

# Exchange Rates in a Behavioural Finance Framework

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## **Abstract**

We develop a simple model of the foreign exchange market in which agents optimize their portfolio and use different forecasting rules. They check the profitability of these rules ex post and select the more profitable one. This model produces two kinds of equilibria, a fundamental and a bubble one. In a stochastic environment the model generates a complex dynamics in which bubbles and crashes occur at unpredictable moments. We also analyse the empirical relevance of the model

# 1 Introduction

The rational expectations paradigm has been important in our understanding of financial markets. However, it has also shown its limitations. This is especially the case in the foreign exchange market where much of the observed short-term dynamics cannot easily be understood in the context of rational expectations models. As will be argued in this paper the difficulties of rational expectations models (RE-models) in correctly describing the short-term dynamics of the exchange market is related to the strong informational burden imposed on individual agents by these models.

In this paper we use an approach which is influenced by the behavioural finance literature. In this literature agents use relatively simple behavioural rules. These rules do not employ all available information, mainly because agents find it difficult to process and to evaluate the wealth and complexity of the available information (Shleifer(2000), Shiller(2001)). Agents are rational, however, in that they use these rules only as long as they are profitable. We will add to this the notion that agents compare the rule they currently use to alternative rules and decide to switch to the alternative if it turns out that it is more profitable. This is tantamount to introducing a fitness criterion in the selection of behavioural rules. In this sense the model is also in the tradition of the literature on evolutionary dynamics (Kirman(1993), Brock and Hommes(1997), Lux(1998)).

The paper proceeds in different steps. We start by presenting the simplest possible version of the model. In this model there is no feedback from the goods market and there are no transactions costs. We show how this leads to endemic bubbles and crashes. In the next step we introduce a feedback from the goods market, first by endogenizing the current account, and then by introducing the assumption that it is costly to trade goods and services internationally. It will be shown that these complications alter substantially the nature of the dynamics in the foreign exchange markets. Finally, we discuss the empirical implications of the model.

# 2 The model

In this section we develop the simple version of the exchange rate model. The model consists of three building blocks. First, agents select their optimal portfolio using a mean-variance utility framework. Here we follow the tradition of mainstream economics in that agents are utility maximizing individuals. Second, these agents make forecasts about the future exchange rate based on simple but different rules. In this second building block we introduce concepts borrowed from the behavioural finance literature. Third, agents evaluate these rules ex-post by comparing their risk-adjusted profitability. Thus, the third building block relies on an evolutionary dynamics.

## 2.1 The optimal portfolio

We assume agents of different types  $i$  depending on their beliefs about the future exchange rate. Each agent can invest in two assets, a domestic and a foreign one. The agents' utility function can be represented by the following equation:

$$U(W_{t+1}^i) = E_t(W_{t+1}^i) - \frac{1}{2}\mu V^i(W_{t+1}^i) \quad (1)$$

where  $W_{t+1}^i$  is the wealth of agent of type  $i$  at time  $t+1$ ,  $E_t$  is the expectation operator,  $\mu$  is the coefficient of risk aversion and  $V^i(W_{t+1}^i)$  represents the conditional variance of the wealth of agent  $i$ . The wealth is specified as follows:

$$W_{t+1}^i = (1 + r^*) s_{t+1} d_t^i + (1 + r) (W_t^i - s_t d_t^i) \quad (2)$$

where  $r$  and  $r^*$  are respectively the domestic and the foreign interest rates,  $s_{t+1}$  is the exchange rate at time  $t + 1$ ,  $d_{i,t}$  represents the holdings of the foreign assets by agent of type  $i$  at time  $t$ . Thus, the first term on the right-hand side of 2 represents the value of the foreign portfolio in domestic currency at time  $t + 1$ , while the second term represents the value of the domestic portfolio at time  $t + 1$ .

Substituting equation 2 in 1 and maximising the utility with respect to  $d_{i,t}$  allows us to derive the standard optimal holding of foreign assets by agents of type  $i$  :

$$d_{i,t} = \frac{(1 + r^*) E_t^i(s_{t+1}) - (1 + r) s_t}{\mu \sigma_{i,t}^2} \quad (3)$$

The optimal holding of the foreign asset depends on the expected excess return corrected for risk. The market demand for foreign assets at time  $t$  is the sum of the individual demands, i.e.:

$$\sum_{i=1}^N n_{i,t} d_{i,t} = D_t \quad (4)$$

where  $n_{i,t}$  is the number of agents of type  $i$ .

Market equilibrium implies that the market demand is equal to the market supply  $Z_t$  which we assume to be exogenous<sup>1</sup>. Thus,

$$Z_t = D_t \quad (5)$$

Substituting the optimal holdings into the market demand and then into the market equilibrium equation and solving for the exchange rate  $s_t$  yields the equilibrium exchange rate:

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<sup>1</sup>The market supply is determined by the net current account and by the sales or purchases of foreign exchange of the central bank. We assume both to be exogenous here. In section we will endogenize the current account.

$$s_t = \left( \frac{1+r^*}{1+r} \right) \frac{1}{\sum_{i=1}^N \frac{w_{i,t}}{\sigma_{i,t}^2}} \left[ \sum_{i=1}^N w_{i,t} \frac{E_t^i(s_{t+1})}{\sigma_{i,t}^2} - \Omega_t Z_t \right] \quad (6)$$

where  $w_{i,t} = \frac{n_{i,t}}{\sum_{i=1}^N n_{i,t}}$  is the weight (share) of agent  $i$ , and  $\Omega_t = \frac{\mu}{(1+r) \sum_{i=1}^N n_{i,t}}$

Thus the exchange rate is determined by the expectations of the agents,  $E_t^i$ , about the future exchange rate. These forecasts are weighted by their respective variances  $\sigma_{i,t}^2$ . When agent's  $i$  forecasts have a high variance the weight of this agent in the determination of the market exchange rate is reduced.

## 2.2 The forecasting rules

We now specify how agents form their expectations of the future exchange rate and how they evaluate the risk of their portfolio.

We start with an analysis of the rules agents use in forecasting the exchange rate. We take the view that individual agents are overwhelmed by the complexity of the informational environment, and therefore use simple rules to make forecasts. Here we describe these rules. In the next section we discuss how agents select the rules.

We assume that two types of forecasting rules are used. One is called a "fundamentalist" rule, the other a "chartist" (technical trading) rule<sup>2</sup>. The agents using a fundamentalist rule, the "fundamentalists", base their forecast on a comparison between the market and the fundamental exchange rate, i.e. they forecast the market rate to return to the fundamental rate in the future. In this sense they use a negative feedback rule that introduces a mean reverting dynamics in the exchange rate. The speed with which the market exchange rate returns to the fundamental is assumed to be determined by the speed of adjustment in the goods market which is assumed to be in the information set of the fundamentalists (together with the fundamental exchange rate itself). Thus, the forecasting rule for the fundamentalists is :

$$E_t^f(\Delta s_{t+1}) = -\psi(s_{t-1} - s_{t-1}^*) \quad (7)$$

where  $s_t^*$  is the fundamental exchange rate at time  $t$ , which is assumed to follow a random walk and  $0 < \psi < 1$ . The fundamental exchange rate is the value of the exchange rate that equilibrates the current account. At this stage, however, we do not model the goods market. As a result, the fundamental exchange rate is exogenous. We return to this issue in a later section. Note also that when fundamentalists forecast the future exchange rate they use information up to period  $t-1$ . This is in the logic of the model. Agents do not know the full model structure. As a result, they cannot compute the equilibrium exchange rate of time  $t$  that will be the result of their decisions made in period  $t$ <sup>3</sup>.

<sup>2</sup>This idea of distinguishing between fundamentalist and chartist rules was first introduced by Frankel and Froot().

<sup>3</sup>Only in a model with rational expectations can this be done.

The agents using technical analysis, the "chartists", forecast the future exchange rate by extrapolating past exchange rate movements. Their forecasting rule can be specified as :

$$E_t^c (\Delta s_{t+1}) = \beta \sum_{i=1}^T \alpha_i \Delta s_{t-i} \quad (8)$$

Thus, the chartists compute a moving average of the past exchange rate changes and they extrapolate this into the future exchange rate change. The degree of extrapolation is given by the parameter  $\beta$ . Note that chartists take into account information concerning the fundamental exchange rate *indirectly*, i.e. through the exchange rate itself. In addition, chartist rules can be interpreted as rules that attempt to detect "market sentiments". In this sense the chartist rule can be seen as reflecting herding behaviour<sup>4</sup>.

It should be stressed that both types of agents, chartists and fundamentalists, use partial information. Thus our approach differs from the approach in the tradition of rational expectations models in which an asymmetry is assumed in the information processing capacity of agents. In the latter approach some agents, the "rational" agents, are assumed to use all available information, while other agents, "noise traders", do not use all available information. Such an asymmetry is made in order to facilitate the mathematical analysis of the models. However, the basis on which such an asymmetry can be invoked remains unclear. In contrast with this tradition, we take the view that the informational complexity is similar for all agents, and that none of them can be considered to be superior on that count.

We now analyse how fundamentalists and chartists evaluate the risk of their portfolio. The risk is measured by the variance terms in equation 6, which we define as the weighted average of the squared (one period ahead) forecasting errors made by chartists and fundamentalists, respectively. Thus,

$$\sigma_{i,t} = \sum_{k=1}^{\infty} \gamma_k [E_{t-k}^i (s_{t-k+1}) - s_{t-k+1}]^2 \quad (9)$$

where  $\gamma_k$  are geometrically declining weights, and  $i = f, c$ .

### 2.3 Fitness of the rules

The next step in our analysis is to specify how agents evaluate the fitness of these two forecasting rules. The general idea that we will follow is that agents use one of the two rules, compare their (risk adjusted) profitability *ex post* and then decide whether to keep the rule or switch to the other one. Thus, our model is in the logic of evolutionary dynamics, in which simple decision rules are selected. These rules will continue to be followed if they pass some "fitness"

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<sup>4</sup>There is a large literature on the use of chartist analysis (technical analysis). This literature makes clear that chartism is widely used in the foreign exchange markets. See Cheung and Chinn(1989), Taylor and Allen(1992), Cheung et al(1999), Mentkhoff(1997) and (1998).

test (profitability test). Another way to interpret this is as follows. When great uncertainty exists about how the complex world functions, agents use a trial and error strategy. They try a particular forecasting rule until they find out that other rules work better. Such a trial and error strategy can be considered to be a rational strategy when agents cannot understand the full complexity of the underlying model.

