Learning to Forecast the Exchange Rate: Two Competing Approaches.

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Abstract

In this paper, we investigate the behavior of the exchange rate within the framework of a standard asset pricing model. We assume boundedly rational agents who use simple rules to forecast the future exchange rate. They test these rules continuously using two learning mechanisms. The first one, the fitness method, assumes that agents evaluate forecasts by computing their past profitability. In the second mechanism, agents learn to improve these rules using statistical methods. First, we find that both learning mechanisms reveal the fundamental value of the exchange rate in the steady state. Second, fitness learning comes closer to mimicking regularities observed in the foreign exchange markets, namely exchange rate disconnect and excess volatility.

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1 Introduction

Exchange rate economics has been dominated by the rational expectations efficient market theory. As the empirical evidence against this theory has tended to accumulate over time[1], researchers have increasingly looked for alternative modelling approaches. One of these approaches challenges the assumptions about the way the agents form their expectations. In this paper, we focus on this approach.

We investigate the behavior of the exchange rate within the framework of a standard asset pricing model. We assume that the market expectations, within this model, are formed by boundedly rational agents. We take the view that the rational expectations assumption puts too great an informational burden on individual agents. Agents experience cognitive problems in processing information. As a result, they use simple forecasting rules (heuristics). We assume that they can use two different forecasting rules and combine them to form their expectations about the future exchange rate. The first one will be called a fundamentalist forecasting rule, the second one a chartist rule (technical analysis). Then, we assume that the agents test these rules continuously. This testing procedure is the mechanism by which we introduce discipline on the behavior of individual agents. We specify two alternative testing procedures (learning mechanisms). In the first one, agents select the rules based on a fitness method in the spirit of Brock and Hommes (1997), (1998). This mechanism assumes that agents evaluate forecasts by computing their past profitability. Accordingly, they increase (reduce) the weight of one rule if it is more (less) profitable than the alternative rule. In the second mechanism, agents learn to improve these rules using statistical methods based on the literature of learning in macroeconomics (e.g. Evans and Honkapohja (2001)).

The purpose of this paper is to analyze the behavior of the exchange rate under different learning rules, and to compare the capacity of these rules to mimic regularities observed in the foreign exchange markets.

The remainder of the paper is organized as follows. In section two, we develop the baseline model of the exchange rate and we specify the way agents form their expectations about the future exchange rate. In section three, we introduce the learning rules of the agents. In section four, we study the steady state properties of the models.

[1] See Sarno and Taylor(2002), De Grauwe and Grimaldi(2006).

Section five presents a numerical analysis of the dynamics of the exchange rate. We carry out sensitivity analysis of the two learning models in section six. Section seven confronts the statistical properties of the exchange rate under the two learning rules with the data. Section eight provides some concluding remarks.

2 Exchange rate model and agents' expectations

2.1 Asset pricing model of the exchange rate

We model the market exchange rate using a standard asset pricing model. This allows us to write the exchange rate as:

$$s_t = s_t^* + b\left(E_t s_{t+1} - s_t\right) \tag{1}$$

where s_t is the log level of the exchange rate in period t, defined as the domestic price of a unit of foreign currency and s_t^* defines the set of fundamentals. Equation (1) expresses the market exchange rate as the sum of the current fundamentals and the expected change of the market rate. We rewrite this equation in the following form:

$$s_t = \left(1 - \alpha\right) s_t^* + \alpha E_t s_{t+1} \tag{2}$$

where $\alpha = \frac{b}{1+b}$, and $1-\alpha = \frac{1}{1+b}$. Thus, the market exchange rate is a convex combination of the fundamental rate and the expectations of the future market exchange rate. Note that α can be interpreted as a discount factor. We also assume that the log fundamental s_t^* is driven by a random walk, i.e.

$$s_t^* = s_{t-1}^* + \epsilon_t \tag{3}$$

where $\epsilon_t \sim iid\left(0, \sigma_\epsilon^2\right)$.

2.2 Expectations of agents

In specifying the mechanism determining expectations we depart from the assumption of rational expectations. We take the view that the rational expectations assumption puts too great an informational burden on individual agents. Agents experience cognitive problems in processing information. As a result, they use simple forecasting rules (heuristics). They are willing to learn however. Their learning process will then lead them to put different weights on the rules they are using.

We start by assuming that agents can use two different forecasting rules. One will be called a fundamentalist forecasting rule, the other a chartist rule (technical analysis). Thus we introduce heterogeneity in the agents' forecasts[2].

When using a fundamentalists rule, agents compare the market exchange rate with the fundamental rate and they forecast the future market rate to return to the fundamental rate:

$$E_t^f\left(\Delta s_{t+1}\right) = -\psi\left(s_{t-1} - s_{t-1}^*\right) \tag{4}$$

or

$$E_t^f\left(s_{t+1}\right) = s_{t-1} + \psi\left(s_{t-1}^* - s_{t-1}\right) \tag{5}$$

In this sense, they follow a negative feedback rule: where $\psi > 0$ is a parameter describing the speed at which the agents expect the exchange rate to return to its fundamental value.

The second forecasting rule agents use is a chartist rule. We assume that this takes the form of extrapolating the last change of the exchange rate into the future, i.e.

$$E_t^c\left(\Delta s_{t+1}\right) = \beta\Delta s_{t-1} \tag{6}$$

or

$$E_t^c\left(s_{t+1}\right) = s_{t-1} + \beta\Delta s_{t-1} \tag{7}$$

The degree of extrapolation is given by the parameter $\beta > 0$. Clearly, more sophisticated rules could be specified. Here we focus on the simplest possible chartist rule.

The agents combine these two rules with their respective weights. As a result, the market forecast, $E_t s_{t+1,}$ is assumed to be a weighted average of the mean-reverting and the extrapolative components, i.e.,

$$E_t s_{t+1} = \omega^f E_t^f s_{t+1} + \omega^c E_t^c s_{t+1} \tag{8}$$

[2] Survey data indicate that the expectations in the exchange market are not homogeneous (Taylor and Allen (1992), Frankel and Froot (1990), Bénassy-Quéré, Larribeau and Macdonald (1999)). These survey data point out that the FOREX traders do not stick to one single trading rule. They alter and even mix the trading rules according to the realized profits.

where $E_t^f s_{t+1}$ and $E_t^c s_{t+1}$ are the mean-reverting (fundamentalist) and the extrapolative (chartist) components, respectively, ω^f is the weight given to the fundamentalist rule, ω^c is the weight given to the chartist rule and $\omega^f + \omega^c = 1$.

The timing in this model should be specified carefully. Since our agents are boundedly rational, they do not know the current exchange rate that will be the outcome of their forecast. The last available information they have about the exchange rate is the one prevailing in the previous period. Thus, when they make a forecast in period t they use the information up to period $t-1$.

We now substitute equation (5) and (7) into equation (8) and the latter into equation (2). This yields:

$$s_t = (1-\alpha)s_t^* + \alpha\omega^f \left[s_{t-1} + \psi\left(s_{t-1}^* - s_{t-1}\right)\right] + \alpha\omega^c \left[s_{t-1} + \beta\Delta s_{t-1}\right] + \eta_t \qquad (9)$$

where $\eta_t \sim iid\left(0, \sigma_\eta^2\right)$ is a white noise which captures unexpected disturbances in the market process.

3 Learning mechanisms of agents

In our world of bounded rationality, agents use simple rules. However, they test these rules continuously. This testing procedure is the mechanism by which discipline is imposed on the behavior of individual agents. We will specify two alternative testing procedures (learning mechanisms). In the first one, agents select the rules based on a fitness method. In the second mechanism, agents learn to improve these rules using statistical methods.

