mark–Yen (panel c) and Yen–Pound (panel d). Both the synthetic and empirical series of returns are characterized by no linear autocorrelation but high absolute and squared autocorrelations persistent up to more than 100 periods. This is another

[3] Similar to Kirman and Teyssiére [19] and Alfarano et al. [12] we distinguish between a microscopic time scale (τ) at which only asynchronous switches can be observed and a macroscopic one (Δt) at which several changes in the socioeconomic configuration may occur.

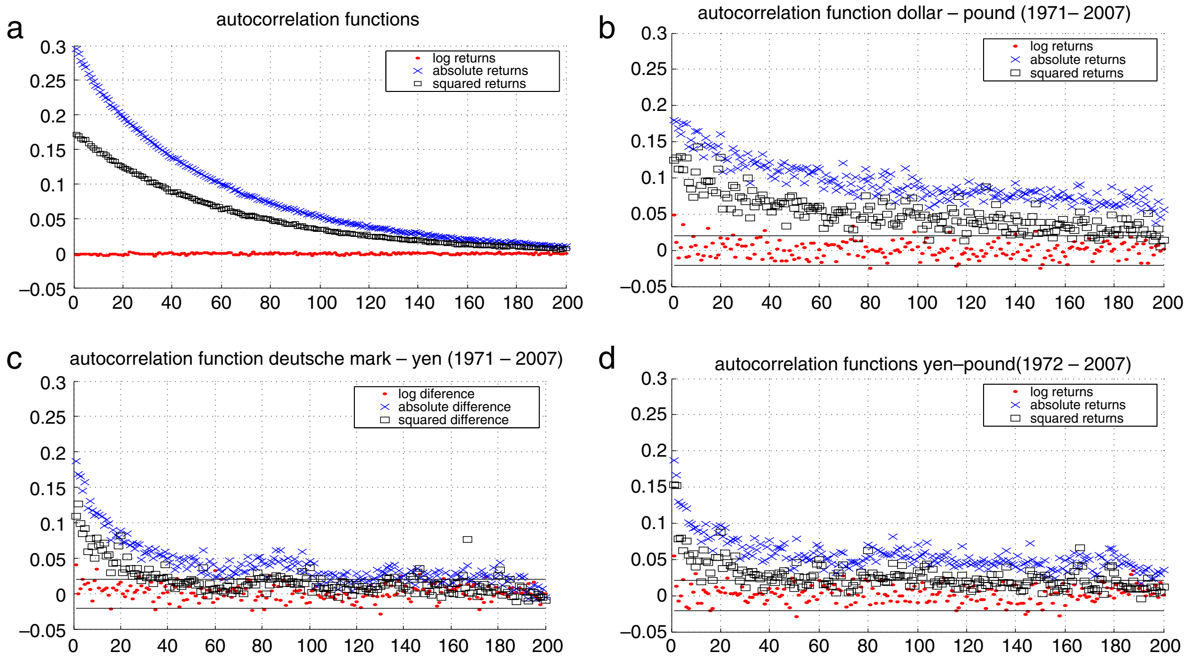
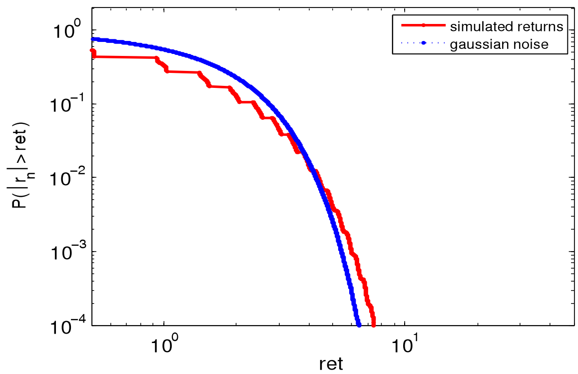
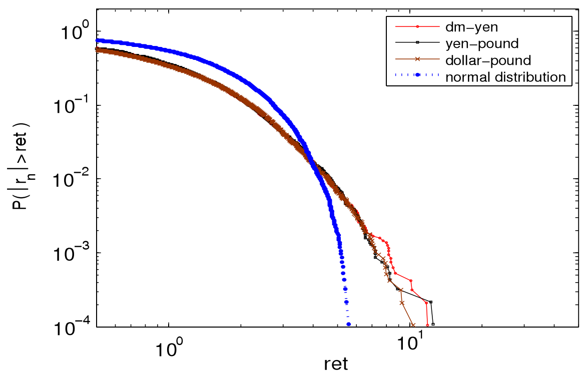


Fig. 2. For the synthetic series the sample size is $1 \times$ e1000000. Other parameter values: $u = 0.01$, $v = 0.2$, $N = 100$ and $a = 0.05$. The empirical time series have the following length: Dollar–Pound (9651), Deutsche Mark–Yen (9488) and Yen–Pound (9364).