In order to implement this idea we use an approach proposed by Brock and Hommes(1997) which consists in making the weights of the forecasting rules a function of the relative profitability of these rules, i.e. <sup>5</sup>:

$$w_{c,t} = \frac{\exp [\gamma \pi'_{c,t-1}]}{\exp [\gamma \pi'_{c,t-1}] + \exp [\gamma \pi'_{f,t-1}]} \quad (10)$$

$$w_{f,t} = \frac{\exp [\gamma \pi'_{f,t-1}]}{\exp [\gamma \pi'_{c,t-1}] + \exp [\gamma \pi'_{f,t-1}]} \quad (11)$$

where  $\pi'_{c,t-1}$  and  $\pi'_{f,t-1}$  are the risk adjusted net profits made by chartists' and fundamentalists' forecasting the exchange rate in period  $t - 1$ , i.e.  $\pi'_{c,t-1} = \pi_{c,t-1} - \mu\sigma_{c,t-1}^2$  and  $\pi'_{f,t-1} = \pi_{f,t-1} - \mu\sigma_{f,t-1}^2$ .

Equations 10 and 11 can be interpreted as follows. When the risk adjusted profits of the chartists' rule increases relative to the risk adjusted net profits of the fundamentalists' rule, then the share of agents who use chartist rules increases in period  $t$  increases, and vice versa. This parameter  $\gamma$  measures the rate with which the chartists and fundamentalists revise their forecasting rules. With an increasing  $\gamma$  agents revise their forecasts very frequently. In the limit when  $\gamma$  goes to infinity agents revise the forecasting rules instantaneously. When  $\gamma$  is low, chartists and fundamentalists revise their forecasts relatively slowly. When  $\gamma$  is equal to zero they do not revise their rules. In the latter case the fraction of chartists and fundamentalists is constant and equal to 0.5. Thus,  $\gamma$  is a measure of inertia in the decision to switch to the more profitable rule. As will be seen, this parameter is of great importance in generating bubbles.

Chartists and fundamentalists make a profit when they correctly forecast the direction of the exchange rate movement. They make a loss if they wrongly predict the direction of its movements. The profit (the loss) they make equals the one-period return of investing \$1.

### 3 Solution of the model

In this section we investigate the properties of the solution of the model. We first study the deterministic solution of the model. This will allow us to analyse the

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<sup>5</sup>This specification of the decision rule is often used in discrete choice models. For an application in the market for differentiated products see Anderson, de Palma, and Thisse(1992). The idea has also been applied in financial markets, by Brock and Hommes (1997) and by Lux(1998).

characteristics of the solution that are not clouded by exogenous noise. Then, we analyse the stochastic version of the model, i.e. we introduce stochastics in the exogenous variables. We use simulation techniques since the non-linearities do not allow for a simple analytical solution. We select "reasonable" values of the parameters, i.e. those that come close to empirically observed values. As we will show later these are also parameter values for which the model replicates the observed statistical properties of exchange rate movements. We will also subject such results to an extensive sensitivity analysis.

We start with an analysis of the deterministic model. In figure 1 we show the solutions of the exchange rate for different initial conditions. These are fixed-point solutions (attractors). We plot such solutions as a function of the different initial conditions<sup>6</sup>. On the horizontal axis we set out the different initial conditions. These are initial shocks to the deterministic system. The vertical axis shows the solutions corresponding to these different initial conditions. The fundamental exchange rate was normalized to 0. We find two types of fixed point solutions. First, for small disturbances in the initial conditions the fixed point solutions coincide with the fundamental exchange rate. We call these solutions the *fundamental solutions*. Second, for large disturbances in the initial conditions, the fixed point solutions diverge from the fundamental. We will call these attractors, *bubble attractors*. It will become clear why we label these attractors in this way. The larger is the initial shock (the noise) the farther the fixed points are removed from the fundamental exchange rate. The border between these two types of fixed points is characterised by discontinuities. This has the implication that in the neighborhood of the border a small change in the initial condition (the noise) can have a large effect on the solution. We return to this issue.

The different nature of these two types of fixed point attractors can also be seen from an analysis of the chartists' weights that correspond to these different fixed point attractors. We show these chartists weight as a function of the initial conditions in figure ??.

We find, first, that for small initial disturbances the chartists' weight converges to 50% of the market. Thus when the exchange rate converges to the fundamental rate, the weight of the chartists and the fundamentalists are equal to 50%. For large initial disturbances, however, the chartists' weight converges to 1. Thus, when the chartists take over the whole market, the exchange rate converges to a bubble attractor. The meaning of a bubble attractor can now be understood better. It is an exchange rate equilibrium that is reached when the number of fundamentalists has become sufficiently small (the number of chartists has become sufficiently large) so as to eliminate the effect of the mean reversion dynamics. It will be made clearer in the next section why fundamentalists drop out of the market. Here it suffices to understand that such equilibria exist. It is important to see that these bubble attractors are fixed point solutions. Once we reach them, the exchange rate is constant. The chartists' expectations are then

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<sup>6</sup>These fixed point solutions of the exchange rate were obtained by running simulations of 100,000 periods. Each time the exchange rate converged to a fixed point.

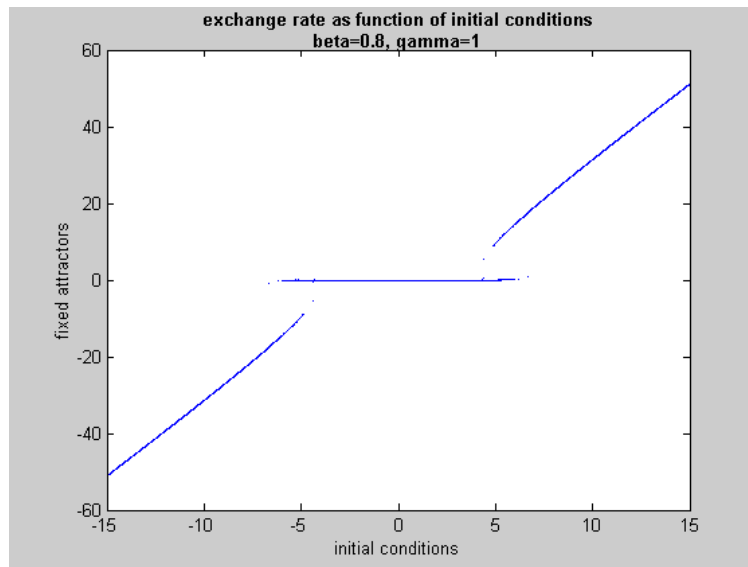


Figure 1:

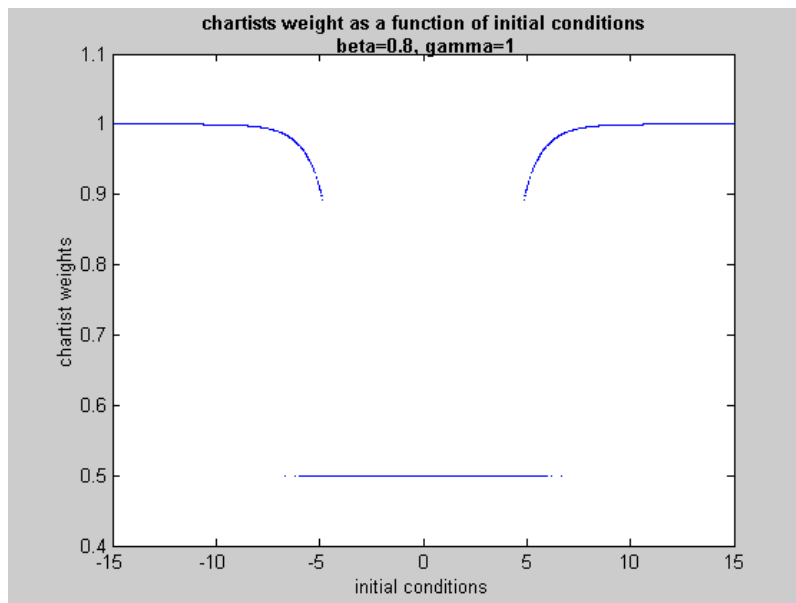


Figure 2:

model consistent, i.e. chartists who extrapolate the past movements, forecast no change. At the same time, since the fundamentalists have left the market, there is no force acting to bring back the exchange rate to its fundamental value. Thus two types of equilibria exist: a fundamental equilibrium where chartists and fundamentalists co-exist, and a bubble equilibrium where the chartists have crowded out the fundamentalists<sup>7</sup>. In both cases, the expectations of the agents in the model are consistent with the model's outcome.

These two types of equilibria differ in another respect. The *fundamental* equilibrium can be reached from many different initial conditions. It is locally stable, i.e. after small disturbances the system returns to the same (fundamental) attractor. In contrast there is one and only one initial condition that will lead to a particular *bubble* equilibrium. This implies that a small disturbance leads to a displacement of the bubble solution. Note also that the border between these two types of equilibria is characterized by discontinuities and complexity, i.e. small disturbances can lead to either a fundamental or a bubble equilibrium. We show the nature of this complexity in the following two figures. These present two successive enlargements of figure 1 around the initial condition 5, where the fundamental and bubble equilibria appear to overlap. We observe that small changes in the initial conditions can switch the equilibrium point from a fundamental to a bubble equilibrium and vice versa. The second enlargement shows the replicating nature of the enlargements. The "holes" in the line segment describing the fundamental equilibrium in the first enlargement are filled up with fundamental equilibria in the second enlargement. This also suggests that these fixed point attractors are surrounded by basins of attraction that are separated by complex borders. The implication of this is that in the border region between fundamental and bubble equilibria there is sensitivity to initial conditions, i.e. trivially small shocks can lead to switches in the nature of the equilibrium to which the exchange rate is attracted.

The previous results also illustrate the nature of the informational problem rational agents face. Even if agents know the structure of the model, they still would have to know the initial conditions to predict whether the exchange rate will be attracted by a fundamental or a (very different) bubble equilibrium. From figure it may appear that if the initial condition is between 5.45 and 5.46, this is enough information to make a precise prediction, i.e. that the exchange rate is attracted by a bubble equilibrium. However as the next figure shows this is not enough information. When this interval is enlarged, there appear to be many initial conditions that lead to a fundamental equilibrium. Focusing on the interval 5.455 and 5.456 we again see that all initial conditions appear to lead to a bubble equilibrium. However, enlargement of this new interval (not shown here) reveals new initial conditions that lead to a fundamental equilibrium. This goes on ad infinitum. Thus, agents need infinite precision in their knowledge of the initial conditions to be able to predict whether the exchange rate will move

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<sup>7</sup>Note that the intermediate points, i.e. when chartists' weight is less than 1 the solution has not converged yet to fixed points. Fundamentalists hold a very small share in the market which exerts some mean reverting force. However their influence is offset by the chartists pressure. In figure xxx the simulation results are for T=100000.