3.1 Fitness mechanism

The first learning mechanism is based on a fitness criterion in the spirit of Brock and Hommes (1997), (1998), which is based on discrete choice theory[3]. This mechanism

[3] This specification is often applied in discrete choice models. For an application in the markets for differentiated goods, see Anderson, et al., (1992). There are other ways to specify a rule that governs the selection of forecasting strategies. One was proposed by Kirman(1993). Another one was formulated by Lux and Marchesi(1999).

assumes that agents evaluate the two forecasting rules by computing their past profitability and to increase (reduce) the weight of one rule if it is more (less) profitable than the alternative rule. We specify this procedure as follows:

$$\omega_t^f = \frac{\exp \delta \pi_t^{f\prime}}{\exp \delta \pi_t^{f\prime} + \exp \delta \pi_t^{c\prime}} \tag{10}$$

$$\omega_t^c = \frac{\exp \delta \pi_t^{c\prime}}{\exp \delta \pi_t^{c\prime} + \exp \delta \pi_t^{f\prime}} \tag{11}$$

where ω_t^f and ω_t^c are the weights given to the fundamentalist and chartist rules, respectively. $\pi_t^{f\prime}$ and $\pi_t^{c\prime}$ are the (risk adjusted) profits. These are defined as:

$$\pi_t^{f\prime} = \pi_t^f - \mu \sigma_{f,t}^2 \tag{12}$$

and

$$\pi_t^{c\prime} = \pi_t^c - \mu \sigma_{c,t}^2 \tag{13}$$

where μ is the coefficient of risk aversion and π_t^f and π_t^c are the profits made in forecasting, while $\sigma_{f,t}^2$ and $\sigma_{c,t}^2$ are the variances of the forecast errors made using fundamentalist and chartist rules, respectively.

We define the profits, π_t^f and π_t^c, as the one-period returns of investing in the foreign asset.

$$\pi_t^i = (s_{t-1} - s_{t-2})\, sgn\left(E_{t-1}^i s_{t-1} - s_{t-2}\right) \tag{14}$$

where $sgn[x] = \begin{cases} 1 & \text{for } x > 0 \\ 0 & \text{for } x = 0 \\ -1 & \text{for } x < 0 \end{cases}$ and $i = c,\ f$

Thus, when agents forecasted an increase in the exchange rate return (the return of the foreign currency) and this increase is realized, their profit is equal to the observed increase in the exchange rate return. If instead the exchange rate retrn declines, they make a loss which equals this decline (because in this case they have bought foreign assets which have declined in return).

Equations (10) and (11) can now be interpreted as follows. When the risk adjusted profits of the extrapolative (chartist) rule increase, relative to the risk adjusted profits of the mean-reverting (fundamentalist) rule, then the weight the agents give to the extrapolative rule in period t increases, and vice versa. The parameter δ measures the intensity with which the agents switch the weights from one rule to the other. With an increasing δ agents react strongly to the relative profitability of the two forecasting rules. In the limit, when δ goes to infinity, the agents choose the forecasting rule which proves to be more profitable. When δ is equal to zero, agents are insensitive to the relative profitability of these rules. In the latter case, the weights of mean-reverting and extrapolative rule is constant and equal to 0.5. Thus, δ is a measure of inertia in the decision to give more weight to the more profitable rule[4].

The weights obtained from equations (10) and (11) are then substituted into the rate equation(9) :

$$s_t = (1-\alpha)s_t^* + \alpha\left[\omega_t^f E_t^f\left(s_{t+1}\right) + \omega_t^c E_t^c\left(s_{t+1}\right)\right] + \eta_t \tag{15}$$

$$E_t^f\left(s_{t+1}\right) = s_{t-1} + \psi\left(s_{t-1}^* - s_{t-1}\right) \tag{16}$$

$$E_t^c\left(s_{t+1}\right) = s_{t-1} + \beta\Delta s_{t-1} \tag{17}$$

Note that in this learning mechanism agents are assumed to use the same values of parameters β and ψ in every period t. However, they give different weights to these parameters each period, depending on how well the forecasting rules underlying these parameters do in terms of profitability.

3.2 Statistical learning

The second learning mechanism that we consider here is statistical learning (See Evans and Honkapoja (2001)). As before, agents' expectations are composed of two components, i.e. a mean-reverting and an extrapolative one. In contrast to the fitness criterion, agents are assumed to have some basic knowledge of econometrics, such that parameters β_t and ψ_t can be interpreted as the estimates based on data up to period $t-1$. As a result, the agents estimate simultaneously, the weights and the parameters in their forecasting rule. Their expectations are formed in the following way:

[4] The logic of the switching weight is the same spirit of the adaptive rules that are now used in game theoretic models (See, for examples, Cheung and Friedman (1997); Fudenberg and Levine, (1998)). In these models, actions that did better in the observed past tend to increase in frequency while actions that did worse tend to decrease in frequency.

$$E_t\left(\Delta s_{t+1}\right) = \psi_{t-1}\left(s_{t-1}^{*} - s_{t-1}\right) + \beta_{t-1}\Delta s_{t-1} \tag{18}$$

At time $t+1$, as the realized values of the market (s_t) and fundamental (s_t^*) exchange rates are available, agents revise their forecasting rule. In particular, according to their forecasting rule, they regress Δs_t on $s_{t-1}^* - s_{t-1}$ and Δs_{t-1}. They update their parameters using recursive methods and the last available information, i.e. s_t :

$$\begin{aligned} \phi_t &= \phi_{t-1} + \gamma_t R_t^{-1} z_{t-1}\left(\Delta s_t - \phi_{t-1}' z_{t-1}\right) \\ R_t &= R_{t-1} + \gamma_t\left(z_{t-1} z_{t-1}' - R_{t-1}\right) \end{aligned} \tag{19}$$

where $\phi_t = (\psi_t, \beta_t)'$ is the vector of parameter estimates, $z_{t-1} = (s_{t-1}^* - s_{t-1}, \Delta s_{t-1})$ is a vector of explanatory variables, $R_t = \gamma_t \sum_{i=1}^{t} z_{i-1} z_{i-1}'$ is a moment matrix and γ_t is the gain. The gain captures the speed of updating in the sense of how much weight the agents put on the new incoming information. We will assume that agents put more weight on new information and thus update their forecasts with a low constant gain. Introducing the perceived law of motion (PLM) as given by equation (18) into equation (2), we obtain the resulting actual law of motion (ALM) of the market exchange rate:

$$s_t = (1-\alpha)s_t^* + \alpha(1+\beta_{t-1}-\psi_{t-1})s_{t-1} - \alpha\beta_{t-1}s_{t-2} + \alpha\psi_{t-1}s_{t-1}^* + \eta_t \tag{20}$$

4 Steady state properties

In this section, we analyze the steady state properties of the market exchange rate under two learning mechanisms. This will allow us to analyze the question of whether these two learning mechanisms are capable of revealing the fundamental value of the exchange rate in the steady state.

4.1 The steady state under fitness learning

In order to analyze the steady state of the model under fitness mechanism, we strip it from its stochastic components. Thus we assume that the fundamental variable is constant. In addition, for the sake of convenience, we set the fundamental rate, $s_t^* = s^* = 0$. As a result, the exchange rate movements can be interpreted as deviations from their fundamental value.