(a) Synthetic data vs normal distribution.



(b) Empirical data vs normal distribution.

Fig. 3. Log–log plot inverse cumulative distribution of the absolute value normalized returns and log–plot absolute value of random draws standard Gaussian distribution. The sample size is $1 \times$ e1000000. Other parameter values: $u = 0.01$, $v = 0.2$, $N = 100$ and $a = 0.05$.

empirical regularity observed in financial time series referred to as 'volatility clustering' or 'volatility persistency' (Ding et al. [21,22]). To conclude, in Fig. 3 we show the log–log plot of the inverse empirical cumulative distribution of the normalized synthetic returns (panel a). Panel (b) plots the same series for the three empirical time series. The figure shows that the model produces extreme returns with a probability that is clearly higher than the random draws from the standard Gaussian distribution. In other words, the distribution of the returns is characterized by the presence of fat tails. However, the tails of the distribution of the synthetic returns are thinner than the tails of the returns distribution of the empirical data. Fat tailed distributed returns approximately following an inverse cubic power law is one of the most pervasive stylized fact of the foreign exchange market and financial markets in general (Cont [23], Koedijk et al. [24] and Mandelbrot [25]). Formally, the distribution of large changes in the exchange rate can be described by $Pr(|r_t| > x) \cong x^{-\alpha}$, where the decay parameter α can be easily estimated by means of its conditional maximum likelihood. Table 1 shows the estimation results of parameter α for both the synthetic and true exchange rate data.[4] The theoretical model produces return series with fat tails for the 10%

[4] The only criterion we have used to select these samples of empirical exchange rate series has been to obtain the longest possible time series of floating rates at daily frequency. In all three cases we have taken care of eliminating all observations corresponding to periods of pegged exchange rates (prior to 1971) leaving a sample size of 9651 observations for the Dollar–Pound rate, 9488 for the Deutsche Mark–Yen rate and 9364 for the Yen–Pound rate. Hence

Table 1
Hill estimator.

	Synthetic data		
	Mean	Minimum	Maximum
10.0%	3.47	2.58	4.16
5.0%	4.25	3.22	5.63
2.5%	5.56	4.26	7.07
	Empirical data		
	Dollar–Pound	Deutsche Mark–Yen	Yen–Pound
10.0%	3.00	3.01	2.93
	(2.81–3.18)	(2.81–3.20)	(2.74–3.11)
5.0%	3.50	3.32	3.26
	(3.19–3.81)	(3.02–3.62)	(2.97–3.56)
2.5%	3.83	3.73	3.96
	(3.34–4.31)	(3.25–4.20)	(3.45–4.47)

Note: The numbers in parenthesis indicate the 95% confidence interval of parameter α.

Table 2
Fractional differentiation.

	Synthetic data		
	Mean	Minimum	Maximum
r	−0.137	−0.313	0.072
r^2	0.370	0.152	0.554
$\|r\|$	0.377	0.143	0.570
	Empirical data		
	Dollar–Pound	Deutsche Mark–Yen	Yen–Pound
r	0.048	0.098	0.129(***)
	(−0.106–0.201)	(−0.053–0.249)	(0.004–0.254)
r^2	0.359(*)	0.216(*)	0.234(*)
	(0.254–0.465)	(0.092–0.339)	(0.138–0.331)
$\|r\|$	0.468(*)	0.279(*)	0.335(*)
	(0.321–0.614)	(0.151–0.408)	(0.202–0.468)

Note: The numbers in parenthesis indicate the 95% confidence interval of parameter α.

and 5% quintiles. At the extreme of 2.5%, however, estimates of parameter α indicate fatter tails than the ones produced by the model.[5]

Table 2 shows the parameter d of fractional differentiation computed with the Geweke and Porter-Hudak [26] estimator for both the artificial and the empirical exchange rate return series. Our results indicate the presence of long term dependency ($d > 0$) for the squared and absolute returns of the synthetic and empirical data. Raw returns are characterized by no persistence in the case of the empirical data but some negative antipersistent behavior ($d < 0$) for the synthetic data. Vandewalle and Ausloos [27] find antipersistency in a group of exchange rate series of countries from the European Monetary System prior to the introduction of the Euro. If a currency is only allowed to freely fluctuate inside of a band, whenever it approaches the border of the band, the probability of observing a reversal increases, provided the exchange rate regime is fully credible. This argument can be well understood in the framework of the Krugman [28] model on target zones.