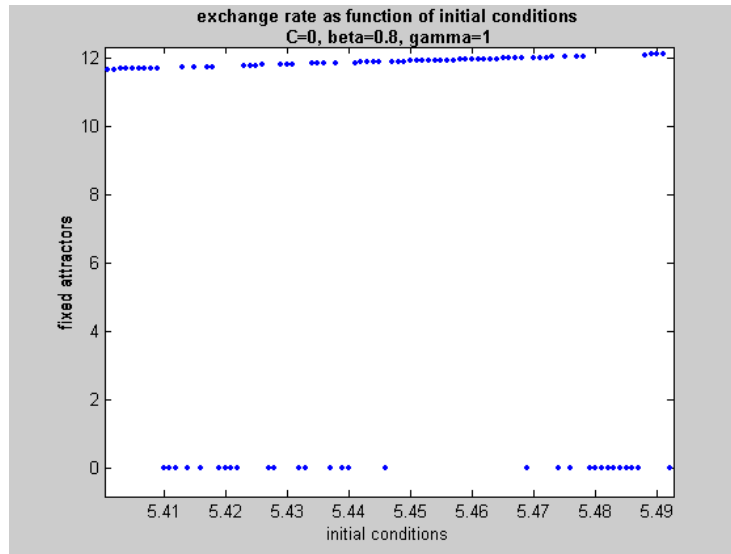


Figure 3:

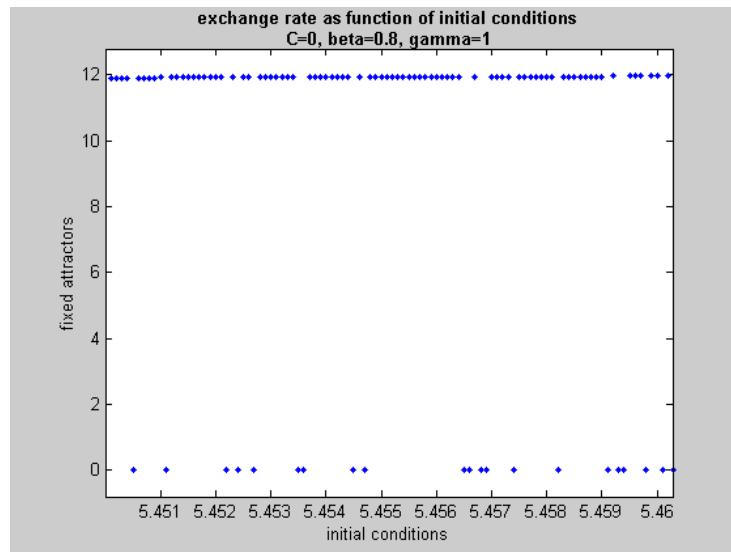


Figure 4:

to a fundamental or a bubble equilibrium. Put differently, infinitesimally small errors in computing the initial conditions lead to very large errors in predicting the exchange rate. We conclude that even in the very simple model we developed here and in the absence of any stochastic shocks, agents face severe informational problems.

## 4 The anatomy of bubbles and crashes

In the previous section we identified the existence of two different types of fixed point solutions, i.e. a fundamental solution characterised by the fact that the exchange rate converges to its fundamental value while chartists and fundamentalists "co-habitate", and a bubble solution in which the exchange rate deviates from its fundamental value and in which chartists dominate the market. In this section we show that in combination with stochastic shocks in the fundamental exchange rate these features of the model lead to the emergence of bubbles and crashes.

The way we proceed is to calibrate the model in such a way that it replicates the statistical properties of observed exchange rate movements. We describe this procedure in section ???. Here we present the results of simulations in the time domain using this calibrated model. (In section 5 we present a sensitivity analysis).

We start by presenting a case study of a typical bubble and crash scenario as produced by the stochastic version of the model. In the next section we will analyse more systematically the factors that determine the frequency with which such bubbles and crashes occur. Figure 5 top panel shows the exchange rate and its fundamental value in the time domain; the bottom panel shows the weight of the chartists in the same time domain. These two pictures allow us to analyse a number of common features of a typical endogenously generated bubble and crashes in a stochastic environment.

First, once a bubble emerges, it sets in motion bandwagon effects. As the exchange rate moves steadily in one direction, the use of extrapolative forecasting rules becomes more profitable, thereby attracting more chartists in the market. This is clearly visible from a comparison of the bottom panel with the top panel of figure 5. We observe that the upward movement in the exchange rate coincides with an increase in the weight of chartists in the market. We have checked this feature in many bubbles produced by the model. In appendix 2 we show another example of a bubble, and we present the results of a causality test which shows that the exchange rate leads the weight of chartists during a bubble and the subsequent crash. Thus, typically a bubble starts after the exchange rate has moved in one direction, thereby attracting extrapolating chartists which in turn reinforces the exchange rate movement.

Second, a sustained upward (downward) movement of the exchange rate will not develop into a full scale bubble if at some point the market does not get sufficiently dominated by the chartists. As can be seen figure 5 at the height of the bubble the chartists have almost 100% of the market. Put differently,

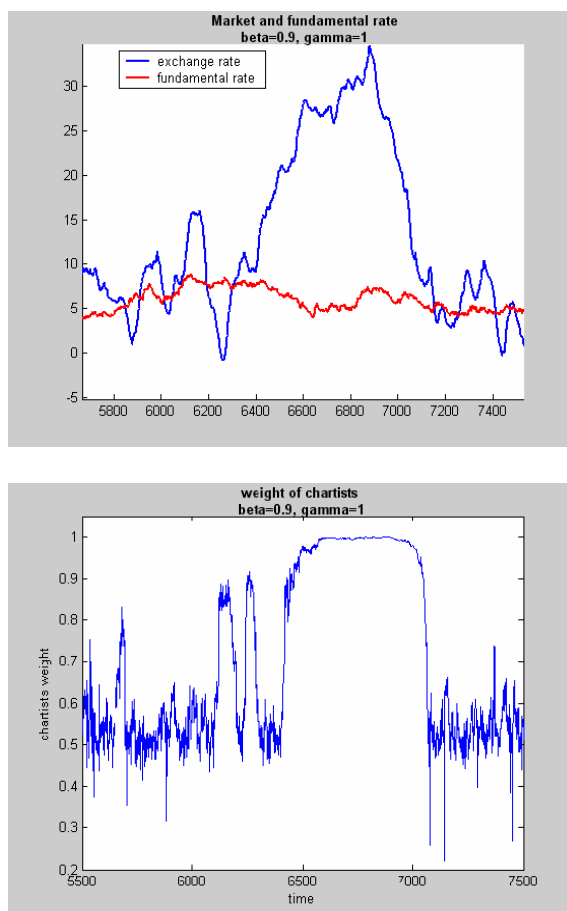


Figure 5:

an essential characteristic of a bubble is that at some point almost nobody is willing to take a contrarian fundamentalist view. The market is then dominated by agents who extrapolate the bubble into the future. This raises the question of why fundamentalists do not take an opposite position thereby preventing the bubble from developing. After all, the larger the deviation of the exchange rate from the fundamental the more the fundamentalists expect to make profit from selling the foreign currency. Yet they do not, and massively leave the marketplace to the chartists. The reason why they do so, is that during the bubble phase the profitability of chartism increases dramatically precisely because so many chartists enter the market thereby pushing the exchange rate up and making chartism more profitable (see James (2003)). In addition, during the bubbles phase fundamentalists make large forecasting errors, reducing their "appetite"

for using fundamentalists forecasting rules. Put differently, during the bubbles phase the riskiness of taking a fundamentalist position (as measured by forecast errors) increases dramatically relative to the riskiness of extrapolative forecasting. As a result, fundamental investors who are continuously acting against the trend will tend to drop out. There is therefore a self-fulfilling dynamics in the profitability of chartism and losses for the fundamentalists.

The limit of this dynamics is reached when chartists have crowded out the fundamentalists. We arrive at our next characteristics of the bubble-crash dynamics. When the chartists' share is close to 100% the self-reinforcing upward movement in the exchange rate and in profitability slows down, increasing the expected relative profitability of fundamentalists. This is so because while the bubble developed, the *expected* profits from fundamentalism also increased. However, these were overwhelmed by the self-fulfilling profitability of chartism. When the latter tends to slow down, fundamentalism becomes attractive again. An exogenous shock, e.g. a shock in the fundamental, can then trigger a fast decline in the share of chartism, back to its normal level of a tranquil market. A crash is set in motion. We come back to issue of why a crash must necessarily occur in this model.

The dynamics of bubbles and crashes we obtain in our simulated data is asymmetric, i.e. bubbles are relatively slow and crashes relatively rapid. An intuitive explanation of this result is that during a bubble chartists' and fundamentalists' rules push the exchange rate in two different directions, i.e. the positive feedback from chartists and the negative feedback from fundamentalists have the effect of slowing down the build-up of a bubble. In a crash the fundamentalists' mean reverting force is reinforced by the chartists' extrapolating behaviour. As a consequence, the speed of a crash is higher than the speed with which a bubble arises.

This asymmetry between bubbles and crashes is a well-known empirical phenomenon in financial markets (see Sornette(2003)). In figure 6 we present the DEM-USD for the period 1980-1987, which is a remarkable example of a bubble in foreign exchange markets. As it can be seen from figure 6 the upward movement in the DEM-USD exchange rate is gradual and builds up momentum until a sudden and much faster crash occurs which brings the exchange rate back to its value of tranquil periods. Our model provides a simple explanation for this empirical phenomenon <sup>8</sup>.

## 5 Sensitivity analysis: the deterministic model

In this section we subject the deterministic model to a sensitivity analysis. This will allow us to describe how the space of fundamental and bubble equilibria is

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<sup>8</sup>Note the contrast with rational expectations models of bubbles and crashes. These predict that bubbles and crashes are symmetric (Blanchard(1979) and Blanchard&Watson (1982)). Moreover, the symmetry of bubbles and crashes neglects the time scale dynamics in which a long term change is an accumulation of short term changes. Thus, the symmetry property in foreign exchange markets is an approximation which holds only in the (very) short-run (see Johansen and Sornette (1999)).

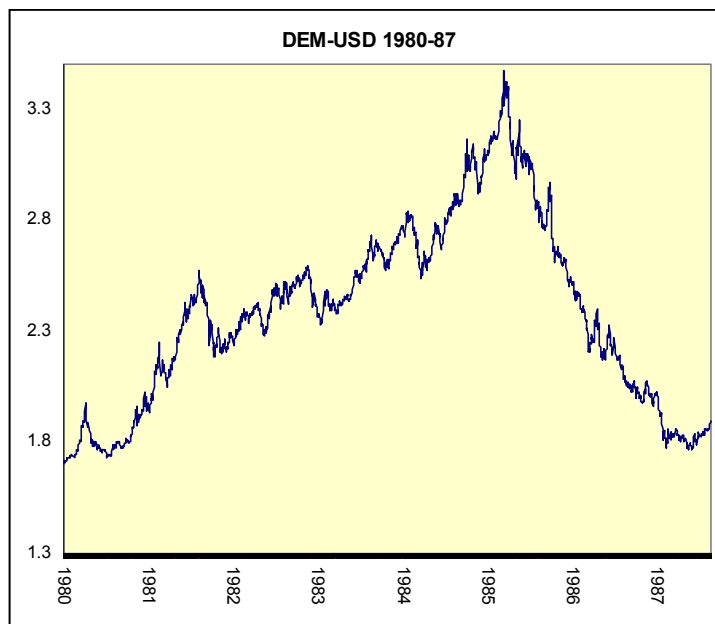


Figure 6:

affected by different values of the parameters of the model. In this section we concentrate on two parameters, i.e.  $\beta$  (the extrapolation parameter of chartists) and  $\gamma$  (the sensitivity of chartists and fundamentalists to relative profitability).