We rewrite equation (15) for as follows:

$$s_t = \alpha \left[s_{t-1} - \omega_{t-1}^f \psi s_{t-1} + (1 - \omega_{t-1}^f)\beta(s_{t-1} - s_{t-2}) \right] \tag{21}$$

where

$$\omega_{t-1}^f = \frac{exp\left[\gamma(\pi_{t-1}^f - \mu\sigma_{f,t-1}^2)\right]}{exp\left[\gamma(\pi_{t-1}^c - \mu\sigma_{c,t-1}^2)\right] + exp\left[\gamma(\pi_{t-1}^f - \mu\sigma_{f,t-1}^2)\right]} \tag{22}$$

Variance terms from equations (12) and (13) can be written as follows:

$$\sigma_{c,t-1}^2 = \left[E_{t-2}^c\left(s_{t-1}\right) - s_{t-1}\right]^2 \tag{23}$$

$$\sigma_{f,t-1}^2 = \left[E_{t-2}^f\left(s_{t-1}\right) - s_{t-1}\right]^2 \tag{24}$$

Using the definition of the forecasting rules (5) and (7) this yields

$$\sigma_{c,t-1}^2 = [-\beta s_{t-3} + (1+\beta)s_{t-2} - s_{t-1}]^2 \tag{25}$$

$$\sigma_{f,t-1}^2 = [(1-\psi)s_{t-2} - s_{t-1}]^2 \tag{26}$$

With suitable changes of variables it is possible to write these equations as a 3-dimensional system. Set:

$$u_t = s_{t-1}$$

$$x_t = u_{t-1} (= s_{t-2})$$

The 3 dynamic variables are (s_t, u_t, x_t). The state of the system at time $t-1$, i.e. $(s_{t-1}, u_{t-1}, x_{t-1})$ determines the state of the system at time t, i.e. (s_t, u_t, x_t) through the following 3-D dynamical system:

$$s_t = \alpha[(1-\omega_{t-1}^f)\left((\psi+\beta)\,u_t - \beta x_t\right)] \tag{27}$$

$$u_t = s_{t-1} \tag{28}$$

$$x_t = u_{t-1} \tag{29}$$

where ω_{t-1}^f is defined in equation (22) and the forecast errors and ex-post profits are defined in following way:

$$\sigma_{c,t-1}^2 = \left[(1+\beta)u_{t-1} - \beta x_{t-1} - s_{t-1}\right]^2 \tag{30}$$

$$\sigma_{f,t-1}^2 = \left[(1-\psi)u_{t-1} - s_{t-1}\right]^2 \tag{31}$$

$$\pi_{t-1}^c = (s_{t-1} - u_{t-1}) \text{ sgn } [\beta(u_{t-1} - x_{t-1})] \tag{32}$$

$$\pi_{t-1}^f = (s_{t-1} - u_{t-1}) \text{ sgn } [-\psi u_{t-1}]$$

We can now analyze the nature of the steady state solution. Since we have normalized the fundamental exchange rate s_t^* to be zero, the fundamental solution implies that $s_t = 0$. As a result, the variance terms go to zero.

The steady state of the system is now obtained by setting:

$$(s_{t-1}, u_{t-1}, x_{t-1}) = (s_t, u_t, x_t) = (\overline{s}, \overline{u}, \overline{x})$$

in the dynamical system (27)-(29).

There is a unique steady state where

$$\overline{s}, \overline{u}, \overline{x} = 0$$

Notice also that at the steady state:

$$\overline{s} = \overline{u} = \overline{x} = s^*, \overline{\omega}^c = \frac{1}{2}, \overline{\omega}^f = \frac{1}{2}, \overline{\pi}^f = 0, \overline{\pi}^c = 0, \overline{\sigma}_f^2 = 0, \overline{\sigma}_c^2 = 0 \tag{33}$$

i.e. the steady state is characterized by the exchange rate being at its fundamental level, by zero profits and zero risk, and by fundamentalist and technical trader fractions equal to $\frac{1}{2}$.

We can also analyze the conditions under which a non-zero steady state solution exists. This is a solution in which the exchange rate is constant and permanently different from its (constant) fundamental value. If such a second steady state solution exists, the model allows for an exchange rate in the steady state that is permanently disconnected from its fundamental.

In order to analyze under what condition such a steady state solution can arise, we use equation (2) and set $s_t = s_{t-1} = s_{t-2} = \overline{s}$, so that

$$\overline{s} = \alpha(1 - \omega_t^f \psi)\overline{s} \tag{34}$$

It can now easily be seen that a solution of the type $\overline{s} > 0$ exists if $\alpha(1 - \omega_t^f \psi) = 1$. This condition is satisfied if $\alpha = 1$ and $\omega_t^f = 0$.The first of these two conditions says that the current fundamental should have no influence on the current exchange rate; the second condition says that the share of the fundamentalists in the market should be zero. The latter, however, can only arise if $\sigma_{f,t}^2 \to \infty$ (This can be seen from the definition of ω_t^f in (22)). As a result, a solution whereby the market exchange rate permanently deviates from the market exchange rate can be ruled out.

We conclude that in the steady state, the exchange rate equals its fundamental value and the learning based on the fitness method reveals the fundamental value.

4.2 The steady state under statistical learning

In this subsection, we analyze the properties of the steady state of the model under statistical learning. The agents' PLM is of the following form:

$$\Delta s_{t+1} = \psi_{t-1}\left(s_{t-1}^{*} - s_{t-1}\right) + \beta_{t-1}\Delta s_{t-1} + \varsigma_{t+1} \tag{35}$$

Accordingly, the agents form their expectations:

$$E_t\left(\Delta s_{t+1}\right) = \psi_{t-1}\left(s_{t-1}^{*} - s_{t-1}\right) + \beta_{t-1}\Delta s_{t-1} \tag{36}$$

where they forecast the market exchange rate return Δs_{t+1}. Since they are assumed to have the information only until $t-1$, the change Δs_{t+1} is defined as $s_{t+1} - s_{t-1}$. We can write equation (35) in the following form:

$$E_t s_{t+1} = s_{t-1} + \psi_{t-1}\left(s^*_{t-1} - s_{t-1}\right) + \beta_{t-1}\Delta s_{t-1} \tag{37}$$

Substituting the PLM into equation (2), yields the resulting ALM of the market exchange rate:

$$s_t = (1-\alpha)s^*_t + \alpha(1+\beta-\psi)s_{t-1} - \alpha\beta s_{t-2} + \alpha\psi s^*_{t-1} + \eta_t \tag{38}$$

Using the ALM for s_{t-1} and the definition of the fundamental rate s^*_t in equation (3), we obtain the following specification of the market exchange rate:

$$\begin{aligned} s_t = {} & \left[(1-\alpha) + \alpha\psi + \alpha(1+\beta-\psi)(1-\alpha)\right] s^*_{t-2} + \\ & \left[\alpha^2(1+\beta-\psi) - \alpha\beta\right] s_{t-2} + \alpha^2\beta(1+\beta-\psi)(s_{t-2} - s_{t-3}) + \\ & \alpha^2\psi(1+\beta-\psi)\left(s^*_{t-2} - s_{t-2}\right) + \left[(1-\alpha) + \alpha\psi + \alpha(1+\beta-\psi)(1-\alpha)\right]\epsilon_{t-1} + \\ & (1-\alpha)\epsilon_t + \left[\alpha(1+\beta-\psi)(1-\alpha)\right]\eta_{t-1} + \eta_t \end{aligned} \tag{39}$$

After substraction s_{t-2} from both sides and carrying out some manipulations, we can rewrite the ALM in the same form as the PLM (equation (18)):

$$\begin{aligned} s_t - s_{t-2} = {} & (\alpha\psi - \alpha + 1)(\alpha + \alpha\beta - \alpha\psi + 1)(s^*_{t-2} - s_{t-2}) + \\ & \alpha^2(1+\beta-\psi)\beta(s_{t-2} - s_{t-3}) + \left[(1-\alpha) + \alpha\psi + \alpha(1+\beta-\psi)(1-\alpha)\right]\epsilon_{t-1} + \\ & (1-\alpha)\epsilon_t + \left[\alpha(1+\beta-\psi)(1-\alpha)\right]\eta_{t-1} + \eta_t \end{aligned} \tag{40}$$

This allows us to define T-map as labeled by Evans and Honkapohja (2001):

$$T\begin{pmatrix} \psi \\ \beta \end{pmatrix} = \begin{pmatrix} (\alpha\psi - \alpha + 1)(\alpha + \alpha\beta - \alpha\psi + 1) \\ \alpha^2(1+\beta-\psi)\beta \end{pmatrix} \tag{41}$$

and proceed to T-mapping from PLM to ALM:

$$\begin{pmatrix} \psi \\ \beta \end{pmatrix} = \begin{pmatrix} (\alpha\psi - \alpha + 1)(\alpha + \alpha\beta - \alpha\psi + 1) \\ \alpha^2(1+\beta-\psi)\beta \end{pmatrix} \tag{42}$$

From the system(42), we can compute the stationary points of $T(\psi,\beta)'$. From the second equation of this system, we obtain two solutions for β i.e., $\beta_1 = 0$ or $\beta_2 = -1 + \psi + \frac{1}{\alpha^2}$.