Moreover, Vandewalle and Ausloos [29] also find antipersistency for the US dollar/Deutsche Mark rate during some periods, mostly associated to particular institutional arrangements like the Louvre Accord. The fact that we find some negative long term persistency in our synthetic series of returns does not come as a surprise given the bounded nature of the exchange rate level series. In our model the imposed reflecting boundary conditions prevent the exchange rate to appreciate/depreciate beyond the level given by the belief bias a. As a result, the exchange rate always bounces back whenever it reaches the boundaries, leading to the reported antipersistency in Table 2.

4. Inside the black-box

In this section the analytical details of the model are presented. Let us compute the exchange rate returns using Eq. (7):

$$r_t = \log\left(\frac{e_{t+\Delta t}}{e_t}\right) \approx \frac{e_{t+\Delta t}}{e_t} - 1 = \left(\sqrt{\frac{e_{opt}}{e_{pes}}}\right)^{y_{t+\Delta t}-y_t} - 1. \quad (8)$$

our decision to compute all statistics for the simulated data into from 100 samples of 10,000 observations, so as to approximately match the length of the three empirical series.

5 Fig. 3 provides evidence of the same.

Defining $y_{t+\Delta t} - y_t = dy_t$ and assuming that dy_t is small it is possible to expand Eq. (8) around dy_t to obtain

$$r_t \approx a\, dy_t. \tag{9}$$

Not surprisingly, the exchange returns are a function of size of the belief bias a and the changes of the sentiment index y. Moreover, since a remains constant over the simulations it only conveys information about the amplitude of the returns. Any nonlinear behavior of the exchange rate returns is not to be found in the belief bias but in the dynamics of the sentiment index. The dynamics of dy_t are better understood after computing the Langevin equation for the sentiment index:

$$dy_t = -2\, u\, y_t\, \Delta t + \sqrt{2\, v\, \Delta t\, \left(1 - y_t^2\right)}\, \eta_t \tag{10}$$

where $\eta(t)$ is a standard normally distributed random variable. It is straightforward to see that the non-Gaussian nature of the exchange rate returns stems from the nonlinear diffusion term present in Eq. (10). The nonlinear component of the diffusion term in (10) acts as an amplifier to the Gaussian noise term, leading to non-Gaussian changes in the sentiment index and therefore to non-Gaussian exchange rate returns as well.

An immediate consequence of the belief bias a being a parameter governing the amplitude of the exchange rate returns is that it has no impact on the estimates of both the decay parameter α (Hill estimator) and the degree of fractional differentiation d of raw, squared and absolute returns. To see this we write down the ML estimator of the decay parameter α for the exchange rate return series

$$\hat{\alpha} = \left(\frac{1}{k} \left[\sum_{i=1}^{k} \ln \left\{ \left| y_i \left(1 + \frac{\sqrt{2\, v\, \Delta t\, \left(1 - y_i^2\right)} \eta_i}{-2\, u\, \Delta t\, y_i} \right) \right| \right\} - \sum_{i=1}^{k} \ln \left\{ \left| y_k \left(1 + \frac{\sqrt{2\, v\, \Delta t\, \left(1 - y_k^2\right)} \eta_k}{-2\, u\, \Delta t\, y_k} \right) \right| \right\} \right] \right)^{-1}. \tag{11}$$

The estimated decay parameter of the exchange rate returns is in fact fully determined by the distribution of the changes in the sentiment index and is totally invariant with respect to a.

Similarly, for the fractional differentiation parameter estimator proposed by Geweke and Porter-Hudak [26] we have

$$\begin{aligned} \log\{\tilde{I}(\omega_{j,T})\} &= \log\{\sigma^2 f_e(0)/2\pi\} - \log\left\{\frac{1}{2\pi T} a^2\right\} - d\, \log\{4\sin^2(\omega_{j,T}/2)\} \\ &\quad + \log\{f_e(\omega_{j,T})/f_e(0)\} + \log\{I(\omega_{j,T})/f(\omega_{j,T})\} \end{aligned} \tag{12}$$

where $\log\{\tilde{I}(\omega_{j,T})\} = \log\{|\sum_{t=1}^{T} \left(-2\, u\, \Delta t\, y_t + \sqrt{2\, v\, \Delta t\, (1 - y_t^2)}\, \eta_t\right) \exp(-\mathrm{i}t\omega_j)|^2\}$. The belief bias a only affects the intercept of the regression model (12) and not its slope. This finding is not surprising given the linear nature of the Fourier transform which is applied in the regression model (12) to estimate the fractional differentiation parameter d.