### 5.1 Sensitivity with respect to $\beta$

We show the result of a sensitivity analysis with respect to  $\beta$  in figure xxz, which is a three-dimensional version of figure 7. The fixed attractors (i.e. the solutions of the exchange rate) are shown on the vertical axis. The initial conditions are shown on the x-axis and the different values for  $\beta$  on the z-axis. Thus, the two-dimensional figure 1 in section 3 is a 'slice' of figure 7 obtained for one particular value of  $\beta$  (0.8 in figure 1).

We observe that for sufficiently low values of  $\beta$  we obtain only fundamental equilibria whatever the initial conditions. As  $\beta$  increases the plane collecting the fundamental equilibria narrows. At the same time, the space taken by the bubble equilibria increases, and these bubble equilibria tend to increasingly diverge from the fundamental equilibria. Thus, as the extrapolation parameter increases, the smaller and smaller shocks in the initial conditions will push the exchange rate into the space of bubble equilibria. Put differently, as  $\beta$  increases, the probability of obtaining a bubble equilibrium increases.

Note also that the boundary between the fundamental equilibrium and the

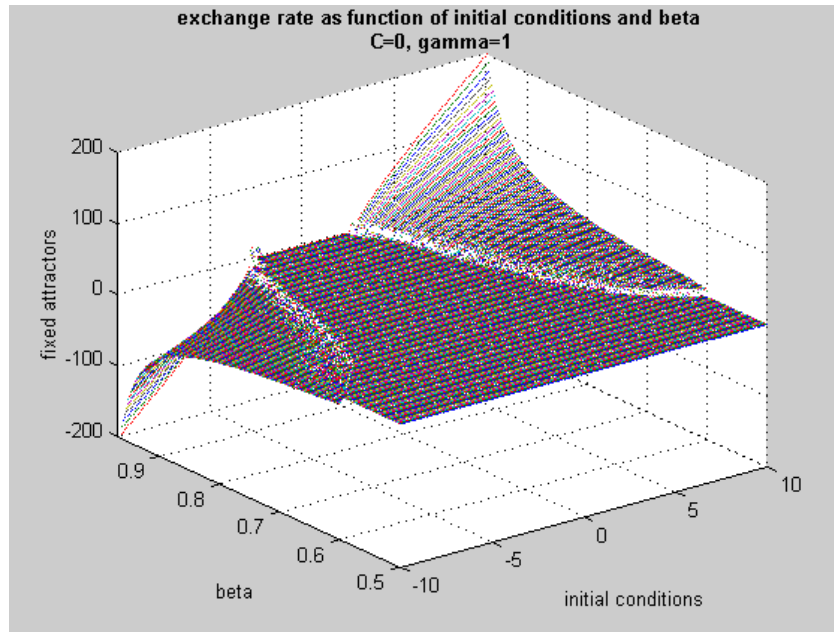


Figure 7:

bubble equilibria is a complex one, i.e. for any given initial condition there is not just one  $\beta$  that forms the boundary between a fundamental and bubble equilibrium. The boundary has a fractal dimension. We return to this issue in section 9.

## 5.2 Sensitivity with respect to $\gamma$

The parameter  $\gamma$  is equally important to determine whether fundamental or bubble equilibria will prevail. We show its importance in figure 8 which presents a similar three-dimensional figure relating the fixed attractors to both the initial conditions and the values of  $\gamma$ . We find that for  $\gamma = 0$  or close to 0, all equilibria are fundamental ones. Thus when agents are not sensitive to changing profitability of forecasting rules, the exchange rate will always converge to the fundamental equilibrium whatever the initial condition. As  $\gamma$  increases, the space of fundamental equilibria shrinks. With sufficiently high values of  $\gamma$ , small initial disturbances (noise) are sufficient to push the exchange rate into a bubble equilibrium. Put differently, as  $\gamma$  increases, the probability of obtaining a bubble equilibrium increases. Finally, as in the case of  $\beta$ , we also observe that the boundary between the bubble and fundamental equilibria is complex.

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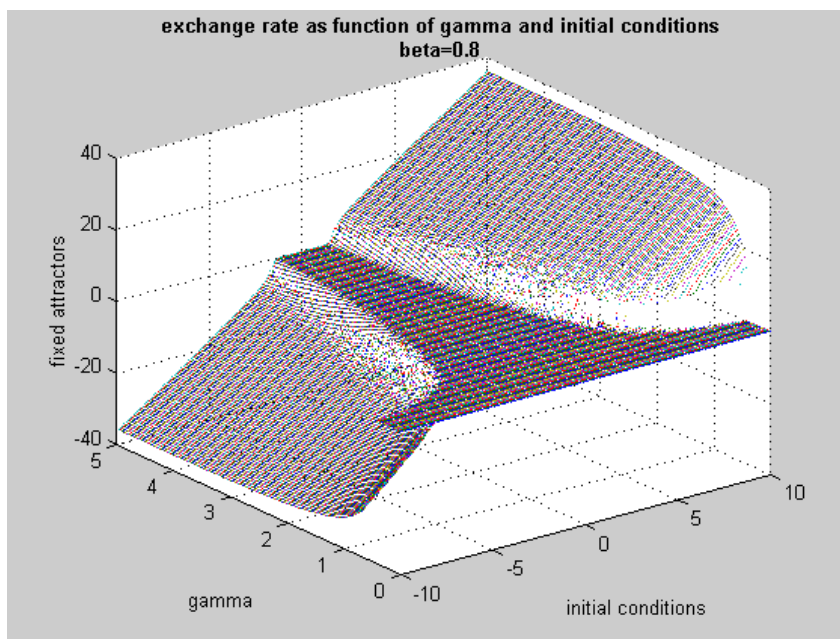


Figure 8:

## 6 The model with an endogenous current account

In the previous sections we used a model in which the exchange rate is often disconnected from its fundamentals. This happens despite the fact that the fundamentalists know the fundamental value of the exchange rate and use that information to forecast the future exchange rate. The selffulfilling expectations of the chartists regularly crowd out the fundamentalists from the market thereby weakening the mean reversion forces in the model.

One drawback of the model is that when the fundamentalists leave the market, there is no mean reverting force present anymore because the goods market is kept outside the model. Incorporating the goods market provides an independent channel through which the exchange rate can be forced back to its equilibrium even if the fundamentalists are temporarily absent. In this section we introduce the goods market into the model and allow for an interaction between the goods market and the foreign exchange market.

We use the same model as in the previous sections, except that we now endogenize the current account. It will be remembered that the current account determines the net supply of foreign assets in the model. Thus we define the current account as

$$\Delta Z_t = X_t \tag{12}$$

The current account consists of the trade balance and the net income from net foreign assets. At this stage of the analysis we will concentrate on the role of the trade balance and we disregard the role of the net income from foreign assets. Thus, we will set the current account equal to the trade balance.

There is a large literature on the determinants of the trade balance. Here we focus on the role of the exchange rate. We postulate that an increase (decline) of the exchange rate leads to an improvement (deterioration) of the trade balance, and thus of the current account, *ceteris paribus*. The reason is that an increase in the exchange rate (a depreciation) stimulates exports and discourages imports. The opposite holds for a decline in the exchange rate (an appreciation). This leads us to postulate the following relationship between the trade account (the current account) and the exchange rate:

$$X_t = \rho_x X_{t-1} + (1 - \rho_x)\epsilon(s_{t-1} - s_{t-2}) \quad (13)$$

The sensitivity of the current account with respect to the exchange rate is given by the parameter  $\epsilon \geq 0$ . This parameter synthesises the reactions of exporters and importers to changes in the exchange rate. It can easily be derived from a model of the export and import markets. We will call this parameter, the trade elasticity, or "elasticity" for short. We also assume that the reactions of the exporters and importers is not instantaneous. The speed with which the trade account adjusts to the exchange rate is given by  $\rho_x$ . Note that  $0 \leq \rho_x \leq 1$ . When  $\rho_x$  is close to 1, there is a lot of inertia in the trade account and the adjustment to the exchange rate is very slow. The opposite holds when  $\rho_x$  is close to 0. Equation 13 can also be rewritten as follows:

$$X_t = \epsilon(1 - \rho_x) \sum_{i=1}^{\infty} \rho_x^{i-1} (s_{t-i} - s_{t-i-1}) \quad (14)$$

i.e., the trade account reacts to all past exchange rates with geometrically declining weights.

Equilibrium in the current account is defined as  $X_t = 0$ , which implies that  $s_{t-1} = s_{t-2}$ . We define the exchange rate that achieves this equilibrium  $s^*$  and call it the fundamental exchange rate. Thus, we have an equivalence between current account equilibrium and fundamental equilibrium of the exchange rate.

We will now assume that the fundamentalists are aware of this. It will be remembered that the forecasting rule of the fundamentalists was assumed to be

$$E_t^f(\Delta s_{t+1}) = -\psi(s_{t-1} - s_{t-1}^*)$$

Taking the equivalence between current account and fundamental equilibrium, the fundamentalists set

$$\psi = -\epsilon(1 - \rho_x) \quad (15)$$

i.e., when the exchange rate deviates from its fundamental value, the fundamentalists know that the current account dynamics will ensure that the speed of adjustment of the exchange rate towards equilibrium is given by equation 15.

We now have all the elements to solve the model. We proceed in the same way as before, i.e. we first analyse the deterministic solution, and then we add stochastics to the model.

## 6.1 Solution of the model

We first set out the solutions for the exchange rate (attractors) as a function of the initial conditions and the parameter  $\gamma$ . We set the elasticity  $\epsilon = 0.5$  (We will perform additional sensitivity analysis in the next section). We show the results in figure 9. The contrast with the results obtained when the current account was exogenous is important. We observe that in the model with exogeneous current account (see equation ??) we obtain bubble equilibria for sufficiently large initial shocks and for sufficiently large values of  $\gamma$ . This is no more the case when the current account is endogenous. We do not find bubble equilibria anymore. For low values of  $\gamma$ , we find that whatever the initial shock, the exchange rate converges to the fundamental exchange rate (which as will be remembered was normalised to 0). For large values of  $\gamma$ , however, we find that for certain initial shocks the exchange rate is attracted by a "strange attractor". (In appendix we show an example of such a strange attractor for a particular parameter configuration). Note, however, that these chaotic attractors are centered around the fundamental exchange rate. Thus the endogeneity of the current account transforms the nature of the dynamics of the exchange rate. On the one hand, it eliminates the bubble equilibria. On the other hand, it introduces the possibility of chaotic dynamics, which is responsible for short-term volatility in the exchange rate.