We calculate the resulting solutions for ψ, for each of the fixed points of β. When $\beta_1 = 0$, we have two possible solutions for $\psi_1 = 1$ or $\psi_2 = 1 - \frac{1}{\alpha^2}$. For $\beta_2 = -1 + \psi + \frac{1}{\alpha^2}$, we find $\psi_3 = 1 - \frac{1}{\alpha^2}$. Substituting this result in β_2, it yields $\beta_2 = 0$. As a result, we have two possible pairs of solutions. The first one is given by the combination $\phi_1 = \begin{pmatrix} \psi_1 \\ \beta_1 \end{pmatrix}$ and means that the agents learn that the extrapolating component does not play a role in determination of the market exchange rate ($\beta_1 = 0$). They find that the market exchange rate will return to the fundamental rate in the next period $(\psi_1 = 1)$. Substituting these values in the ALM (38), we obtain :

$$s_t = s_t^* + \eta_t - \alpha v_t \tag{43}$$

Thus we find that this set of fixed points leads to the rational expectations solution of the asset pricing model[5]. In the steady state, if we assume $s_t^* = s_{t-1}^* = \bar{s}^* = 0$ and $\eta_t = v_t = 0$, $s_t = 0$. This means that in the steady state the adaptive learning model leads the exchange rate to its fundamental value. The second pair of steady state solutions is given by $\phi_2 = \begin{pmatrix} \psi_2 \\ \beta_2 \end{pmatrix}$ and indicates that the agents again learn that extrapolating parameter to be zero $(\beta_2 = 0)$ and a negative value of ψ_2. This means that the fundamentalists learn to extrapolate the difference between market and fundamental exchange rates. We plug the values of the second solution i.e., $\psi_2 = 1 - \frac{1}{\alpha^2}$ and $\beta_2 = 0$ into the ALM (38). We find that the current market exchange rate is a sum of the fundamental rate and the extrapolated difference between past market and fundamental rates:

$$s_t = s_t^* + \frac{1}{\alpha}\left(s_{t-1} - s_{t-1}^*\right) - \alpha v_t + \eta_t \tag{44}$$

If we again assume that in the steady state $s_t^* = s_{t-1}^* = \bar{s}^* = 0$ and $v_t, \eta_t = 0$, we find that $s_t = 0$. We conclude that the only existing equilibrium is the one when the market exchange rate equals the fundamental rate. The two solutions however imply different short run dynamics. The first set of fixed points ϕ_1, leading to rational expectations solution implies that the market rate is permanently connected to the fundamental rate. The second solution ϕ_1, allows for some short run disconnection from the fundamental

[5]If we assume in the framework of asset pricing model that the agents are rational, solve this model forward and assume transversality condition, we find that the market exchange rate is equal to the fundamental rate.

rate. We study the short dynamics of the market rate within the proposed model in the following section.

We check the stability of the two possible solutions by calculating the eigenvalues of the Jacobian matrix $DT\begin{pmatrix}\psi \\ \beta\end{pmatrix}$:

$$DT\begin{pmatrix}\psi \\ \beta\end{pmatrix} = \begin{pmatrix}\alpha^2(2+\beta-2\psi) & \alpha(\alpha\psi-\alpha+1) \\ -\alpha^2\beta & \alpha^2(1+2\beta-\psi)\end{pmatrix} \tag{45}$$

The characteristic polynomial of the above matrix is:

$$\begin{aligned} p(\lambda) &= \det(DT-\lambda I) = \det\begin{pmatrix}\alpha^2(2+\beta-2\psi)-\lambda & \alpha(\alpha\psi-\alpha+1) \\ -\alpha^2\beta & \alpha^2(1+2\beta-\psi)-\lambda\end{pmatrix} \\ &= \left(\alpha^2(2+\beta-2\psi)-\lambda\right)\left(\alpha^2(1+2\beta-\psi)-\lambda\right)+(\alpha(\alpha\psi-\alpha+1))\alpha^2\beta \end{aligned} \tag{46}$$

which must be equal to zero. Hence, we find two solutions for λ :

$$\lambda = \frac{1}{2\alpha}\left(3\alpha(1-\psi+\beta) \pm \sqrt{\alpha^2(1-\psi(2-\psi+2\beta)+\beta(2+\beta))-4\alpha\beta}\right) \tag{47}$$

We now plug into equation (47) two possible sets of solutions $\phi_1 = \begin{pmatrix}\psi_1 \\ \beta_1\end{pmatrix}$ and $\phi_2 = \begin{pmatrix}\psi_2 \\ \beta_2\end{pmatrix}$. For the first set of solutions $\phi_1 = \begin{pmatrix}1 \\ 0\end{pmatrix}$, we find that both eigenvalues are equal to zero. Thus, the stability of this equilibrium solution cannot be determined analytically[6]. However, as it will be shown in the following section, the numerical exercises provide the evidence that this solution is stable. The second set of solutions: $\phi_2 = \begin{pmatrix}1-\frac{1}{\alpha^2} \\ 0\end{pmatrix}$ yields the eigenvalues of the form $\lambda_1 = \frac{1}{2\alpha^2}\left(\alpha\sqrt{\frac{1}{\alpha^2}}+3\right)$ and $\lambda_2 = -\frac{1}{2\alpha^2}\left(\alpha\sqrt{\frac{1}{\alpha^2}}-3\right)$ which are both positive (remember that $0<\alpha<1$) and, as a result this solution is not stable. The numerical exercises performed in the following section confirm this result.

5 Dynamic analysis

From the previous analysis it follows that both learning models produce the same steady state properties allowing the exchange rate to converge to its fundamental value. Thus,

[6] See M. Braun (1975).

both learning mechanisms are efficient in revealing the fundamental value of the exchange rate in the steady state. These steady state properties, however, do not guarantee the same dynamic properties. As a result, these two learning mechanisms could produce very different short-term behavior of the exchange rate. We study these properties within two different cases. First, we simulate the model with a discount factor $\alpha = 0.99$; in a second simulation we set $\alpha = 0.999$. The low α implies an interest rate of 1% per period. This suggests that the units of time implicit in this simulation should be thought of as months or quarters. The simulations using the high α should be thought of representing high frequency data, e.g. daily observations. In addition, we calibrate the size of the shocks in the fundamental and in the noise to be of the same magnitude as the shocks in the fundamentals observed in real life. These will be described in more detail in Section 7.

5.1 Low frequency observations

We show the contrast in the dynamics produced by the two learning mechanisms by simulating the model in the time domain over 10000 periods with $\alpha = 0.99$. In the fitness learning model, we set the value of the parameter β at 0.95 and ψ at 0.1. This choice may seem arbitrary but it is consistent with empirical evidence suggesting that in the short run agents expect the past change will be almost entirely extrapolated into the future, while they believe that, in the longer run, e.g. 1 to 2 years, the market rate will return to the fundamental value[7]. In the statistical learning model, we assume that the agents learn the underlying process of the market exchange rate using a constant gain $\gamma = 0.01$. This assumption is a sensible way to model the preference of the traders to put more weight on the more recent data[8]. However, under the constant gain learning, the estimated coefficients do not converge to a single point, as suggested by the steady state analysis from the previous section, but to a distribution centered around the estimates[9]. As a result, we expect that the estimated coefficients will be more volatile and with the means corresponding to the solutions ϕ_1.