In order to compute the unconditional moments of the exchange rate returns we first compute the equilibrium distribution of $y(t)$. Using the transition rates defined in (1) it is possible to compute the master equation. For large N the sentiment index y can be regarded as continuous, in which case also $P(y, t)$ will be a continuous function of y. This allows to write the Fokker–Planck equation of the sentiment index density

$$\frac{\partial}{\partial t} P(y, t) = -\frac{\partial}{\partial y}[K(y)P(y, t)] + \frac{1}{2}\frac{\partial^2}{\partial y^2}[Q(y)P(y, t)] \tag{13}$$

where $K(y) = -2\, u\, y$ and $Q(y) = 2v(1 - y(t)^2) + 4\frac{u}{N}$ are the drift and diffusion terms respectively.[6] Notice the similarity in the diffusion term of the Langevin equation and the Fokker–Planck equation. In both cases the state variable $y(t)$ enters nonlinearly confirming again that the fluctuations of $y(t)$ are governed by the nonlinearity introduced through the contagion between agents (see Eq. (1)). Clearly as $y(t) \to \pm 1$ the fluctuations of the system tend to diminish, reducing as well the fluctuations of the exchange rate.

Finally, the equilibrium distribution $P_y(y)$ of the stochastic process $P(y, t)$ is determined by solving Eq. (13):

$$P_e(y) = \left(1 - y^2\right)^{\frac{u}{v} - 1} \frac{\Gamma(2\,\frac{u}{v})}{2^{(2\frac{u}{v} - 1)}\, \Gamma(\frac{u}{v})^2} \tag{14}$$

where $\Gamma(z)$ is the gamma function. Eq. (14) allows to determine the unconditional moments of y and e respectively

$$\mathrm{E}[(1 - y^2)^k] = 2^{2k} \frac{\Gamma(2\frac{u}{v})}{\Gamma(2k + 2\frac{u}{v})} \left[\frac{\Gamma(k + \frac{u}{v})}{\Gamma(\frac{u}{v})}\right]^2. \tag{15}$$

[6] The term $4\frac{u}{N}$ in the diffusion term can be neglected for large N.

Eq. (15) shows that all even moments of the sentiment index are well defined. Additionally, taking expectations from Eq. (10) one can readily see that all uneven moments of the exchange rate return process are defined and equal to 0. Combining (10) and (15) we can compute

$$\mathrm{E}[r^{2k}] = a^{2k}(2v\Delta t)^{2k}\,\mathrm{E}\left[(1-y_t^2)^k\right]\mathrm{E}[\eta^{2k}]$$
$$= 2^{2\,k}\,a^{2k}\,(2\,v\,\Delta t)^{2k}\,\frac{\Gamma(2\frac{u}{v})}{\Gamma(2k+2\frac{u}{v})}\left[\frac{\Gamma(k+\frac{u}{v})}{\Gamma(\frac{u}{v})}\right]^2 \tag{16}$$

and for $k = 1$ we obtain the unconditional variance of the exchange rate return

$$\mathrm{E}[r^2] = 4\,a^2\,\frac{u\,\Delta t}{\left(1+2\frac{u}{v}\right)}. \tag{17}$$

We have just shown the existence of both all even and uneven moments up to infinity of the theoretical exchange rate returns. This feature of the model however stands in contradiction with the empirical evidence. To see this we refer to the Gnedenko theorem which states that (after a proper centering and normalization) if an *iid* sequence converges to a non-degenerate distribution, then the limit distribution of the sequence can only be the Generalized Extreme Value (GEV). In the Cramer–von Mises representation the GEV takes the following functional form

$$H_\xi(x) = \exp[-(1+\xi\,x)^{-\frac{1}{\xi}}] \tag{18}$$

where $-\infty < \xi < \infty$ is a shape parameter indicating the speed with which the tails of the distribution will approach 0. For $\xi \to 0$ the GEV belongs to the Gumbel distribution family (exponential decay) and for $\xi > 0$ it corresponds to the Fréchet distribution family which has fatter tails than the Gumbel family.[7] A distribution described by a power-law tail with exponent α corresponds to the Fréchet family with decay parameter $\xi = 1/\alpha > 0$. As we have mentioned in Section 3 empirical exchange rate returns closely follow an inverse cubic power law. That is, empirical estimates of the decay parameter α typically lie around 3 (see Table 1). Furthermore, since the order of the highest absolute moment of a Pareto distribution is given by the decay parameter α, a value $\alpha = 3(\xi > 0)$ indicates that the exchange rate returns distribution belong to the Fréchet family. The fact that for the returns of the theoretical exchange rate model infinite moments exist indicates that $\xi = 0$ and thus a rejection of a power-law behavior, in favor of a Gumbel type extreme value distribution. It is not surprising then that the Hill estimator of the decay parameter α shown in Table 1 displays thinner tails for the theoretical returns than for the empirical ones.