## 6.2 Sensitivity analysis: the importance of the elasticity

In this section we present the result of applying an extensive sensitivity analysis. We start with an analysis of the importance of the sensitivity of the current account to the exchange rate (the elasticity,  $\epsilon$ ). We show the results in figure 10. It plots the exchange rate solutions (attractors), as a function of the elasticity and the initial conditions. When the elasticity is zero (or close to zero), we obtain both bubble and fundamental attractors depending on the initial conditions. This result was also found when the current account was exogenous. When the elasticity exceeds zero by a sufficient margin the bubble equilibria disappear. We then only have fundamental equilibria. However, for particular values of the initial conditions the exchange rate moves within the confines of a chaotic attractor centered around the fundamental value of the exchange rate. It is also noteworthy that zones of chaotic attractors and fixed point attractors alternate. This phenomenon is sometimes called "intermittency".

## 6.3 Sensitivity analysis: the importance of $\beta$

Finally, we performed a similar sensitivity analysis with respect to  $\beta$ , the chartists' extrapolation parameter. Figure 11 shows such a sensitivity analysis. The re-

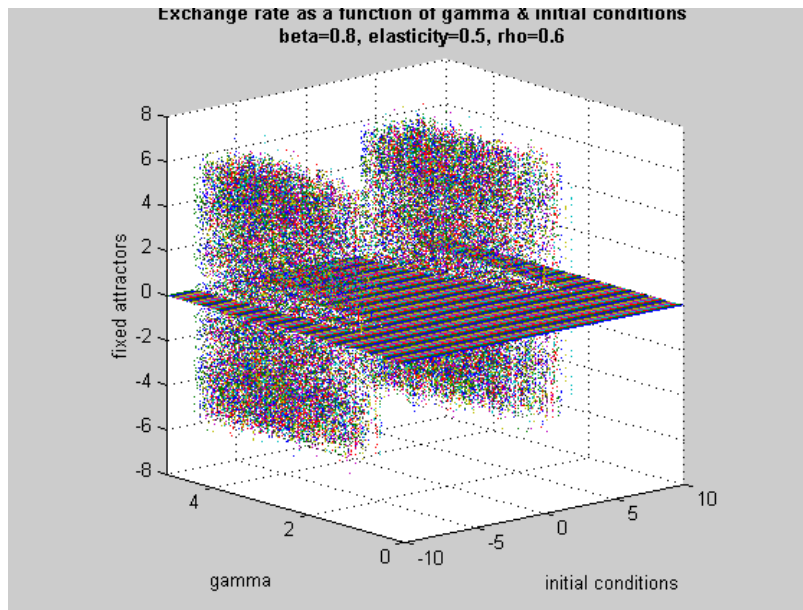


Figure 9:

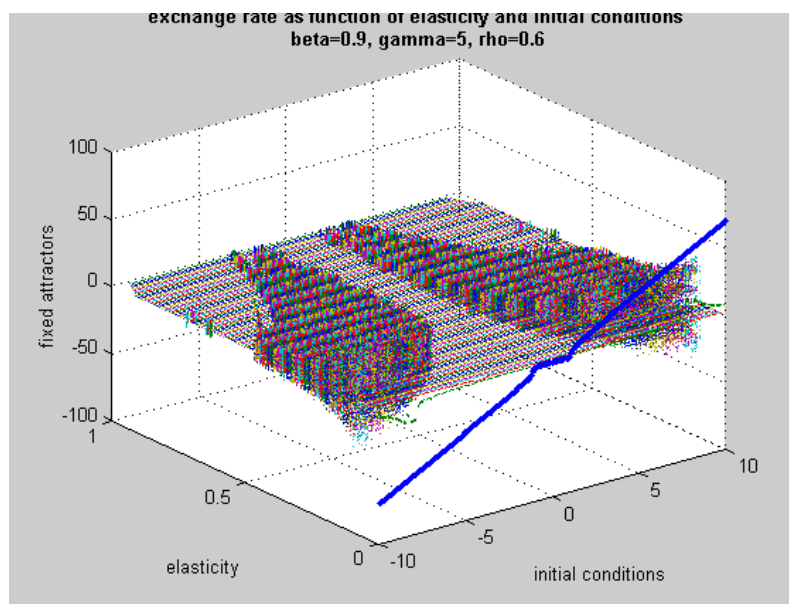


Figure 10:

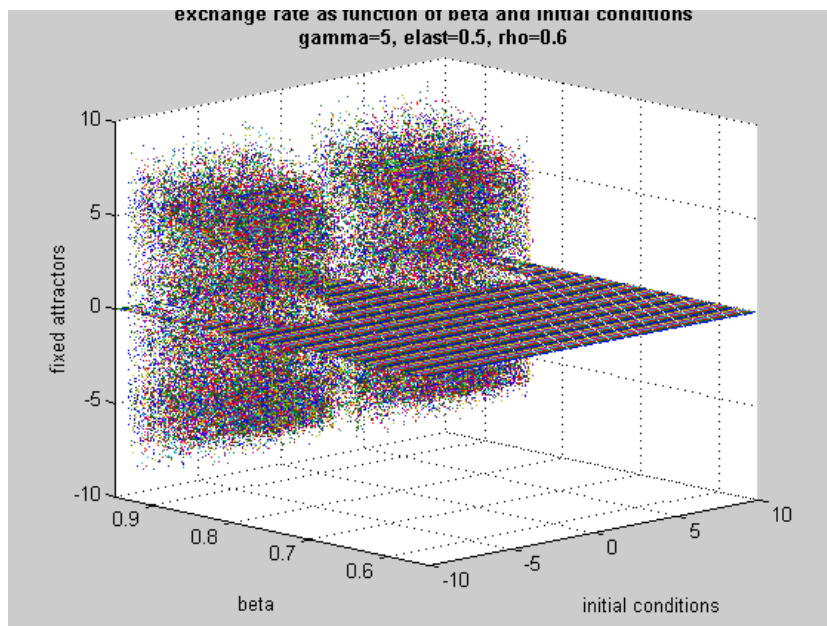


Figure 11:

sults are very similar to those discussed in the previous paragraphs. It can be seen that with an increasing  $\beta$ , we move from fixed point attractors into a zone of chaotic attractors. This contrasts with the case of an exogenous current account. In that case, increases in  $\beta$  lead to bubble equilibria.

We conclude from this sensitivity analysis that the endogeneity of the current account changes the dynamics of the exchange rate profoundly. In general, we find that in a model without a feedback from the current account, bubbles and crashes are an endemic feature of the exchange rate dynamics. When, however, the current account reacts to exchange rate changes, this dynamics is changed. The bubbles and crashes dynamics is weakened. Instead a complex, sometimes, chaotic dynamics takes the place of the bubbles dynamics.

## 7 The model with transactions costs

In this section we introduce transaction costs. There is an increasing body of theoretical literature stressing the importance of transactions costs in the goods markets as a source of non-linearity in the determination of the exchange rate (Dumas(1992), Sercu, Uppal and Van Hulle(1995), Obstfeld and Rogoff(2000)). The importance of transaction costs in the goods markets has also been confirmed empirically (Taylor, Peel, and Sarno(2001), Kilian and Taylor(2001)). In addition, several recent empirical studies report the continued existence of

large price differentials for the same traded goods across borders (see Haskel and Wolf(2001) and Engel and Rogers(1995)). These authors document price differentials of up to 40% for identical products in different countries. This indicates that producers apply ‘pricing to market’. Such pricing strategies, however, can only be applied successfully if transaction costs prevent arbitrage. Thus, the large observed price differentials suggest that transactions costs for traded goods are large. In addition, for many services, which are non-traded goods, transactions costs are even higher. (See Obstfeld and Rogoff(2001) who argue that transactions costs are key to understanding the major puzzles in international economics).

We assume that the existence of transactions costs creates a band of inaction, i.e. when the deviation of the exchange rate from its fundamental is lower than the transactions costs, goods arbitrage is not profitable. Put differently, when the exchange rate is within this band, export and import decisions are not sensitive to changes in the exchange rates (as long as these changes do not move the exchange rate outside the band). We, therefore, write that

$$X_t = \rho_x X_{t-1} + (1 - \rho_x)\epsilon(s_{t-1} - s_{t-2}) \quad (16)$$

holds if  $|s_{t-1} - s_{t-1}^*| \geq C$ , where  $C$  is the transaction cost in the goods market assumed to be of the iceberg type. Otherwise  $X_t = 0$ .(In the stochastic version of the model we will then set  $X_t = \varepsilon_t$ , where  $\varepsilon_t$  is white noise).

In addition, we assume that fundamentalists are aware of the existence of this band of inaction. As a result, when the exchange rate deviation from its fundamental value is larger than the transaction costs  $C$ , then the fundamentalists follow the forecasting rule as in equation 7. More formally,

when  $|s_{t-1} - s_{t-1}^*| \geq C$  holds, then equation 7 applies<sup>9</sup>.

However when the exchange rate deviations from the fundamental value are smaller than the transaction costs in the goods markets, there is no mechanism that drives the exchange rate towards its equilibrium value. As a result, fundamentalists expect the changes in the exchange rate to follow a white noise process and the best they can do is to forecast no change. More formally,

when  $|s_t - s_t^*| < C$ , then  $E_t^f(\Delta s_{t+1}) = 0$  or  $E_t^f(\Delta s_{t+1}) = \eta_t$ , and  $\eta_t$  is white noise in the stochastic version of the model.

## 7.1 Solution of the model

We proceed as before, i.e. we solve the deterministic part of the model, and present the attractors as a function the parameter  $\beta$  and of the initial conditions. We have selected two values of the elasticity,  $\epsilon$ , of the current account, i.e.  $\epsilon = 0$  and  $\epsilon = 0.5$ . The other parameter combinations are the same as in

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<sup>9</sup>Note that since  $\psi < \infty$  market inefficiencies other than transaction costs continue to play a role when the exchange rate moves outside the transaction costs band. As a result, these inefficiencies prevent the exchange rate from adjusting instantaneously.

the basic model (see figure 1). We show the results in figures 12 and 13. From figure 12, where we have set  $\epsilon = 0$ , we observe that the existence of transactions costs reduces the space of fundamental equilibria relative to the space of bubble equilibria (compare figure 12 with figure 7 in section 5). In addition, there are more discontinuities in the borders between fundamental and bubble equilibria, making the emergence of bubbles less predictable. Thus, the existence of transactions costs increases the probability of bubbles, and these are less predictable. The intuition of this result is that when the exchange rate is in the transaction cost band, fundamentalists stop making forecasts because they know that there is no mean reversion at work, the exchange rate process can more easily be gripped by extrapolating forecasting. This then triggers bubbles.