[7] For the detailed description of the traders' forecasts see Cheung and Chinn (2001) and Cheung et al. (2004).

[8] See Carceles-Poveda and Giannitsarou (2005).

[9] See Evans and Honkapohja (2001) for the necessary conditions for the converegence of the constant gain algorithms.

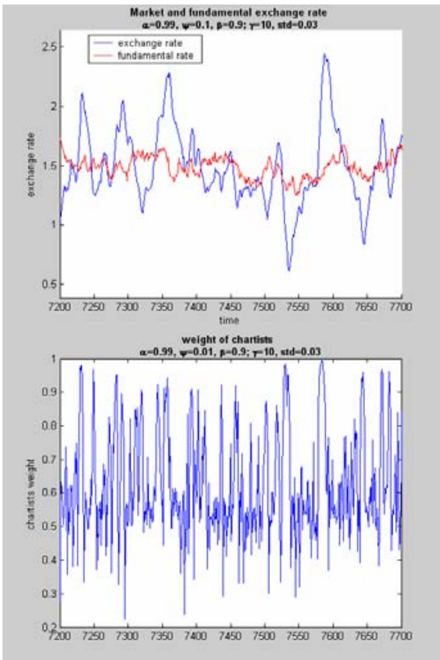

Figure 1: **Exchange rate at low frequency under fitness learning and weight on chartists' rule**

We present the results of fitness learning in Figure 1. The upper panel shows the market and the fundamental exchange rates in the time domain for a subsample of 500 periods. We find that the market exchange rate is often disconnected from the fundamental one. As can be seen in Figure 1, the market exchange rate moves around the fundamental in a cyclical way. These cyclical movements have the appearance of bubbles and crashes. A comparison of the upper and lower parts of the left panel of Figure 1 allows us to understand the nature of these cycical movements. The lower part shows the weights on the chartists' rule. We find that periods of sustained deviations of the exchange rate from the fundamental coincide with periods during which chartists' rule dominates the market expectations. We have analyzed this feature in the framework of similar model in De Grauwe and Grimaldi (2005). Our interpretation of this result is that a series of stochastic shocks can lead to increased profitability of extrapolative (chartist) forecasting rule thereby leading to an increased popularity of this rule at the expense of fundamentalist rule. The latter can be less frequently used because, during the upward phase of the bubble, fundamentalist rule becomes increasingly lossmaking. It creates a self-fulfilling dynamics, since chartist rule becomes more profitable and thus it gets more weight in the market forecast. At some point, movements in the fundamental have the effect of pulling back the exchange rate to its fundamental. We will come back to this feature later and apply a sensitivity analysis to check under what conditions this dynamics occurs.

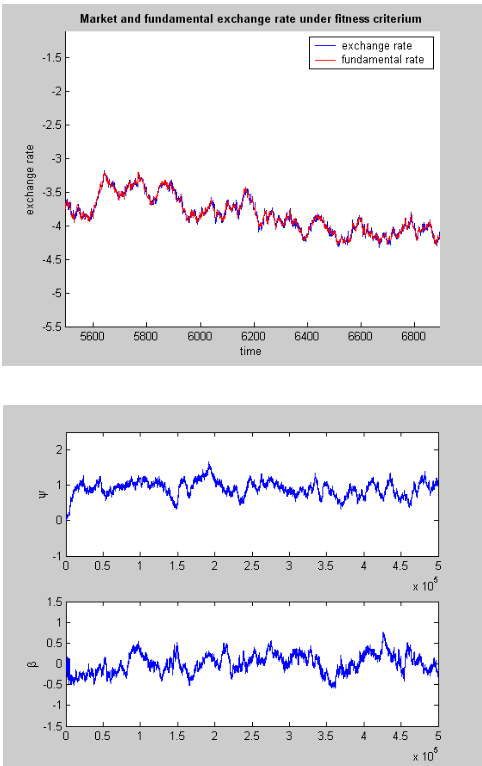

Figure 2: **Exchange rate at low frequency under statistical learning and estimated parameters**

The results of statistical learning, shown in Figure 2, lead to a different conclusion. First, the difference between market and fundamental rates is very small. The mean difference is equal to 0.0011 in the sample shown in Figure 2. Thus, there does not seem to be disconnection nor excess volatility as the exchange rate closely follows the fundamental rate. The lower panel of Figure 2 shows the parameters ψ and β estimated by the agents. As expected, they are very volatile and they fluctuate around 1 and 0. Thus, the agents learn the values of parameters ψ and β distributed around the means leading to the rational expectations solution (see equation (43)). As a result, the market exchange rate is connected to the fundamental rate.

5.2 High frequency observations

In this subsection we analyzed the model in an environment of high frequency observations. We now set the discount factor $\alpha = 0.999$. This implies that we can interpret the time periods over which forecasts are made to be close to days.

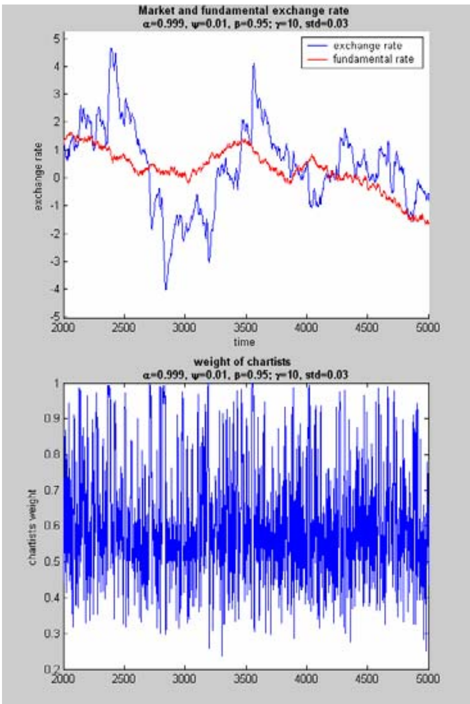

Figure 3: **Exchange rate at high frequency under fitness learning and weight of chartists' rule**

In the fitness learning mode we now set the parameter $\psi = 0.01$. As will be remembered, this parameter measures the speed with which fundamentalists expect the market rate to return to its fundamental. If the model relates to daily observations we should assume that agents expect the return to the fundamental during the next day to be very small. Here we assume that the expected return movement over the next day is 1% of today's deviation. We show an example of a simulation in the time domain in Figure 3. Note that we show the results of more periods than in the previous section, since they now correspond to a much shorter time span. The results are similar to those reported in the previous subsection. However, we now obtain even stronger disconnection. The exchange rate deviates from its fundamental for longer periods. Also the excess volatility appears to be stronger. The results of the statistical learning are shown in Figure 4. We find, as before, that most of the time the exchange rate stays close to its fundamental value. We do not observe sustained disconnections.