Finally, for the model presented here we obtain the following closed form solutions for the autocorrelation function of raw returns $C_{r_t} \approx -u\Delta t\exp(-2ut)$ and squared returns $C_{r_t^2} = (4\frac{u}{v}+6\frac{u}{v}+3)^{-1}\exp(-2vt(2\frac{u}{v})+1)$. These expressions are identical to the one obtained by Alfarano et al. [12], meaning that the inclusion of the parameter a of belief bias does not affect the dynamics of both autocorrelation functions. Therefore, as in their case, high persistency in volatility does not come as a surprise. At the same time, the model produces some negative autocorrelation in raw returns which can be neglected as long as the value of u is not too high.

5. Conclusion

Knowing the value of economic fundamentals is key to determine whether assets are aligned or misaligned with respect to their long-run equilibrium value. Economic fundamentals however have to be estimated and several methods are available. As a consequence there is a large uncertainty about their true value and economic agents base their decisions based on beliefs and judgements about fundamentals. In this paper a parsimonious heterogeneous agent model of the exchange rate has been presented. The model extends on the work of Alfarano et al. [12] and therefore several features of their model are present here. Our model however innovates in several ways. Firstly, we show a very simple way in which uncertainty about fundamentals can be modeled. Secondly, we show the relevance of the parameter a of belief bias and how it affects the dynamics of the stochastic process. We show that it affects the size of the returns but not their dynamic properties which are totally governed by the changes in the sentiment index. Additionally, we demonstrate that in the current setup the parameter a has no influence on both the Hill estimator and the fractional differentiation parameter proposed by Geweke Porter-Hudak. By determining the unconditional moments of the return distribution it was also shown that the stochastic process does not belong to the Fréchet type family to which empirical exchange rate returns do belong.

The model offers a very intuitive explanation for both the disconnect and excess volatility puzzles. Even in a world absent of trend followers, who exert a self-fulfilling force (positive feedback mechanism) in the market, cyclical fluctuations of the exchange rate around its fundamental can emerge. No doubt, traders live in a world where trend followers do exist. We have decided not to consider trend following trading strategies in our analysis in order to make sure that our results stem only from the fact that there is uncertainty about the fundamental exchange rate. Additional dynamics might be obtained by including trend following trading rules.

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[7] A third possibility is given when $\xi < 0$ in which case GEV belongs to the Weibull family.

Finally, traders' changes of opinion from optimism to pessimism and vice versa are fully governed by a stochastic process in the model. In other words, traders in the model do not use an evolutionary fitness criterion to evaluate the different trading strategies. Our approach is admittedly extreme, as any economic rationality has been dropped from the analysis leaving only the stochastic contagion to drive the results. This should not be interpreted as a suggestion of an absolute lack of rationality of traders when dealing and taking positions in financial markets. It should rather be seen as attempt to show the importance of intuitive decision making in financial markets when accompanied by heterogeneous views about fundamentals and how these two elements help to understand the emergence of some financial economic stylized facts. Clearly the presence of excess volatility, volatility clustering and disconnection in the model has to do with two things: heterogeneous beliefs about the fundamental value of the currency (optimism and pessimism) and the nonlinear type of herding that drives traders' changes of opinion. If traders could perfectly recognize the equilibrium value of a currency there would be no room for heterogeneity in our model, and therefore no room for excess volatility and disconnection. This is probably also true in the real world. If everyone knew the true fundamental at every time, any misalignment could be corrected almost immediately by traders. Uncertainty about fundamentals makes the life of real traders more difficult. Furthermore, as we have stressed in Sections 3 and 4, the clusters of volatility are associated precisely with periods in which the market is on the look for a consensus about the equilibrium value of the currency. Once a consensus has been reached there is less uncertainty, volatility decreases and the clusters of volatility disappear as well. This is true regardless of whether the consensus reached is the optimistic or the pessimistic one.

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