In figure 13 we show the results when  $\epsilon = 0.5$ . Bubble equilibria now have disappeared. Instead, the complexity of the space of equilibria has increased significantly. For low values of  $\beta$  all the equilibria are contained within the transaction cost band (see appendix 3).

One major aspect of these results is that complexity has increased. This can be seen by the fact that exchange rate solutions are more sensitive to initial conditions. In the absence of transaction costs there are large zones in which the exchange rate solution is not sensitive to initial conditions. This is the case for low  $\beta$ 's where we obtain fundamental equilibria. In the presence of transactions costs this is no longer the case. We find sensitivity to initial conditions for all values of  $\beta$ . We show this by "cutting a slice" from figure 13 at  $\beta = 0.9$ . We show the result in figure 14. We observe that the attractors are arranged in a complex manner. Some attractors are located on a continuous line, others are spread around in a discontinuous way. As a result, small changes in the initial conditions can lead to large displacements in the attractors. As it will be shown, this feature of the attractors creates a complex dynamics in a stochastic environment characterised by sensitivity to initial conditions. Note that in the simulations reported in figure 14 there is no chaotic dynamics, as each attractor is a fixed point. These fixed point attractors are separated by basins of attraction with complex features. These are responsible for the sensitivity to initial conditions.

## 7.2 The model in a stochastic environment

We now proceed in solving the model in a stochastic environment. We show the results in the time domain for a particular combinations of parameters that do not produce bubbles and crashes, i.e. with an elasticity of the current account that is sufficiently large ( $\epsilon = 0.5$ ). We show the results in figure 15. Figure 15 shows the simulated exchange rate together with the fundamental exchange rate. We observe that the exchange rate is disconnected most of the time from its underlying fundamental. This feature is present despite the absence of bubble equilibria. Note that sometimes the exchange rate dynamics has the appearance of bubbles and crashes although there is no such underlying dynamics. We will analyse the nature of this "disconnect puzzle" in greater detail when we discuss the empirical relevance of the model. We also observe that the

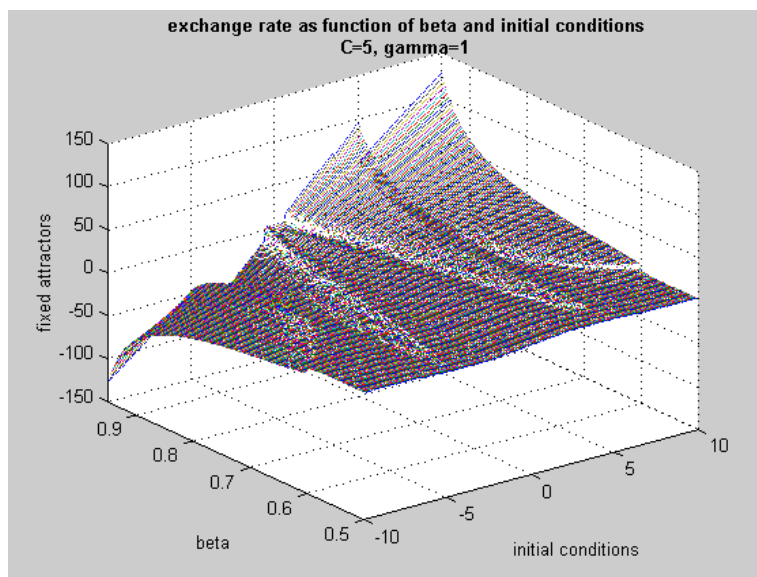


Figure 12:

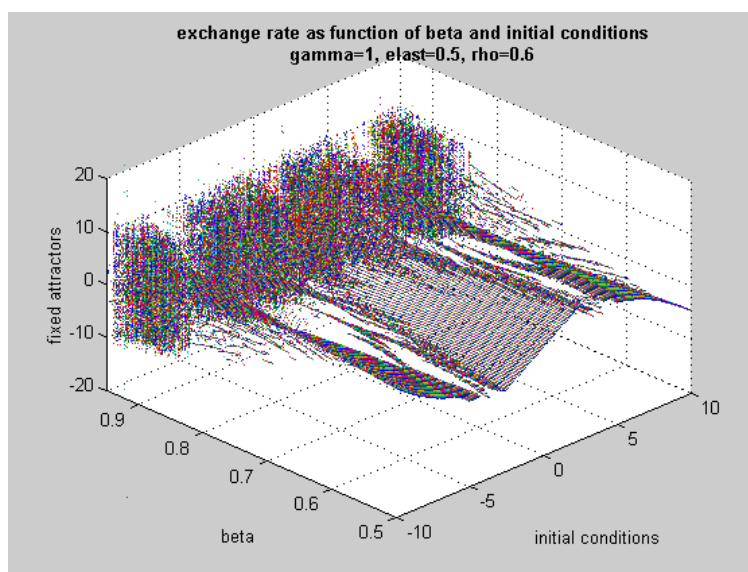


Figure 13:

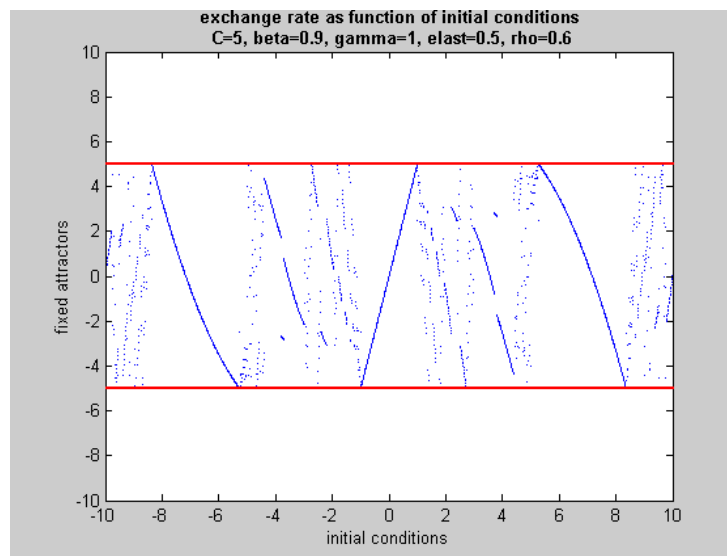


Figure 14:

short-term volatility of the simulated exchange rate appears to be significantly higher than the volatility of the underlying fundamental. This feature is related to the sensitivity to initial conditions dynamics: small changes in the underlying stochastics can lead to large displacements of the attractor. This sensitivity to initial conditions can also be illustrated in another way. We simulated the model with two different initial conditions in the exchange rate. In one simulation we set the initial exchange rate equal to 4, in the other to 4.01. All the rest is kept identical in the two simulations (the parameters, the underlying stochastics driving the exogenous variables). The results of figure 16 illustrate the power of the sensitivity to initial conditions. A small disturbance in the initial exchange rate leads to sustained deviations between the two exchange rates, despite the fact that the underlying fundamentals are identical. It appears that the two exchange rates follow a different "history".

One can conclude this section as follows. Without a feedback from the current account the model produces bubbles and crashes. These bubbles and crashes tend to disappear when we allow the current account to react to exchange rate changes. The introduction of transactions costs does not change this result. However, transactions costs create a new, and complex dynamics whereby the exchange rate appears to be disconnected most of the time from the underlying fundamental creating a resemblance with bubbles and crashes. In addition, by creating a band of inaction transactions costs are also responsible for the appearance of sensitivity to initial conditions that make it possible for small changes in the initial conditions to have profound effects on the future movements of the exchange rate. Thus, in such a world, history seems to matter.

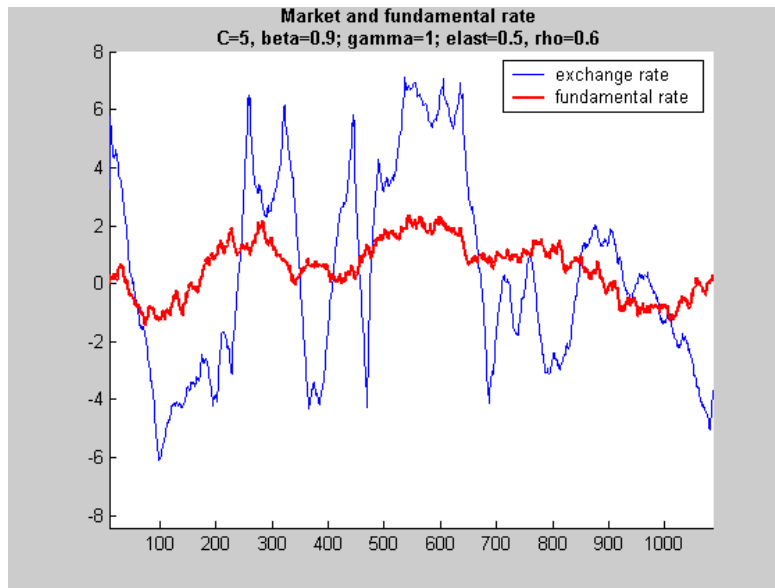


Figure 15:

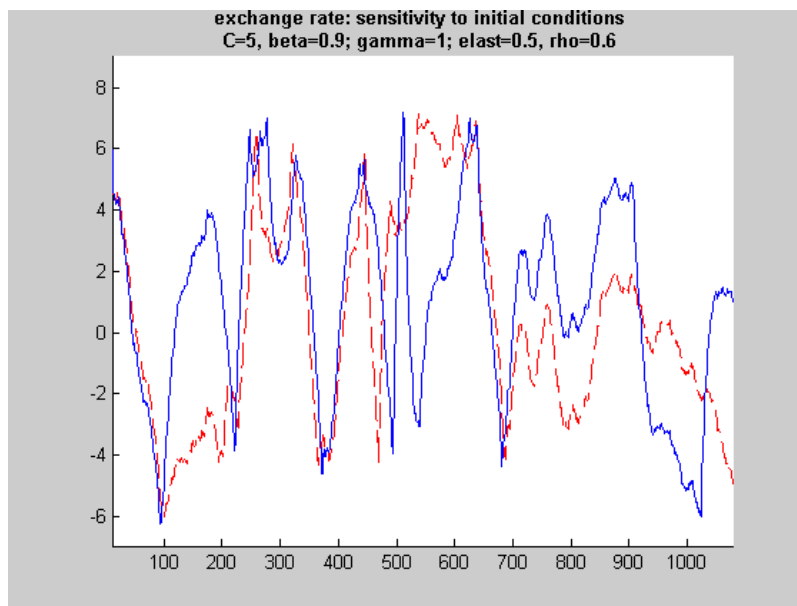


Figure 16:

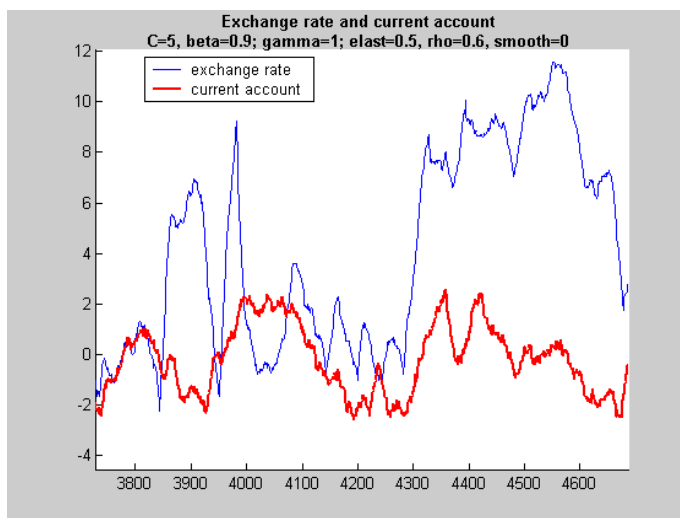


Figure 17:

## 8 More on the disconnect puzzle

In the previous section we showed how the model is capable of replicating a widely observed phenomenon, i.e. that the exchange rate is often disconnected from its underlying fundamentals. In that analysis we stressed one dimension of the disconnect puzzle, i.e. the causality (or the lack of it) from the fundamental to the exchange rate. There is, however, another dimension to the disconnect puzzle, which relates to the causality running from the exchange rate to the current account. From the empirical evidence it appears that the exchange rate has a weak and unpredictable influence on the fundamentals, in particular on the current account. (see Obstfeld and Rogoff for a formulation of this puzzle). The version of the model in which the current account reacts endogenously to the exchange rate can be used to shed light on this puzzle. In order to do so we simulated the model assuming an elasticity of 0.5. Thus, we assume that there is a causality running from the exchange rate to the current account. We show two examples of such a simulation for different parameter configurations in figures 17 and 18. The interesting aspect of this simulation is that there are periods in which the current account is influenced by the exchange rate movements. These periods, however, alternate with other periods during which this influence is almost totally absent, making the effect of exchange rate changes on the current account very unpredictable. The intuition of this result is that during periods of turbulence, the effect of exchange rate changes on the current account is weakened. Since turbulent and tranquil periods alternate in unpredictable fashion, the effects of exchange rate changes on the current account also alternate from being predictable to becoming unpredictable.

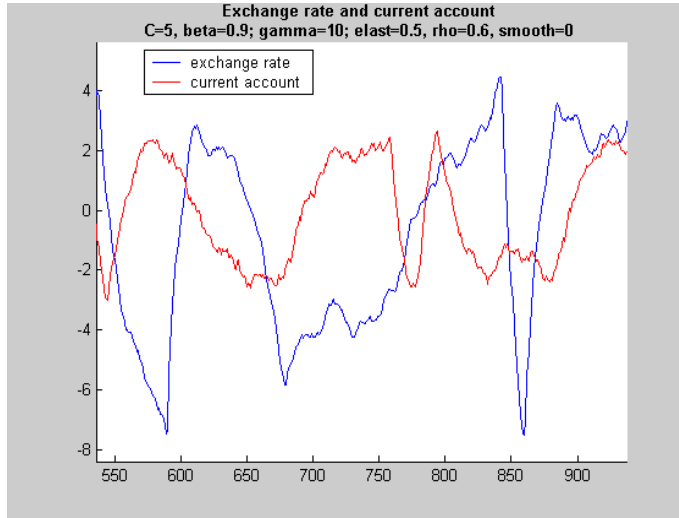


Figure 18:

In order to be more precise about the nature of the disconnect puzzle we analyzed the simulated exchange rate and current account econometrically. We first tested for cointegration of the two series, and found that in general the two series are cointegrated. We then specified a vector error correction (VEC) model in the following way:

$$\Delta s_t = \mu (s_{t-1} - \gamma X_{t-1}) + \sum_{i=1}^n \lambda_i \Delta s_{t-i} + \sum_{i=1}^n \phi_i \Delta X_{t-i} \quad (17)$$

$$\Delta X_t = \mu' (s_{t-1} - \gamma' X_{t-1}) + \sum_{i=1}^n \lambda'_i \Delta s_{t-i} + \sum_{i=1}^n \phi'_i \Delta X_{t-i} \quad (18)$$

The first term on the right hand side in both equations are the error correction terms. We have estimated this model for a broad range of parameter values. The result of estimating equation 17 for selected parameter values is presented in table 1 where we have set  $C = 5$ ,  $\beta = 0.9$ ,  $\gamma = 1$ ,  $\epsilon = 0.5$  and  $\rho_x = 0.6$  and number of lags  $n = 5$ .

We find that the error correction coefficients ( $\mu$  and  $\mu'$ ) are low in both equations. This suggests that the mean reversion towards the equilibrium exchange rate and current account takes a long time. In particular, only 0.7% and 0.6% of the adjustments take place each period. It should be noted that in the simulations we have assumed a speed of adjustment in the current account equation of 0.2 ( $\rho = 0.6$  and  $\epsilon = 0.5$ ), implying that structurally the 20% of the disequilibrium in the current account should be corrected. Instead, our model generates a much slower adjustment in the current account. This slow adjustment in the

Table 1: error correction model

Error correction		$\Delta \mathbf{s}_{t-i}$					$\Delta \mathbf{X}_{t-i}$				
$\mu$	$\gamma$	$\lambda_1$	$\lambda_2$	$\lambda_3$	$\lambda_4$	$\lambda_5$	$\varphi_1$	$\varphi_2$	$\varphi_3$	$\varphi_4$	$\varphi_5$
0.007	-3.5	0.38	0.17	0.13	0.03	-0.03	0.02	0.06	0.04	0.05	0.08
<i>2.2</i>	<i>-3.2</i>	<i>8.4</i>	<i>3.5</i>	<i>2.7</i>	<i>0.5</i>	<i>-0.7</i>	<i>0.4</i>	<i>1.2</i>	<i>0.7</i>	<i>1.1</i>	<i>1.6</i>
$\mu'$	$\gamma'$	$\lambda'_1$	$\lambda'_2$	$\lambda'_3$	$\lambda'_4$	$\lambda'_5$	$\varphi'_1$	$\varphi'_2$	$\varphi'_3$	$\varphi'_4$	$\varphi'_5$
0.006	-3.5	0.24	0.04	0.08	0.09	0.13	-0.10	-0.07	-0.07	-0.02	0.01
<i>2.5</i>	<i>-3.2</i>	<i>6.2</i>	<i>0.9</i>	<i>1.9</i>	<i>2.0</i>	<i>3.0</i>	<i>-2.3</i>	<i>-1.5</i>	<i>-1.6</i>	<i>-0.4</i>	<i>0.2</i>

current account is due to the chartists' extrapolation behaviour which adds a lot of noise in the exchange rate movements. Thus, our model helps to explain the disconnect puzzle. As stressed by Obstfeld and Rogoff(2000) the disconnect phenomenon runs in both directions: the exchange rate is disconnected from the fundamentals (in this case the current account) and the fundamental (the current account) is disconnected from the exchange rate most of the time.

From table 1, we also note an asymmetry in the disconnect puzzle. We observe that the past changes in the current account have a weak and insignificant effect on the current exchange rate changes. The converse is not true. Past changes in the exchange rate have a significant effect on the current changes in the current account.

The picture that emerges from this analysis can be summarized as follows. There is a long run cointegration relationship between the exchange rate and the current account .However, the speed of adjustment of both the exchange rate and the current account towards this long run equilibrium relationship is very weak despite the fact that we have built into the structure of the model relatively strong speeds of adjustment. It is in this sense that there is a disconnect puzzle that runs in both directions. There is an asymmetry though. Past exchange rate changes have a significant effect on today's changes in the current account. The reverse does not seem to be the case.

We have estimated similar error correction models on simulated exchange rates and current accounts for other parameter values of the model. We show the estimated  $\mu$ 's and  $\gamma$ 's in table 5. We find similarly low speeds of adjustment, and an asymmetry in the coefficients of the past changes in the exchange rate and the current account (not shown).

## 8.1 Fat tails and excess kurtosis

It is well known that the exchange rate changes do not follow a normal distribution. Instead it has been observed that the distribution of exchange rate changes has more density around the mean than the normal and exhibits fatter tails than the normal (see de Vries(2001)). This phenomenon was first discovered by Mandelbrot (1963), in commodity markets. Since then, fat tails and excess kurtosis have been discovered in many other asset markets including the

Table 2: ecm for different parameter values

parameter values	EC-coefficient		Error correction
	$\mu$	$\mu'$	$\gamma$
$C = 5, \beta = 0.8, \gamma = 1, \epsilon = 0.5$	-0.003	0.01	-1.66
	-0.5	1.6	-3.8
$C = 5, \beta = 0.8, \gamma = 10, \epsilon = 0.5$	-0.01	0.01	-1.45
	-1.7	1.9	-4.7
$C = 5, \beta = 0.8, \gamma = 10, \epsilon = 1$	-0.01	0.01	-0.64
	-1.6	2.5	-4.7
$C = 5, \beta = 0.9, \gamma = 10, \epsilon = 1$	-0.02	0.1	-0.65
	-2.8	0.9	-5.7
$C = 5, \beta = 0.8, \gamma = 1, \epsilon = 0.5$	-0.02	0.01	-0.05
	-4.2	1.4	-0.04

Table 3: Kurtosis and Hill index USD-DEM and JPY-DEM 1975-98

Exchange rate	kurtosis	median Hill index		
		2.5% tail	5% tail	10% tail
USD-DEM	12.1	4.0	3.6	3.1
JPY-DEM	19.6	3.7	3.6	2.9

exchange market. In particular, in the latter the returns have a kurtosis typically exceeding  $3^{10}$  and a measure of fat tails (Hill index) ranging between 2 and 5 (see Koedijk, Stork and de Vries (1992), Huisman, et al.(2002)). It implies that most of the time the exchange rate movements are relatively small but that occasionally periods of turbulence occur with relatively large exchange rate changes.

In table 3 we show the kurtosis and the Hill index of the USD-DEM and the JPY-DEM exchange rate returns for the period 1975-1998. We computed the Hill index for different cut-off points of the tails (2.5%, 5%, 10%) and for 4 different subsamples of the original series. We find that these exchange rates exhibited excess kurtosis and fat tails during the sample period.

Another empirical finding that has been observed is that the kurtosis is reduced under time aggregation (see Lux(1998), Calvet and Fisher(2002)). We checked this finding for the same exchange rates. In table 4 we show the results for USD-DEM and JPY-DEM exchange rates, and we confirm that the kurtosis declines under time aggregation.