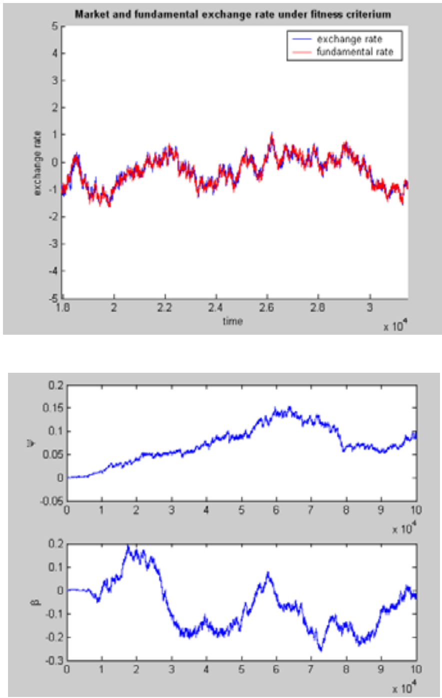

Figure 4: **Exchange rate at high frequency under statistical learning and estimated parameters**

One intriguing result obtained with statistical learning is that the agents do not learn the values of parameters ψ and β distributed around steady state values (i.e. 1 and 0). Our interpretation of this result is that in a high frequency environment, say days, the attempt to update the estimated coefficients on a daily basis is likely to run into difficulties. This has to do with the fact that the high discount rate $\alpha = 0.999$ has the effect of giving a very low weight $(1 - \alpha = 0.001)$ on the current fundamental. As a result, the agents face a near observational equivalence problem. They are unable to recognize whether the weight on the fundamental is equal to 0 or to 0.001. This can also be understood by invoking equation 42, which is reproduced here:

$$T\begin{pmatrix} \psi \\ \beta \end{pmatrix} = \begin{pmatrix} (\alpha\psi - \alpha + 1)(\alpha + \alpha\beta - \alpha\psi + 1) \\ \alpha^2(1 + \beta - \psi)\beta \end{pmatrix} \tag{48}$$

If we assume for the sake of simplicity that $\alpha = 1$ (agents percieve that $\alpha = 1$), we obtain the following T map:

$$T'\begin{pmatrix} \psi \\ \beta \end{pmatrix} = \begin{pmatrix} \psi(2 + \beta - \psi) \\ (1 + \beta - \psi)\beta \end{pmatrix} \tag{49}$$

We see that the only possible fixed point of the system is when $\psi = 0$ and $\beta = 0$. Thus, at the high frequency, when α is close to 1, the agents find that the market exchange rate follows a random walk process. We show in Figure 5 that parameters ψ and β indeed converge both to 0[10]. Note that these are agents who face a near observational equivalence problem while the market exchange rate equation includes $\alpha < 1$. Thus, the fundamental rate still influences the dynamics of the market rate. The resulting market exchange rate is now a convex combination of the current fundamental rate and the past market rate:

$$s_t = (1 - \alpha)s_t^* + \alpha s_{t-1} + \eta_t \tag{50}$$

We see in Figure 5 that the market rate follows the process around the fundamental. The reason for this is that the weight on the fundamental is actually different from 0.

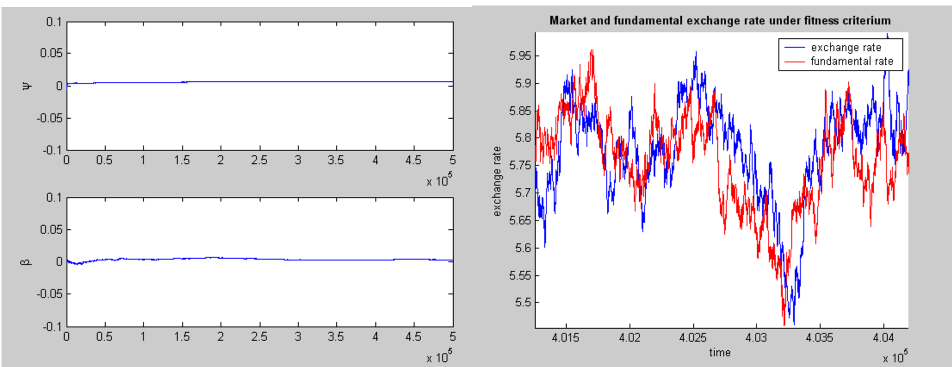

Figure 5: **Exchange rate at high frequency under statistical learning and LS parameters**

[10]We assumed in this simulation the decreasing gain and thus Least Squares updating, in order to show explicitly that both coefficiens coverge to the point 0.

Since agents continue to estimate the exchange rate process, which is not stationary, they can find the whole set of parameter values which do not correspond to the stable solution of the market rate. In particular, we found that with other realizations of the stochastic components of the model, the exchange rate can be explosive or it can be pulled by an attractor creating extreme turbulence. This is because we sometimes find that β does not converge to 0, but tends to go to 1 producing explosive or highly turbulent behavior of the market exchange rate. We show some examples of such attractors in Appendix.

These dynamics occure because the solution $\{\psi = 0, \beta = 0\}$ is unstable. Using the system of equations (49), we write a Jacobian matrix DT' :

$$DT' = \begin{pmatrix} 2(1-\psi)+\beta & \psi \\ -\beta & 1+2\beta-\psi \end{pmatrix}$$

The characteristic polynomial of this matrix is:

$$\begin{aligned} p'(\lambda) &= \det\left(DT' - \lambda I\right) = \det\begin{pmatrix} 2(1-\psi)+\beta-\lambda & \psi \\ -\beta & 1+2\beta-\psi-\lambda \end{pmatrix} \\ &= (2(1-\psi)+\beta-\lambda)(1+2\beta-\psi-\lambda)+\psi\beta \end{aligned} \tag{51}$$

Setting the determinant equal to zero and solving the quadratic equation we obtain two following eigenvalues:

$$\lambda = \left(-\frac{1}{2}\right)\left(3(\psi-\beta) \pm \sqrt{(\beta-\psi)^2 - 2(\beta+\psi)+1} - 3\right)$$

We plug in $\psi = 0$ and $\beta = 0$, and we find eigenvalues $\lambda_1 = 1$ and $\lambda_2 = 2$. Clearly, the solution with both parameters equal to zero is unstable.

6 Sensitivity analysis

In the previous section, we found that the fitness learning model can generate a market exchange rate disconnected from the fundamental rate, and that the statistical learning model can lead to explosive solutions of the exchange rate. In this section, we analyze these characteristics in more details. In particular, we check under what conditions this disconnection and explosive solutions emerge.

6.1 The role of the discount factor

First, we analyze the sensitivity of the results of the different models to the discount factor values. We simulated the model assuming that the stochastic realizations of the fundamental was identical for all the different values of the discount factor α and for all initial conditions. We varied the values of the discount factor α between 0.99 and 0.9999. We show in Figure 6 the results of the model simulated during 100 periods. On the vertical axis we set out the deviation of the exchange rate from its fundamental. On the x-axis we show the different values of the parameter α, and on the y-axis the different initial conditions. The left panel corresponds to the model under fitness learning, the right one under statistical learning.

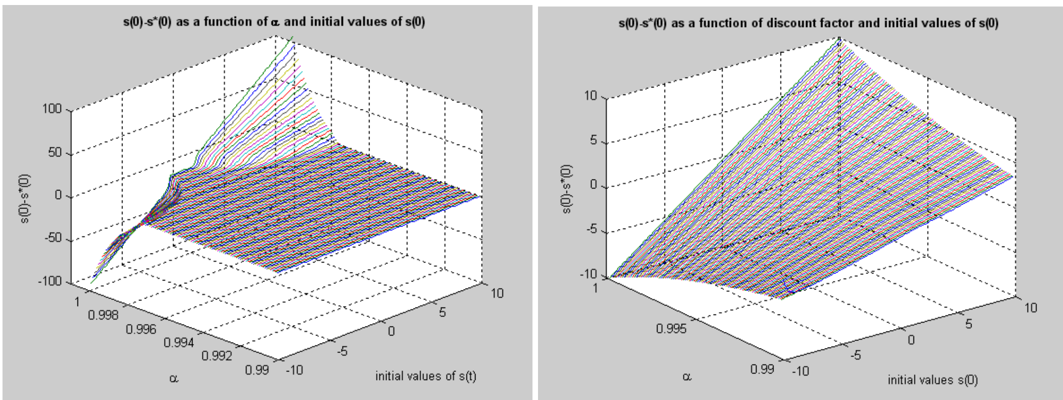

Figure 6: **Difference between market and fundamental rates as a function of discount factor**

We find first that when the model is simulated over short periods of time (T=100), the exchange rate can deviate significantly from its fundamental under fitness and statistical learning cases. Thus, it appears that when the initial shock is large enough and when α is large enough (high frequency), the exchange rate remains disconnected from its fundamental value. In case of fitness learning, the interpretation is following. Given that, in a stochastic environment, it takes a relatively long period for the exchange rate to return to its fundamental value, the exchange rate will often be attracted by temporary equilibria that deviate from the fundamental. This will then lead to relatively long episodes of disconnection. These phenomena become more pronounced when the size of the shocks increases and when the discount factor α increases. It is also for these values that we obtain the bubble and crash scenarios described in the previous section. When agents use the statistical learning and the discount factor is close to one, we found in the previous section that the solution of the process is given by the parameters ψ and β equal to zero. Then, the market rate is described by the process in equation (50). On the high frequencies, the weight on the past values of the market weight is very high. As a result, as shown in Figure 6, the initial high values of market rate are followed by high deviations from the fundamental rate (normalize to zero). We find this scenario when the agents estimate both parameters ψ and β to be zero. These solutions are however unstable (see previous section) and we also find the strong departures from these values. For this reason, we also carried out simulations of the model under statistical learning assuming different initial values of the exchange rate and different values of the discount factor α, and checked the resulting values of parameters ψ and β. Although we realize that the constant gain learning describes better traders' behavior, we assumed in this exercise the decreasing gain. The reason is that we wanted to check the precise values that the parameters ψ and β converge to.

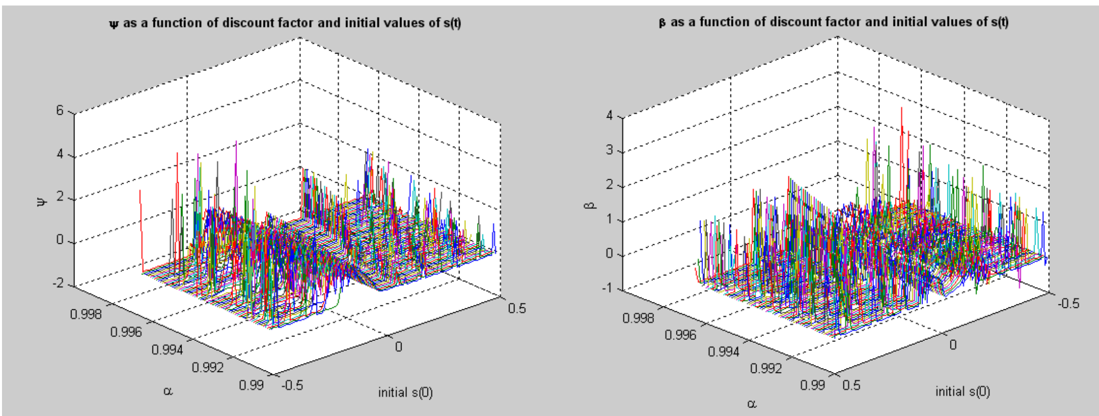

Figure 7: **Difference between market and fundamental rates as a function of initial conditions and disount facor**

We computed the difference of the exchange rate and the fundamental after 100 periods and plotted this value in a three-dimensional figure. This figure has the initial value of market rate (s_0) on the y-axis and the different values of α on the x-axis (see Figure 7). We find that very often the parameter values strongly deviate from their steady state values. We observe two cases. First, both parameters can be very close to zero and this produces the exchange rate close to the random walk (see section above and equation(50)). Second, they can take very high values which lead to explosive of high-turbulence solutions for the exchange rate, which we presented in Appendix.

6.2 Sensitivity to β

In order to better understand the cyclical nature of the short-term dynamics in the fitness learning, we performed a sensitivity analysis whereby we allowed parameter β and the initial conditions to change. We simulated the model assuming that the stochastic realization of the fundamental was identical for all the different values of the parameter β and for all initial conditions. We then simulated the model for different time lengths going from T=50 to T=1000 and we collected the deviation of the exchange rate from its fundamental in period T. We show the results in Figure 8. On the vertical axis we

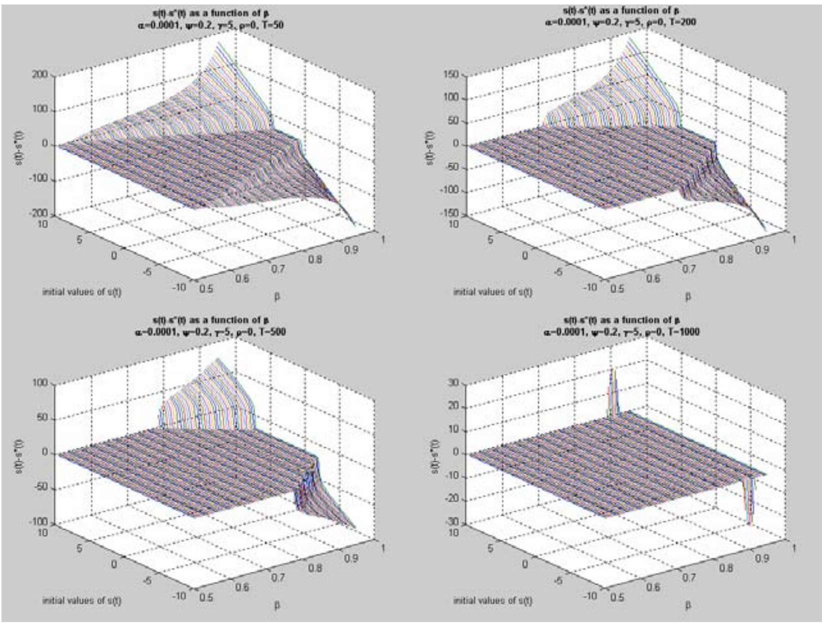

Figure 8: **Difference between market and fundamental rates as a function of initial conditions and beta**

set out the deviation of the exchange rate from its fundamental after 50, 200, 500 and 1000 periods respectively. On the x-axis we show the different values of the parameter

β, and on the y-axis the different initial conditions. We find first that when the model is simulated over short periods of time, the exchange rate can deviate significantly from its fundamental. When for example the model is simulated for T=50, the exchange rate in period T deviates from its fundamental when initial shocks are sufficiently large and when β increases. Thus, it appears that when the initial shock is large enough and when β is large enough, the exchange rate remains disconnected from its fundamental value even after 50 periods. As the simulation period is extended, the area of disconnections shrinks. When T exceeds 1000, the exchange rate has returned to its fundamental value for all initial conditions and for all values of β. This result is consistent with our steady state analysis which showed that in the steady state the exchange rate is equal to its fundamental. However, given that it takes a relatively long period for the exchange rate to return to its fundamental value, in a stochastic environment, the exchange rate will often be attracted by temporary equilibria that deviate from the fundamental. This will then lead to relatively long episodes of disconnection. These phenomena become more pronounced when the size of the stochastic shocks increases and when the chartists' extrapolation parameter β increases. It is also for these values that we obtain the bubble and crash scenarios illustrated in the previous section.

7 Descriptive statistics

In this section, we analyze the variability characteristics of the market exchange rate produced by the model under two different learning mechanisms. For this purpose, we use monthly market rates of the German Mark, Japanese Yen and UK Pound, during the period between the sixth month of 1982 and the twelfth month of 1998. These market rates are expressed as units of these currencies per US dollar and they are provided by International Financial Statistics. The monthly fundamental rates are constructed on the basis of the monetary model:

$$s_t^* = (m_t - m_t^*) - \phi(y_t - y_t^*) \tag{52}$$

where s_t^* denotes the log fundamental rate expressed as units of national currency per US dollar, m_t, m_t^* and y_t, y_t^* are money supplies and real incomes in the home and foreign country, respectively. The income elasticity of money demand ϕ is assumed to be 1. The money supply is proxied by seasonally adjusted M2 aggregates coming from OECD Main Economic Indicators Database. For the real income data, we use seasonally adjusted industrial production from the same database. We have 200 data points and simulate the model with the same number of periods under two learning rules assuming $\alpha = 0.995$.

Table 1: **Volatility in the exchange rate market**

	Market volatility	*Fundamental volatility*	*Excess Volatility*
DM	0.027	0.015	0.012
JY	0.030	0.014	0.016
UKP	0.027	0.013	0.014
Statistical Learning	0.015	0.015	0.000
Fitness Learning	0.021	0.015	0.006

Return volatility is measured by a sample standard deviation of the returns. DM denotes the German Mark, JY the Japanese Yen and UKP the UK Pound. The excess volatility is calculated as a difference between the market return volatility and the fundamental return volatility.

Table 1 shows the unconditional volatility of the market and fundamental rates for three currencies and of the simulated series of the market exchange rate under fitness and statistical learning. The unconditional volatility is measured as a sample standard deviation of the returns. In the simulations we set the standard deviation of the fundamental rate equal to 0.015 (remember that the fundamental rate follows a random walk eq.3), which is a number corresponding to the standard deviation found in the data (see the first three columns of Table 1). We notice first that the model under statistical learning does not generate excess volatility of the exchange rate returns. This is not surprising since, agents learn the solution distributed around the rational expectations solution of the asset pricing model. As a result, the market exchange rate moves around the fundamental rate and its volatility is very close to the one of the fundamental rate. We also see from Table 1 that the model under fitness learning produces excess volatility of the market rate. This excess volatility is equal to 0.006. Although it is lower that the one found in the data, the fitness learning outperforms the statistical learning in reproducing the excess volatility feature of the exchange rate.

7.1 Conclusion

In this paper, we investigated the behavior of the exchange rate within the framework of a standard asset pricing model. We introduced into this model boundedly rational agents who use simple rules to forecast the future exchange rate. However, they test these rules continuously. This testing procedure is the mechanism by which discipline is imposed on the behavior of individual agents. We specified two alternative testing procedures (learning mechanisms). In the first one agents select the rules based on a fitness method. This mechanism assumes that agents evaluate the two forecasting rules by computing their past profitability and to increase (reduce) the weight of one rule if it is more (less) profitable than the alternative rule. In the second mechanism, agents learn to improve these rules using statistical methods. They are assumed to have some basic knowledge of econometrics, such that they estimate the parameters of the rules they use. In order to investigate which of these learning rules generates more realistic dynamics of the exchange rate, we carried out a number of analytical and numerical exercises.

First, we found that both learning mechanisms are efficient in revealing the fundamental value of the exchange rate in the steady state. Second, they can generate very different short-term behavior of the exchange rate. In particular, we find that fitness learning comes more closely to mimicking important regularities in the foreign exchange markets than the statistical learning model.

The fitness learning model produces the disconnect phenomenon of the exchange rate. The disconnections produced by the model are large and persistent, indicating the presence of bubbles and crashes. The statistical learning model does not reproduce the disconnection phenomenon of the exchange rate. When the model is applied to low frequency observations (low discount rate), we found that the market rate moves closely around the fundamental rate, in line with the rational expectations solution. For high frequency observations (high discount rate), the statistical learning model can generate a near random walk behavior of the exchange rate. It can then, however, also lead to an explosive solution. This is because, in a high frequency environment, the weight attached to the fundamental is very low. As a result, agents using statistical learning do not easily recognize the signal coming from the fundamental in the exchange rate process.

Finally we analyzed in detail the variability characteristics of the market exchange rate produced by the model under the two different learning mechanisms, and we compared these with those observed in the data. We found that the fitness learning model comes closer to mimicking the excess volatility observed in the market exchange rates than the statistical learning model.

More research needs to be done to test the validity of these two learning methods. In particular we intend to analyze other statistical properties of the exchange rate movements generated by the two models (e.g. volatility clustering and excess kurtosis) and to confront these to the observed movements of the exchange rate. This will allow us to evaluate the two modelling approaches with more confidence.

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8 Appendix

In Figures 9 and 10, we present an example of a simulation of the proposed model under statistical learning assuming high-frequency observations. We simulated the model over

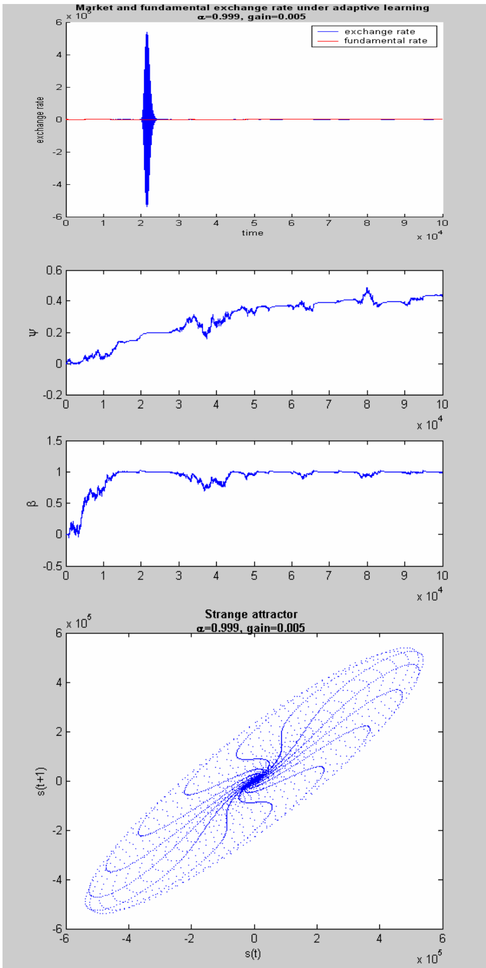

Figure 9: **Explosive solution under statistical learning**

100.000 periods. We have set $\alpha = 0.999$. The upper part of the figure shows the exchange rate in the time domain. It can be seen that the exchange rate is sometimes gripped by episodes of extreme turbulence. In the top part of the figure, there is one such episode visble. However, when we blow up periods of apparent tranquility, it appears that these are also periods of high turbulence. We show one such blow-up in Figure 10. The underlying reason is that agents learn too high value of the extrapolative parameter

β (which tends to converge to 1) and too low a value of the mean-revering parameter ψ (see the second and third panels of Figure9). This is due to the fact that statistical updating in the high frequency observations environment gives a very low weight to the signals comming from the fundamental. As a result, the extrapolative part in the dynamics of the exchange rate gets an unduly large weight in the updating.

The lower panel in Figure 9, shows the phase diagram corresponding to the simulations of the exchange rate (top panel). We obtain a very complex attactor. Attractors obtained under other stochastic realisations of the noise and the fundamental are all different in shape but have a similar structure of spiraling towards the equilibrium value.

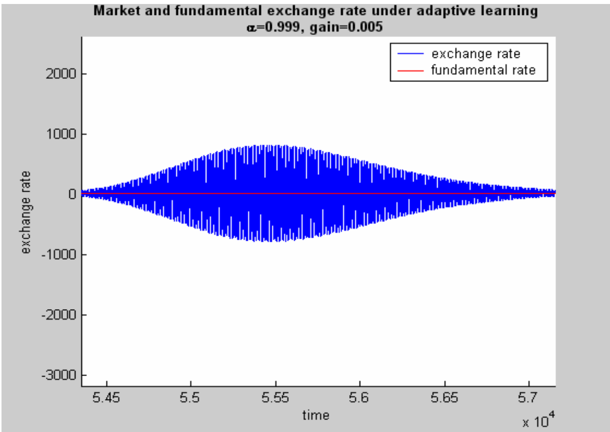

Figure 10: **Market and fundamental rates under statistical learning**