The next step in the analysis was to check whether these empirical features are also shared by the simulated exchange rate changes in our model.

The model was simulated using normally distributed random disturbances (with mean = 0 and standard deviation = 1). We computed the kurtosis and the Hill index of the simulated exchange rate returns. We computed the Hill index for 4 different samples of 2000 observations. In addition, as before, we

<sup>10</sup>The normal distribution has a kurtosis index equal to 3.

Table 4: Kurtosis and time aggregation USD-DEM and JPY-DEM 1975-98

<b>Parameter values</b>	5 period returns	10 period returns	50 period returns
USD-DEM	7.4	5.3	3.4
JPY-DEM	14.9	5.7	2.7

considered three different cut-off points of the tails (2.5%, 5%, 10%). We show the results of the kurtosis and of the Hill index in table 5. We find that for a broad range of parameter values the kurtosis exceeds 3 and the Hill index indicates the presence of fat tails.

Table 5: Kurtosis and Hill index

<b>Parameter values</b>	<b>kurtosis</b>	<b>median Hill index</b>		
		<b>2.5% tail</b>	<b>5% tail</b>	<b>10% tail</b>
$C=5, \beta=0.9, \gamma=1; \epsilon=0$	21.5	6.1	5.3	4.3
$C=5, \beta=0.9, \gamma=0.5; \epsilon=0.5$	7.9	3.5	3.4	3.2
$C=5, \beta=0.9, \gamma=1; \epsilon=0.5$	20.5	3.1	2.8	3.0
$C=5, \beta=0.9, \gamma=5; \epsilon=0.5$	35.9	3.4	3.6	3.5
$C=5, \beta=0.8, \gamma=1; \epsilon=0$	15.5	5.6	5.0	4.0
$C=5, \beta=0.8, \gamma=0.5; \epsilon=0.5$	6.0	3.3	3.8	3.5
$C=5, \beta=0.8, \gamma=1; \epsilon=0.5$	7.3	3.5	3.7	3.6
$C=5, \beta=0.8, \gamma=0.5; \epsilon=0.8$	11.9	2.9	3.3	3.1

## 9 Conclusion

In this paper we provide a framework for analysing the dynamics of exchange rate movements. The special feature of our model is that individual agents recognize that they are not capable of understanding and processing the complex information structure of the underlying model. As a result, they use simple rules to forecast the exchange rates. None of these rules is rational in the technical sense. Yet we claim that these agents act rationally within the context of the uncertainty they face. That is, agents check the 'fitness' (profitability) of the forecasting rule at each point in time and decide to reject the rule if it is less profitable (in a risk adjusted sense) than competing rules. Our model is in the tradition of evolutionary dynamics where agents use trial and error strategies. We assume that some of the forecasting rules are based on extrapolating past exchange rate movements (technical trading) and others are based on mean reversion towards the fundamental rate (fundamentalism).

We analysed this model first within a framework where the current account (the fundamental) is exogenous. The model then generates two types of equilibria. The first one, which we called a fundamental equilibrium, is one in which the exchange rate converges to its fundamental value. The exchange rate, how-

ever, can also converge to a second type of equilibrium, which we called a bubble equilibrium, and which is reached in a self-fulfilling manner. An important feature of the bubble equilibrium is that chartism (extrapolative forecasting) takes over most of the market. We simulated the model in a stochastic environment and generated complex scenarios of bubbles and crashes. One interesting aspect of the model is that it explains both the emergence of the bubble and its subsequent crash. The model also predicts that bubbles and crashes are asymmetric, i.e. the bubble phase is slower than the subsequent crash. This asymmetry has been widely observed in financial markets. It cannot be explained by RE-models of bubbles and crashes which predict symmetry (Blanchard and Watson(1982)).

We also analysed under what conditions bubbles and crashes occur. We find that when agents react quickly to changing relative profitabilities of the different forecasting rules, the frequency of bubbles increases. In such an environment chartists will make large profits and will tend to dominate the market, crowding out fundamentalists who have a poor forecasting record and make losses. It will then be quite rational to be a chartist.

In a second stage we extended the model to allow for a feedback from the current account. We found that when the elasticity of the current account with respect to exchange rate changes is sufficiently high the bubble equilibria disappear.

In a third stage, we introduced transactions costs in the goods market. These transactions costs create a band of inaction within which exports and imports do not react to exchange rate changes. The implications of introducing such transactions costs for the dynamics of the exchange rate are far-reaching. We found that this version of the model is capable of generating the disconnect phenomenon (misalignment), whereby the exchange rate is disconnected most of the time from the underlying fundamentals. Interestingly, these misalignments resemble the bubbles and crashes dynamics, although the deterministic part of the model does not produce bubble equilibria.

Finally, we also found that the model is capable of mimicking a widely observed empirical phenomenon in financial markets, i.e. the existence of excess kurtosis and fat tails. This phenomenon implies that periods of tranquility and turbulence alternate in unpredictable way. Our model generates this feature suggesting that it has empirical support.

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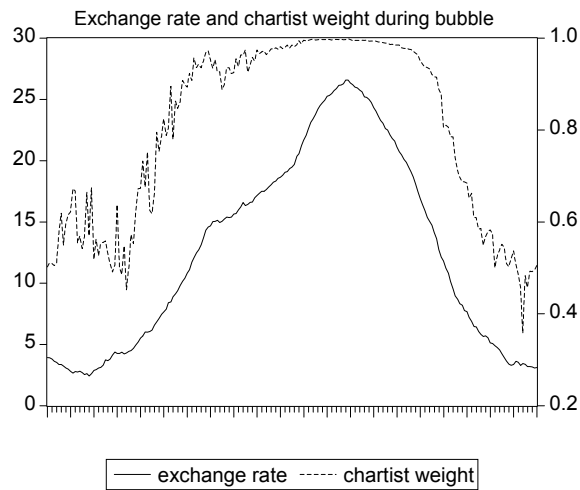
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## A Causality tests between exchange rate and chartist weight

In this appendix we present the results of causality tests between the exchange rate and the weight of chartists during a bubble and crash episode. We simulated the model using the standard set of parameters, and we selected an episode during which a bubble and crash occurred. We show such an episode in figure A2. A visual inspection of the graph reveals that the exchange rate appears to lead the chartist weight. at least when the bubble starts and later when the bubble bursts. Note also that the crash occurs faster than the bubble phase, a feature we often find in our simulated bubbles and crashes. This has also been found in empirical data (see Sornette(2003))



Next we performed a Granger causality test on the exchange rate and the chartist weight during the bubble and crash episode represented in figure A2<sup>11</sup>. The result of this causality test is presented in table A1. We observe that we cannot reject the hypothesis that the exchange rate leads the chartists' weight during the bubble and crash episode, while we can reject the reverse. We find this feature in most bubble and crash episodes.

<sup>11</sup>We checked for stationarity and could not reject that the two series are stationary during the sample period.

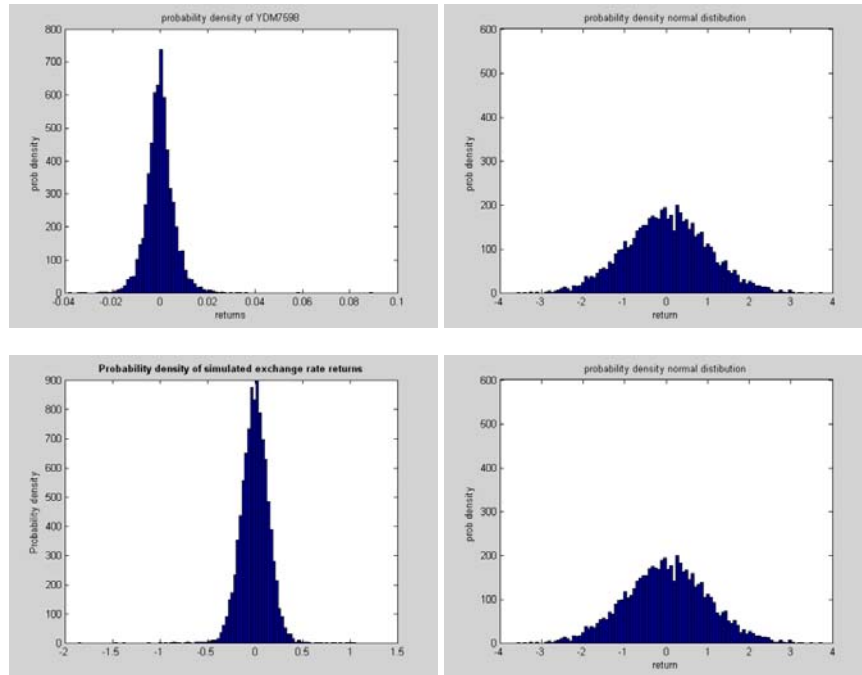
Table 6: Granger causality tests

<b>Null Hypothesis:</b>	<b>F-statistic</b>	<b>Probability</b>
cw not Granger cause exchange rate	0.377	0.865
exchange rate not Granger cause cw	6.85	6.4E-06

**Note:** obs=211, lags=5.

## B Stylised Facts of JPY-DEM Exchange Rate

The up left panel shows the distribution of the JPY-DEM returns. The bottom left panel represents the distribution of our simulated returns. The right panels show the distribution of normally distributed exchange rate returns.



## C The model with transaction cost: additional results

It can be seen that for values of  $\beta < 0.85$  the equilibria fit exactly within the transaction cost band. For higher values of  $\beta$  we obtain chaotic attractors. Note that the critical value of  $\beta$  depends on the initial conditions and the other parameters of the model.

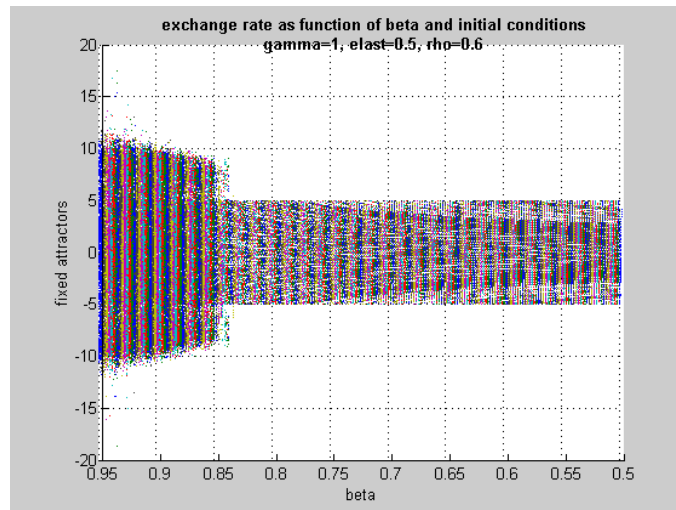


Figure 19: