Preliminary draft

Exchange Rates, Non-Linear Dynamics and Chaos

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1. Introduction

The large econometric literature analysing the Purchasing Power Parity (PPP) has established that deviations from PPP take a very long time to die out (Rogoff (1995), Obstfeld & Rogoff (2000), Cheung & Lai (2000)). Put differently, when shocks occur in the real exchange rate the time needed to return to the equilibrium value is surprisingly long. The half-life of the PPP deviations has been estimated to be of the order of 4 to 5 years. Some researchers have found even longer half-lives (Lothian & Taylor (1998), O' Connell (1998)). This long time needed to achieve PPP after a shock has been called the PPP puzzle.

Some researchers (Dumas (1992)) have stressed that the long time needed to adjust to PPP might be due to the existence of transaction costs. The latter imply that when shocks to the exchange rate are small relative to transaction costs, arbitrage fails and therefore the mean reversion process in the real exchange rate does not operate. Only when the shocks are large enough, the forces of arbitrage impose mean reversion. The transaction cost hypothesis implies a non-linearity in the adjustment process. Recent empirical research based on time series confirms that the non-linearities are important in explaining the PPP puzzle (Michael, Nobay, Peel (1997), Kilian & Taylor (2001)).

In this paper we pursue two objectives. First we provide additional evidence about the non-linear nature of the adjustment towards PPP. Second we analyse the implications of this non-linear adjustment dynamics in the goods market for the modelling of the exchange rate. In section 2 we provide the empirical evidence about the non-linearity in the adjustment towards PPP using data of approximately 100 countries during 1970-99. Section 3 formulates some implications for the specification of dynamic adjustment rules, and section 4 presents a simple model of the exchange rate, which incorporates the non-linear adjustment dynamics towards PPP.

2. Cross-section analysis over different time horizons

In this section we investigate the PPP-relationship over different time horizons using a cross-section of about 100 countries. We proceed as follows. We define the PPP relationship over different time horizons starting from one year, two years, five years, seven years and finally ten years.

More formally, we specify the PPP relationship as follows:

$$e_{it} = b_1 + b_2 p_{it} + b_3 y_{it} + v_{it} \qquad (1)$$

where e_{it} is the rate of depreciation of currency i relative to the dollar over the time horizon t. Similarly, p_t and y_{it} are the inflation rate and output growth of country i over the same time horizon t. The growth rates of prices and output are expressed as differences with the US. We add the output growth rate to the PPP relationship in order to capture the effects that the real sector has on the exchange rate. Ideally we would like to have productivity growth as an explanatory variable, but the data were not available for all the countries included in the sample.

We apply the cross-section analysis over six different time horizons, i.e. t=1, t=2, t=3, t=5, t=7 and t=10 so that we obtain six separate cross-section analyses. The sample period is 1971-99. All the data are from IMF, IFS.

In order to test for possible non-linearities in the adjustment process, we ordered the data according to the size of the shocks and applied a recursive estimation procedure, which allows us to find out whether the coefficient of inflation in equation 1 changes with the size of the shocks The transaction cost hypothesis predicts that these coefficients will increase when the size of the shocks increases. This should hold over the different time horizons used in the cross-section regressions. The results of the regression analysis are presented in the following table 1.

Table 1 Regression results of equation 1 (OLS with White correction)

	coefficients					
Variable	**T=1**	**T=2**	**T=3**	**T=5**	**T=7**	**T=10**
Constant	-0.003	0.0	-0.01*	-0.002*	0.004	-0.006
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
cpi	1.014*	0.967*	0.776*	0.911*	0.873*	0.946*
	(0.13)	(0.06)	(0.05)	(0.06)	(0.03)	(0.04)
Gdp	-0.250*	-0.236*	-0.074	0.137	0.139	-0.025
	(0.08)	(0.09)	(0.09)	(0.14)	(0.14)	(0.13)
Number of observations	2169	1047	619	320	242	152
Adjusted R-squared	0.62	0.79	0.76	0.88	0.90	0.94

Note : * statistically significant at 5% level

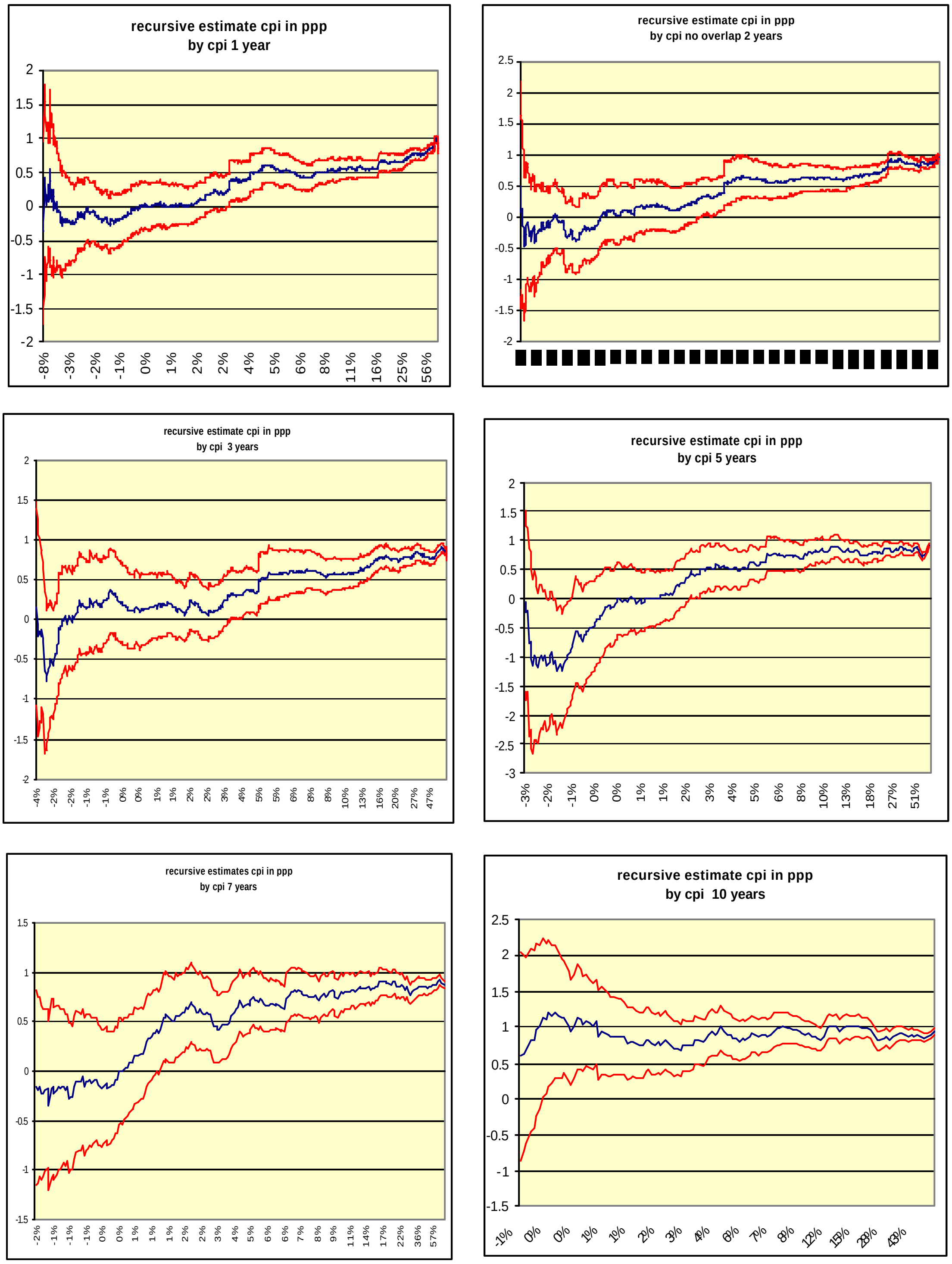
From table 1 we observe that the coefficients of inflation are close to one and statistically significant. This is the case for all time horizons. It is worthwhile to note that the explanatory power of the regressions (as measured by the R^2) increases as the time horizon increases. This confirms the intuition that in the short run the PPP relationship holds less precisely than in the long run.

The recursive estimates presented in figure 1 can be interpreted as follows. First, we observe that for all time horizons the coefficient of inflation increases with the size of the shocks in inflation. Another way to interpret this result is that large shocks in inflation produce smaller effects on the real exchange rate than small shocks. Second, only when the size of the shock is large enough does the coefficient come close to one. For low and intermediate sizes of shocks we reject that the coefficient of inflation is equal to one. Third, we note that when the time horizon increases the curve shifts upwards. This means that a given shock in inflation leads to a larger adjustment in the exchange rate as the time horizon increases. Put differently, the mean-reverting process in the real exchange rate becomes stronger as time goes by.

Figures 1 suggests the existence of an inverse relationship between the size of the shocks in inflation and the time it takes for the real exchange rates to return to their equilibrium. In order to investigate this relationship more formally we construct a graph relating the size of the shocks and the time it takes to reach a given adjustment in the real exchange rates. We show this graph in figure 2, which was constructed from the recursive estimates of figure 1 as follows. Each graph in figure 1 can be interpreted as the percentages of the adjustment in the real exchange rate after shocks in inflation. In each of these graphs we selected the coefficient 0.5 and the corresponding value of the shock in inflation. These numbers were plotted in figure 2. We obtain a non-linear relationship that can be interpreted as follows. We find that a shock of 40% in inflation necessitates 1 year to lead to a change in the real exchange rate representing half of the total adjustment. In other words, the half-life of the adjustment process is 1 year when the shock size is

40%. When the size of the shock is 25% the half-life increases to two years, for a shock of 10% the half-life is almost seven years.

Figure 1



Note: the thin lines represent ± 2σ

We also performed the analysis with different values of the recursive coefficients. We set the coefficient values equal to 0.2, 0.4, 0.6 and 0.8. We found similar non-linear relationship as in figure 2. These are presented in figure 3. This suggests that this non-linear relationship between size of shock and time to adjust holds independently of whether 20%, 40% 60% or 80% of adjustment has occurred.

Figure 2

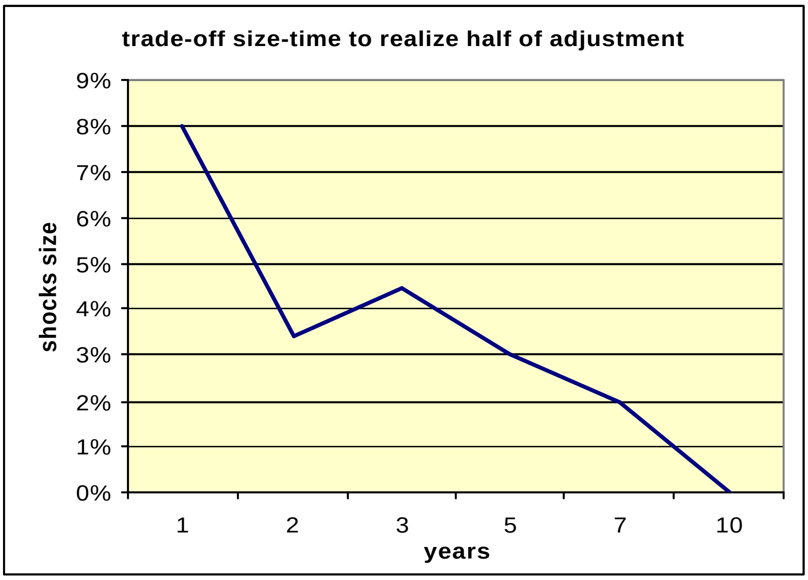
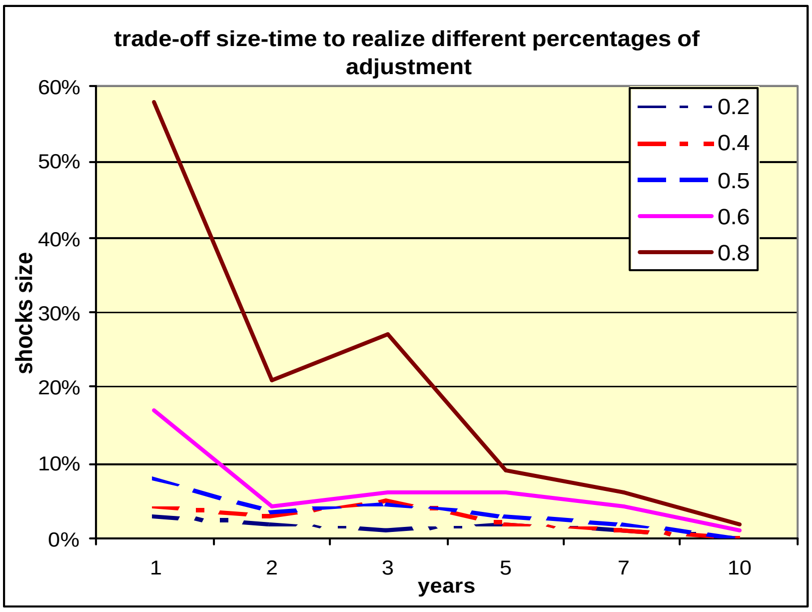


Figure 3



An interesting aspect of the graphs in figures 2 and 3 is that they approximate hyperbolas. This means that the product between the two dimensions of time and size is approximately a constant. Therefore a shock of x% that takes 1 year to achieve half-adjustment is approximately equivalent to a shock of half of x% that takes two years to achieve the same adjustment. This feature has an important implication for the speed of adjustment of prices in goods markets. We discuss this issue in the next section.

3. Theoretical issues

The econometric results derived in the previous section have theoretical implications about the dynamics of price adjustments. More precisely, we will show that the dynamics of the adjustment is not linear. Before doing this, we analyse the implications of the linear price adjustment rule that is traditionally used in macroeconomic models and we will show how this contradicts the empirical results we found.

3.1 The linear price adjustment dynamics

The traditional specification of the dynamics of the adjustment of prices is in the following *linear* way, i.e.

$$\frac{dp}{dt} = -q\left(p - p^*\right) \qquad (1)$$

where p* is the equilibrium value of prices. According to this specification the rate of change in prices is linearly related to the gap between the actual value of p and its equilibrium value p*. θ is a positive constant determined by the speed of adjustment in the goods market.

The important implication of this rule is that the relative change of the gap over time is equal to a constant. We can see this by rewriting equation (1) in the following way:

$$\frac{dz}{z}\frac{1}{dt} = -q \qquad (2)$$

where z = p-p*, and can be interpreted as the gap between actual and equilibrium prices.

Equation (2) says that the speed with which the *relative* gap between the actual and the equilibrium value of prices is reduced, is constant over time. This notably implies that the time it takes the variable to reach the equilibrium is independent of the size of the absolute gap.

The solution of the differential equation (2) is given by:

$$z_t = z_0 e^{-qt} \qquad (3)$$

where z_0 is the initial value of z . We can rewrite equation (3) as follows

$$\frac{z_t}{z_0} = e^{-qt} \qquad (4)$$

the ratio z_t/z_0 measures how much the initial shock is adjusted in period t. If we set this ratio equal to 0.5 we can compute the half-life of the adjustment.

$$0.5 = e^{-qt} \tag{5}$$

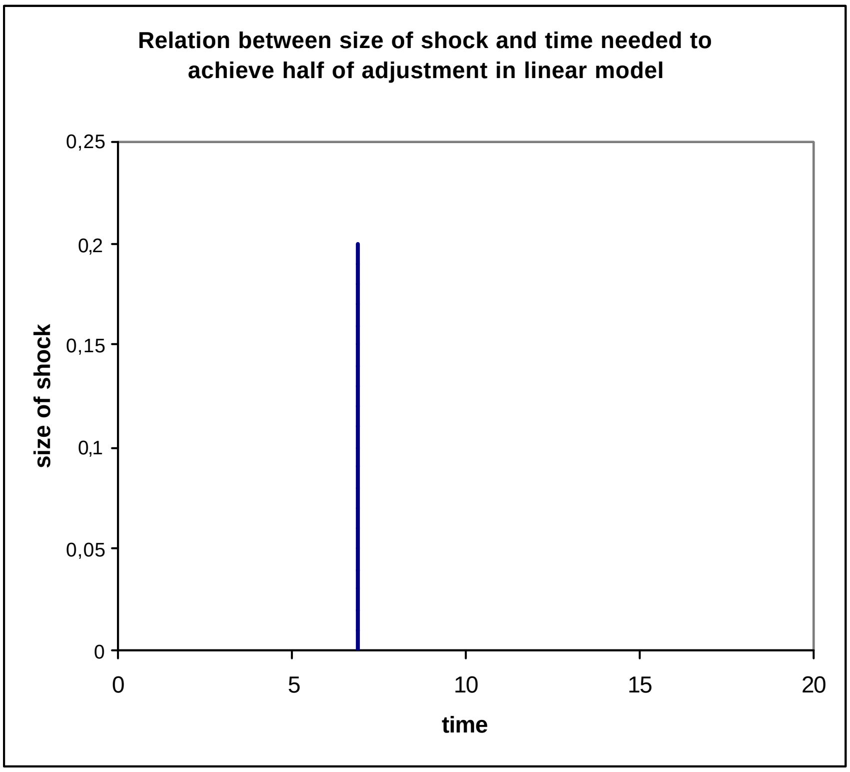
Solving for t we obtain

$$t = \frac{0.69}{q} \tag{6}$$

Thus, we can conclude that the time needed to achieve half of the adjustment depends on the constant θ and is independent of the size of the initial shock.

This feature of the linear adjustment dynamics is shown in figure 3 where we plot the relation between the size of the initial shock and the time to achieve half of the adjustment (half-life of the adjustment) in the linear model. It can immediately be seen that the linear model is not appropriate to explain our empirical results. The latter suggest that the relationship between the size of the shocks and the half-life of the adjustment is non-linear (hyperbolic, see figure 1). In order to deal with this problem, we analyse a simple non-linear model of price adjustment in the next section.

Figure 3



3.2 A simple quadratic price dynamics

We now turn to a non-linear specification of the price dynamics. We assume a simple quadratic specification, i.e.:

$$\frac{dp}{dt} = q(p - p^*)^2 \qquad \text{if } p < p^* \tag{7}$$

and

$$\frac{dp}{dt} = -q(p - p^*)^2 \qquad \text{if } p > p^* \tag{8}$$

This specification of the price dynamics assumes that the rate of change in prices accelerates when the gap between actual and equilibrium price becomes larger. The underlying logic is that arbitrage forces increase as the profit opportunities become larger. The reason is that to undertake arbitrage some fixed cost must borne (e.g. buying a truck, chartering an airplane). Therefore, as arbitrage profits increase more agents are willing to undertake such investments, so that the resources used in arbitrage activities increase.

Taking the integral and solving this equation for the gap z = p-p* we obtain

$$z = -\frac{1}{qt + c} \qquad \text{if } z<0$$

(9)

$$z = \frac{1}{qt + c} \qquad \text{if } z>0$$

where c is an arbitrary constant . Setting t=0 we find that

$$c = -\frac{1}{z_0} \qquad \text{if } z<0 \qquad (10)$$

$$c = \frac{1}{z_0} \qquad \text{if } z>0 \qquad (11)$$

In the following we concentrate on positive shocks. The solution is then given by

$$z_t = z_0 \frac{1}{z_0 qt + 1} \qquad (12)$$

We rewrite equation (12) as follows:

$$\frac{z_t}{z_0} = \frac{1}{z_0 qt + 1} \tag{13}$$

Like in the linear case we compute the half-life of the adjustment process setting z_t/z_0 =0.5 and solving equation (13) for t. We obtain

$$t = \frac{1}{qz_0} \tag{14}$$

We find that the half-life of the adjustment process decreases with the size of the initial shock z_0. In other words, when shocks are large, the time to achieve half of the adjustment is shorter than the time needed to achieve the same adjustment when shocks are small.

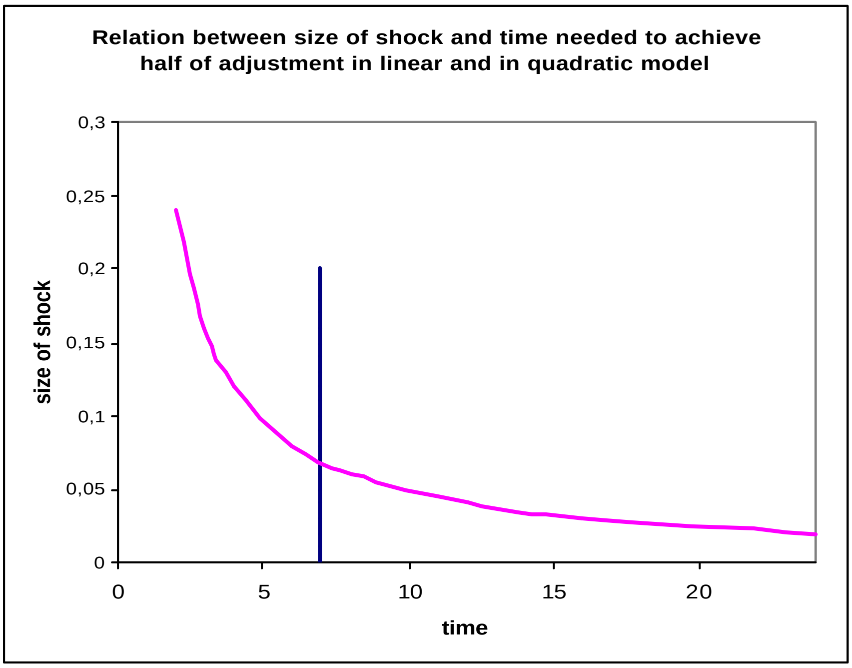
The equation (14) can be rewritten as:

$$z_0 t = \frac{1}{q} \tag{15}$$

Equation (15) states that the product between size of shocks and time to adjust is equal to a constant. This is the approximate empirical relation we have found. Thus, a simple non-linear price adjustment rule is capable of reproducing an empirical regularity found in the data.

The contrast between the linear and the non-linear price adjustment rules becomes obvious in figure 4 where we present the relation between the size of the shock and the time it takes to reach half of the adjustment in both the linear and the non-linear models. The vertical line parallel to the y-axis represents the trade-off between size and time in the linear model and the hyperbolic line represents this trade-off in the non-linear price dynamics model. We see that the simple quadratic model is capable of mimicking the empirical result that the speed of adjustment depends on the size of the shocks. Other more complicated non-linear equations could also be used to reproduce the empirical evidence. Here we have concentrated on the simplest possible one. (For other specifications, see e.g. Kilian and Taylor(2001)).

Figure 4



4. **A Simple non-linear exchange rate model**

We discovered from the empirical evidence of the previous section that the adjustment towards PPP is not linear in nature. We now analyse the implications of this non-linear dynamics for the behaviour of the exchange rate. In order to do so, we develop a very simple non-linear exchange rate model. The main characteristic of this model is that some agents ("fundamentalists") take into account that the goods markets dynamics is non-linear. We analyse whether this simple model can generate a complex (chaotic) dynamics.

4.1 The model

We assume that there are two types of agents in the foreign exchange market, chartists and fundamentalists, using different information sets[1]. The chartists forecast the future exchange rate by extrapolating the past behaviour of the exchange rate. In this sense they use a *positive feedback rule*. The fundamentalists base their forecasts of the future exchange rate on a model of the economy. More particularly, they compute the equilibrium value of the exchange rate, e.g. the PPP value. When the exchange rate deviates from its equilibrium value then they forecasts that it returns to it in the future. In this sense they use a *negative feedback rule*.

We specify these two rules in the following way.

The chartists forecast is written as:

[1] This way of modelling the foreign exchange market was first proposed by Frankel and Froot (1988). It was further extended by De Long et al. (1990) and De Grauwe et al. (1993) and more recently Kilian and Taylor (2001). For evidence about the use of chartism see Allen and Taylor (1989).

$$E_{t-1}^{C}(\Delta e_t) = b\sum_{i=1}^{T} a_i \Delta e_{t-i} \tag{15}$$

where E_t^C is the forecasts made by chartists using information up to time t. Δe_t is the change in exchange rate. As can be seen, the chartists compute a moving average of the past exchange rate changes and they extrapolate this into the future exchange rate change. The degree of extrapolation is given by the parameter β.

The fundamentalists' behaviour can be represented as follows:

$$E_{t-1}^{F}(\Delta e_t) = q\left|e_{t-1} - e_{t-1}{*}\right|\left(e_{t-1} - e_{t-1}{*}\right) \tag{16}$$

where E_{t-1}^F represents the forecasts made by the fundamentalists at time t-1. $e_{t-1}*$ is the equilibrium exchange rate. This equilibrium value might be the PPP equilibrium value but also the equilibrium value obtained by a more complex model. We do not model this equilibrium value but we assume that it follows a random walk. We assume that the fundamentalists know that the adjustment process is non-linear in nature and therefore they expect that the return to the equilibrium will follow the non-linear dynamics. We use a simple non-linear specification to model their forecasts, i.e. the same one as specified in section 3 (see equations 7 and 8 for the model in differential form).

We assume that the market forecast is an average between the forecasts made by chartists and fundamentalists assuming equal weights[2],i.e. :

$$E_{t-1}(\Delta e_t) = 0.5E_{t-1}^C(\Delta e_t) + 0.5E_{t-1}^F(\Delta e_t) \tag{17}$$

where $E_{t-1}(\Delta e_t)$ is the market expectation of the changes in the exchange rate.

Assuming market efficiency, the change in the market exchange rate from t-1 to t is equal to the change forecasted in t-1 plus a white noise error, i.e.

$$\Delta e_t = E_{t-1}(\Delta e_t) + e_t \tag{18}$$

As stressed in the previous chapters, the transaction costs are important to explain the dynamics of the adjustment. Therefore, we introduce the transaction costs in the model represented by equations (15)-(18). If there are transaction costs the fundamentalists will take this information into account. Therefore, if the exchange rate is within the transaction costs band the fundamentalists will behave differently than if the exchange rate moves out of the transaction costs band. Consider the first case, when the exchange rate deviation from its equilibrium value is larger than the transaction costs C. Then the fundamentalists follow the same forecasting rule as in equation (16). More formally, when $|e_{t-i} - e_{t-i}*| > C$ holds, then equation (16) applies. In the second case the

[2] Note that this is a simplification. More complex weight schemes could be used (see De Grauwe et al. (1993) that give a varying weight to chartists and fundamentalists.

exchange rate deviations from the equilibrium value are smaller than the transaction costs. Then the fundamentalists know that arbitrage in the goods market does not apply. As a result, they expect the changes in the exchange rate to follow the white noise process ε_t. The best they can do is to forecast no change. More formally, when $|e_{t-1} - e_{t-1}^*| < C$, then $E^F_{t-1}(\Delta e_t) = 0$.

4.2 Solution of the model

The non-linear structure of our model does not allow us to derive analytic solutions. Therefore we provide results with simulation techniques using plausible values of the parameters. We will also analyse how sensitive the results are with respect to these parameter values.

In the first step we look at the deterministic part of the model, i.e. we eliminate all stochastic variables. In the second step we introduce the stochastic variables (*news*).

4.2.1 Solution of the deterministic model

In order to better understand the implications from the deterministic model we first analyse a simplified version where we set all the lags in the exchange rate beyond 2 periods equal to zero. This yields the following simplified version of the model:

$$e_t = \left(1 + \frac{b}{2}\right) e_{t-1} - \frac{b}{2} e_{t-2} + \frac{q}{2} e_{t-1}^2 \qquad (19)$$

This simplified version of our model has a logistic structure[3], i.e. for a given value of e_{t-2} we obtain a logistic equation. We first represent the logistic curve for θ =-0.3 and β =5 in figure 5.

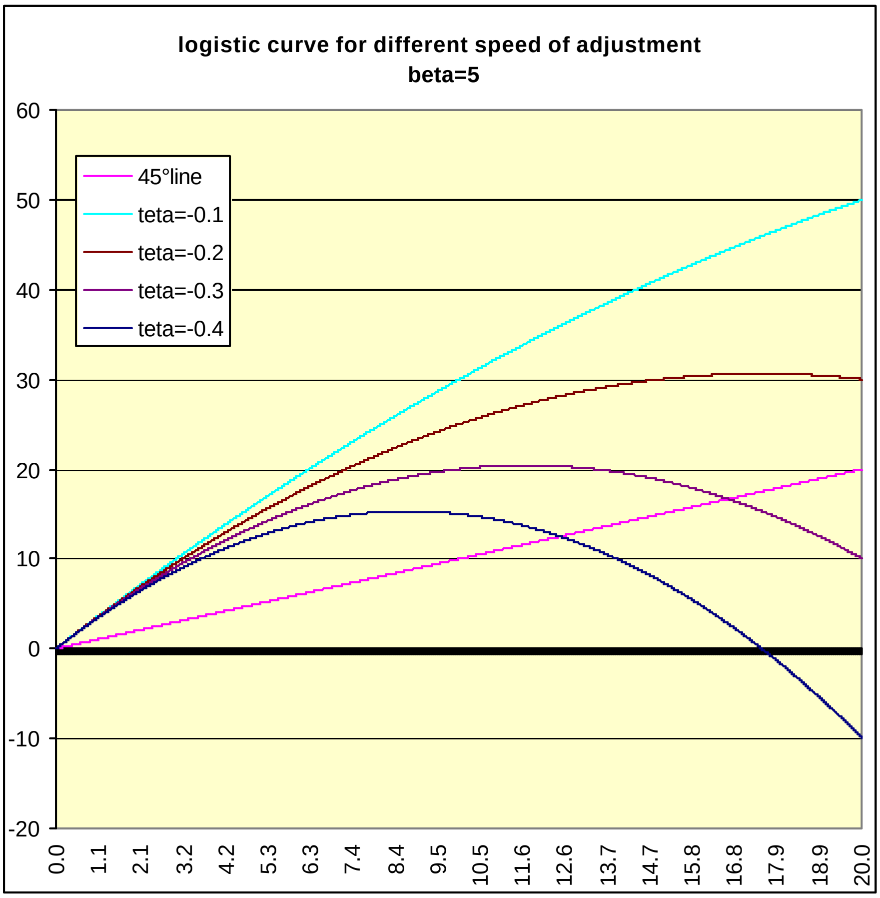
Figure 5

Figure 5 shows the logistic structure of the model, which is the result of the different behaviour of the chartists and fundamentalists. When the exchange rate is close to its equilibrium value, i.e. zero, the exchange rate movements are driven by the chartists who extrapolate the past into the future. Therefore the curve is upward-sloping. When the exchange rate deviates more and more from its equilibrium value the

[3] See Baumol and Benhabib (1989)

fundamentalists become more important and overwhelm the chartists. Therefore the curve becomes downward-sloping. The exact shape of the logistic curve depends on the value of the parameters. In figure 6 we show the logistic curve for the given value of β =5 and different values of θ. When the speed of adjustment increases the downward action of the fundamentalists occurs sooner and the intersection point with the 45° line also occurs sooner.

Figure 6



We now return to the full model represented by equations (15)-(18) and we simulate it using different combinations of parameters. Our main result is that a relatively simple model is capable of generating very complex exchange rate behaviour. In appendix 1 we produce a table where we present the nature of the solution for different combinations of parameters. It can be seen that for some combinations we obtain a fixed-point solution, for other combinations we have limit-cycle solutions, and other combinations give chaotic solutions. In fact we find that the exchange rate follows a chaotic pattern for a relatively broad range of parameter values. We show some examples of chaotic dynamics in figures 7 and 8. Figure 7 presents results when we assume transactions costs (C=5) and figure 8 shows results in the absence of transactions costs (C=0). In the panels of figures 7 and 8 a we show the strange attractors. In panels b we show the results of performing a sensitivity analysis, which consisted in increasing slightly (0.01) the size of the shocks in the initial exchange rate. Note that we have normalised the equilibrium value of the exchange rate to be equal to zero.

The strange attractor's panels in both figures 7 and 8 show that our model has a potential chaotic structure. For given combinations of parameters the exchange rate follows a chaotic path designed by the shape of the strange attractor. The tests of sensitivity to initial conditions confirm the intrinsic chaotic nature of the model.

Figure 7.a

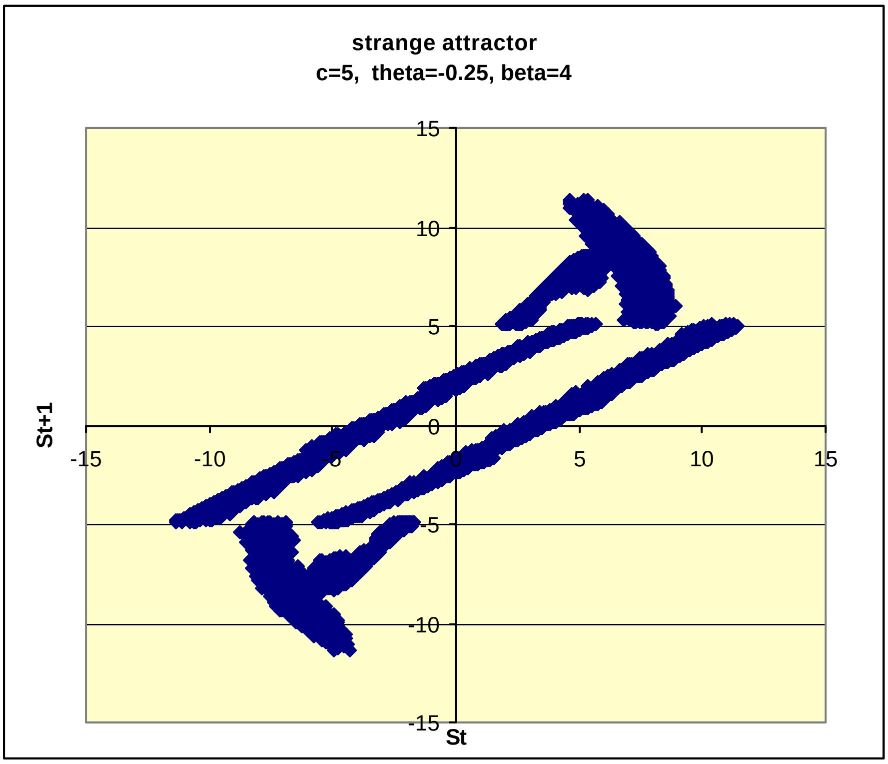


Figure 7.b

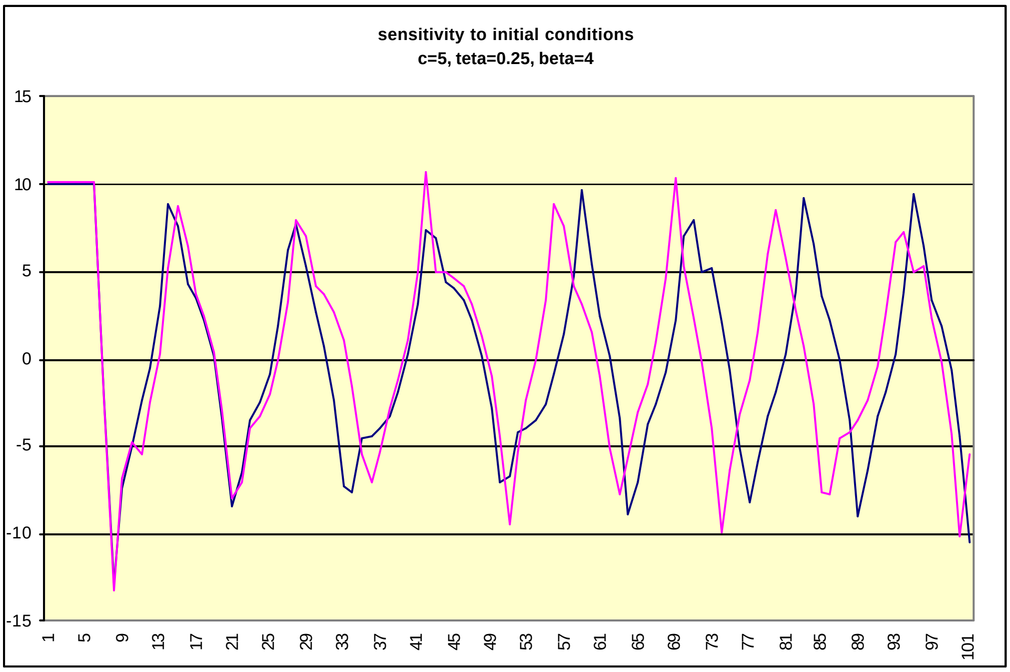


Figure 8.a

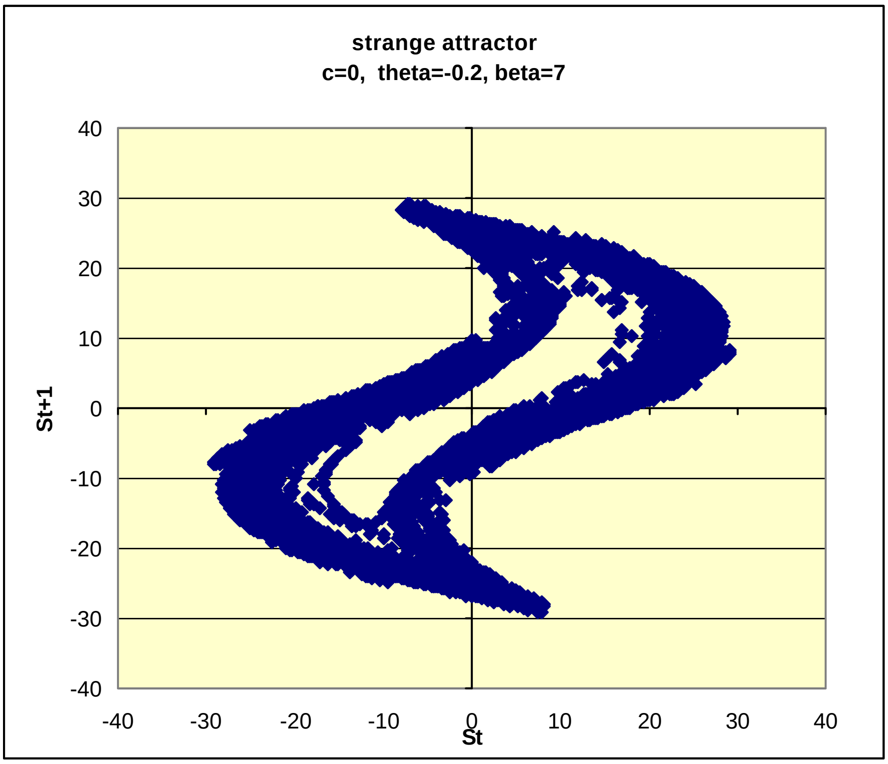
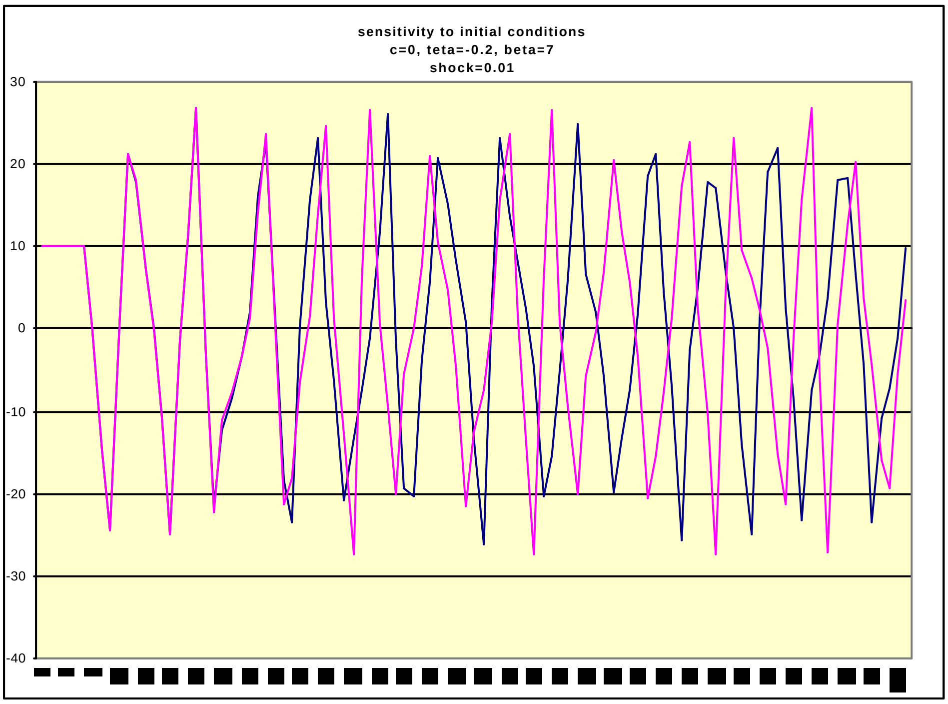


Figure 8.b

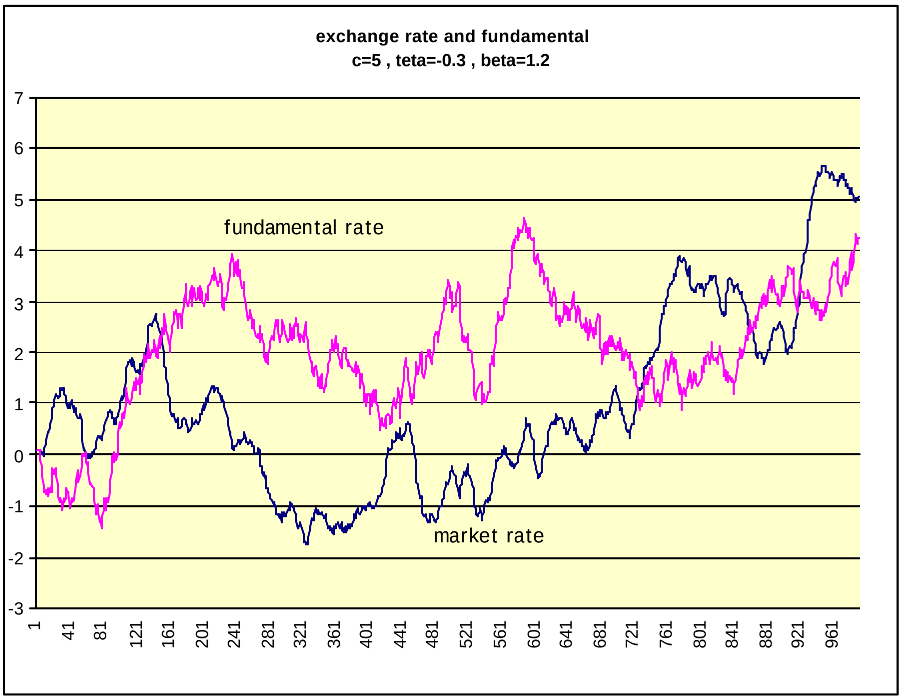


4.2.2 Solution of the model with random shocks (news)

In this section we investigate the solution of the model when random shocks in the equilibrium exchange rate are present. This will allow us to combine the model with random shocks (news model) with the complex dynamics described in the previous section. We will not restrict the analysis to the cases where the deterministic part of the model produces a chaotic dynamics.

The first question we analyse is how the exchange rate behaves relative to the equilibrium exchange rate (fundamental). In figure 9 we show the two variables, for a combination of parameters that does not produce deterministic chaos.

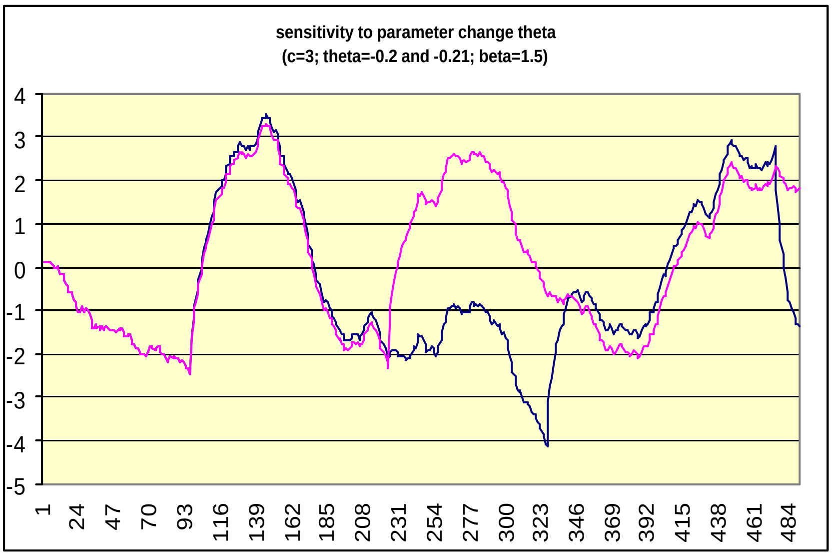
Figure 9



We observe that the model predicts that the market rate can deviate from the fundamental value substantially and in a persistent way. Moreover, it appears that the exchange rate movements are often unrelated to the movements of the underlying fundamental. Thus, the model is able to generate empirical regularities that have also been observed in reality[4]. We will return to this result later to analyse how sensitive it is to particular parameter values, like speed of adjustment and transaction costs.

Sensitivity to initial conditions is a crucial feature of the complex dynamics. We illustrate its power showing how a very small change in the speed of adjustment in the goods market can affect the market exchange rate. In Figure 10 we show the market exchange rate dynamics under two assumptions about the speed of adjustment. The first case is our standard case where θ is equal to –0.2, in the second case the value of θ is equal to –0.19. A way to interpret this simulation is to think of the case where the fundamentalists make a small error (0.01) in estimating the speed of adjustment. As figure 10 shows, this small error will lead after some time to a completely different time-path of the exchange rate, producing the appearance of large structural breaks.

Figure 10



[4] See De Grauwe(2000) for a survey of the empirical evidence. In De Grauwe and Vansteenkiste(2001) we present additional empirical evidence.

4.2.3 Sensitivity analysis

We now investigate how sensitive our results are when we change the value of certain parameters. We first analyse the effect of assuming different speeds of adjustment in the goods market. The results are presented in the following figures 11a and 11b which compare the movements of the market exchange rate under two assumptions about the speed of adjustment.

Figure 11a

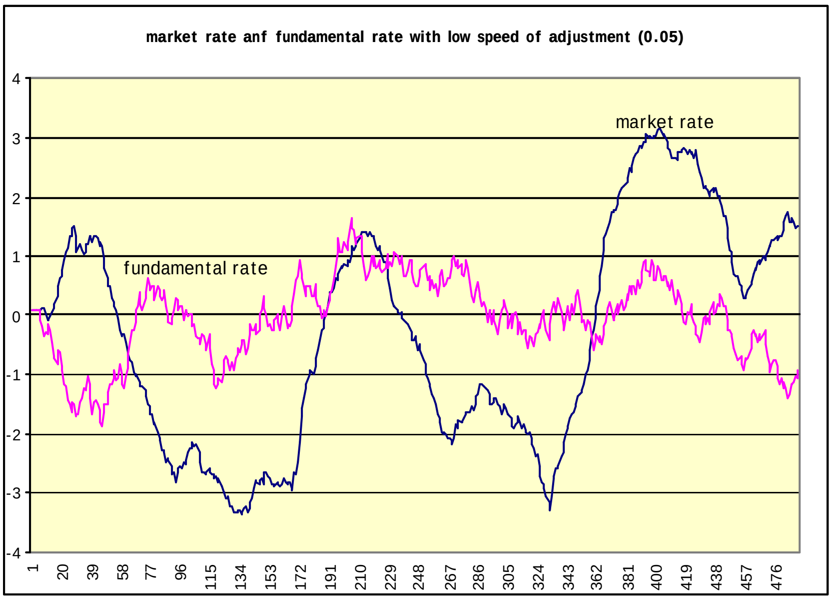
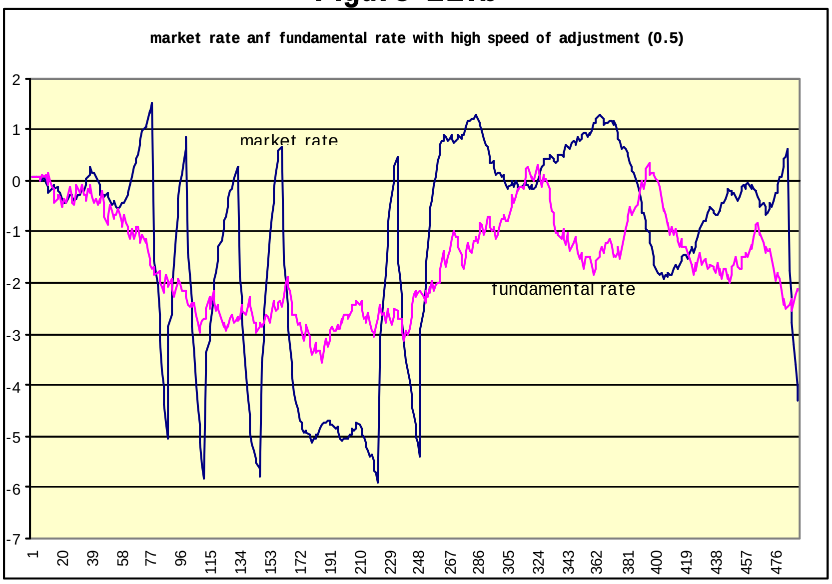


Figure 11.b



The comparison of panels a and b in figure 11 allow us to conclude the following. When the speed of adjustment is low (θ= -0.05) the deviations of the exchange rate from its equilibrium value show persistence for a long time. In contrast, when the speed of adjustment is high (θ= -0.5) the deviations from the equilibrium value are short-lived, and we also obtain substantial short–term volatility in the market exchange rate. Thus, the contrast between a world of low and high speeds of adjustment in the goods market is that in the former case the exchange rate can deviate from the fundamentals for a long time, while in the case of speedy adjustments these sustained misalignments are less likely, but short-term turbulence becomes more prevalent.

In the next figure 12 we compare the movements of the market exchange rate under two assumptions about transaction costs. In panel a we assume that transaction costs are zero and in panel b we assume them to be high, i.e. equal to 5.

Figure 12.a

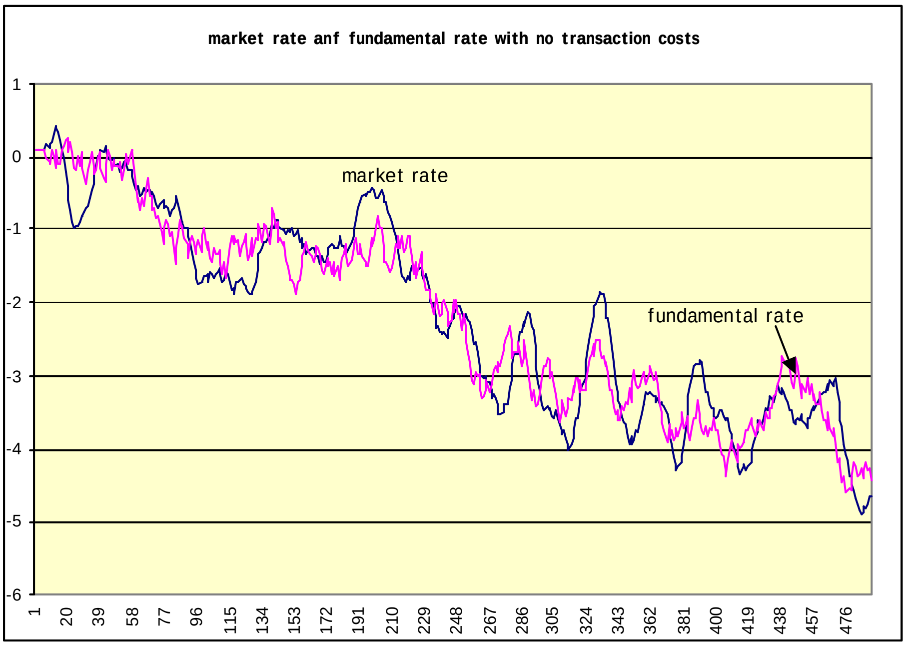
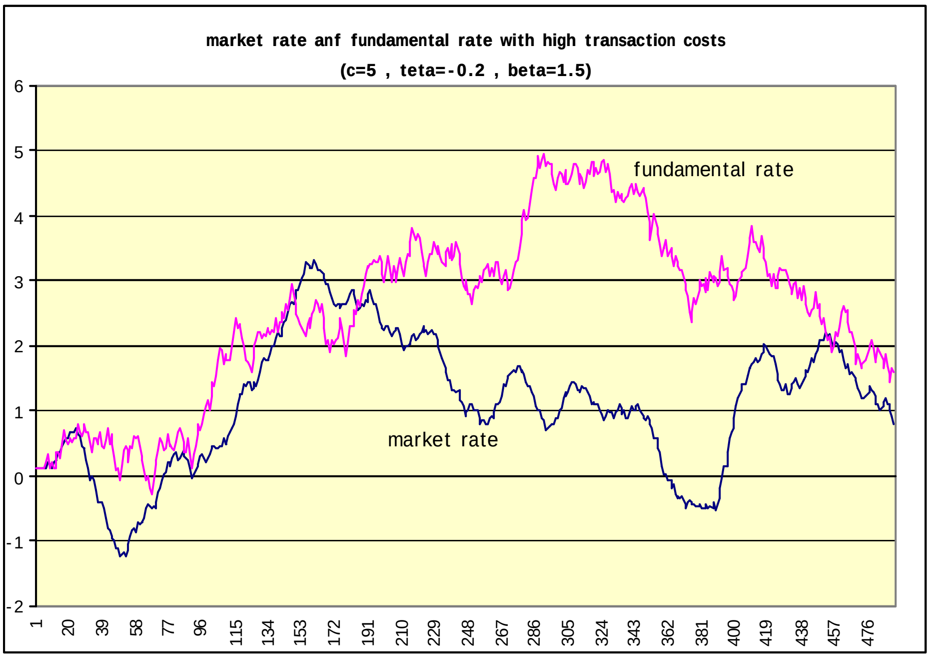


Figure 12.b



The contrast between the two panels is striking. When transaction costs are zero, the market exchange rate does not deviate substantially for a long period of time from its fundamental value. In contrast, when transaction costs are present the deviations of the market exchange rate from its equilibrium value are large and persistent. Thus transactions costs have similar effects on the dynamics of the exchange rate as the speed of adjustment.

Transaction costs have also other important implications for the dynamics of the exchange rate. We show this in figure 13, where we introduce a negative and permanent shock (-0.01) in the fundamental exchange rate change. Thus, over time the “new” fundamental exchange rate progressively but slowly departs from the “old” one.

Figure 13

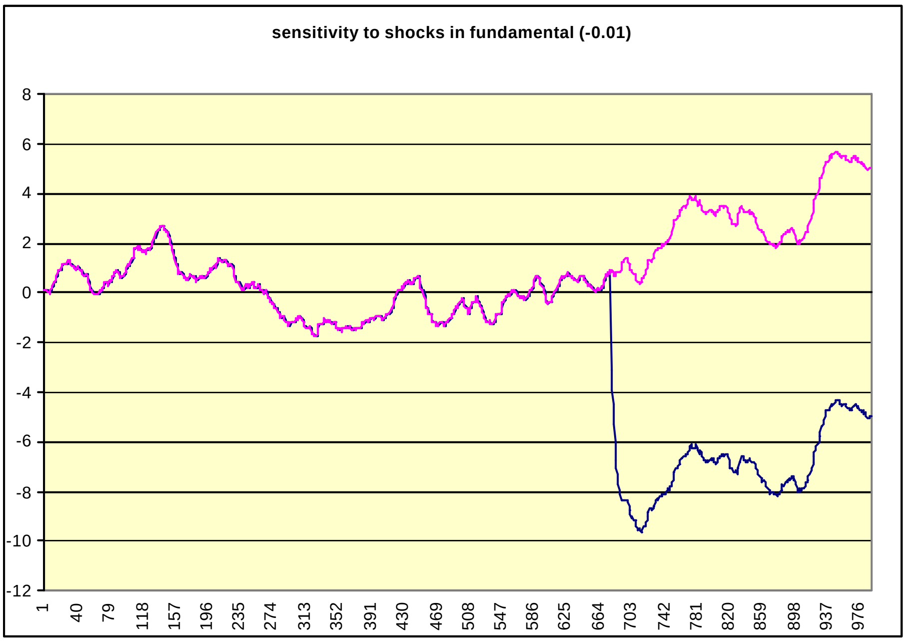
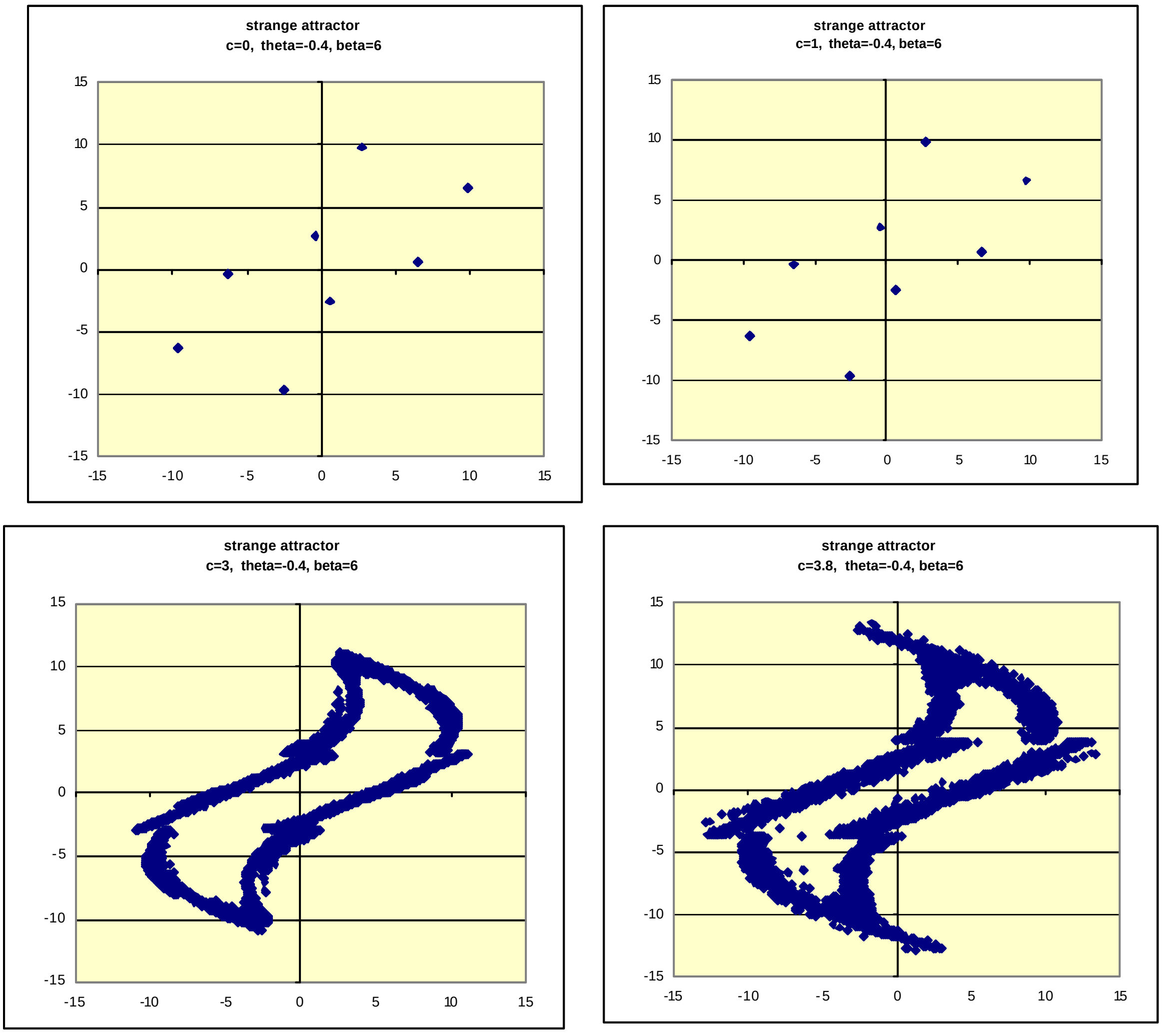


Figure 13 allows us to see how this accumulating small change in the equilibrium value of the exchange rate, which occurs in period 1, leads to a large jump in the exchange rate many periods later. This change has the appearance of regime's shift in spite of the fact that the change in the fundamental exchange rate is very small. This feature is much related to the existence of transaction costs, which implies that the effect of the accumulated changes in the fundamental exchange rate will be visible only when it will overcome the transaction costs band.

The existence of a transaction costs band has another remarkable implication, i.e. it affects the nature of the dynamics. This is shown in figure 14, which presents the strange attractor for different values of the transaction costs band (given the value of the other parameters).

From figure 14 it can be seen that for low transaction costs we obtain an eight-period cycle. When the transaction costs band increases to three and beyond we obtain a strange attractor, which increases in complexity when the transaction costs increase. Thus, increasing transaction costs leads to an increasing complexity into the dynamics of the exchange rate.

Figure 14



4.3 "Excess volatility"

The model discussed in the previous sections is driven by exogenous news in the fundamentals and by the noise produced by the non-linear dynamics embedded in the model. As a result, the non-linear dynamics is capable of producing "excess volatility" in the exchange rate, i.e. volatility that exceeds the volatility of the underlying fundamental. In this section we analyse the sources of this excess volatility. We do this by computing the noise to signal ratio in the simulated exchange rate. We define this noise to signal ratio as follows:

$$\text{var(e)} = \text{var(f)} + \text{var(n)} \qquad (30)$$

where var(e) is the variance of the simulated exchange rate, var(e) is the variance of the fundamental and var(n) is the residual variance (noise) produced by the non-linear speculative dynamics which is assumed to be uncorrelated with var(f). Rewriting (30) we obtain

$$\frac{\text{var}(n)}{\text{var}(f)} = \frac{\text{var}(e)}{\text{var}(f)} - 1$$

The ratio var(n)/var(f) can be interpreted as the noise to signal ratio. It gives a measure of how large the noise produced by the non-linear dynamics is with respect to the exogenous volatility of the fundamental exchange rate. We simulate this noise to signal ratio for different values of the parameters of the model. We show the results in figures 15-17. We observe that the noise to signal ratio is very sensitive to the extrapolation parameter of the chartists and to a lesser degree to the transactions costs. We observe that the noise produced by the non-linear dynamics can become very large relative to the volatility of the fundamental exchange rate when the extrapolation parameter is large and when transactions costs are large. The noise to signal ratio is much less sensitive to the speed of adjustment in the goods market. Overall our model is capable of generating volatility of the exchange rate that is much in excess of the volatility of the underlying fundamental.

Figure 15

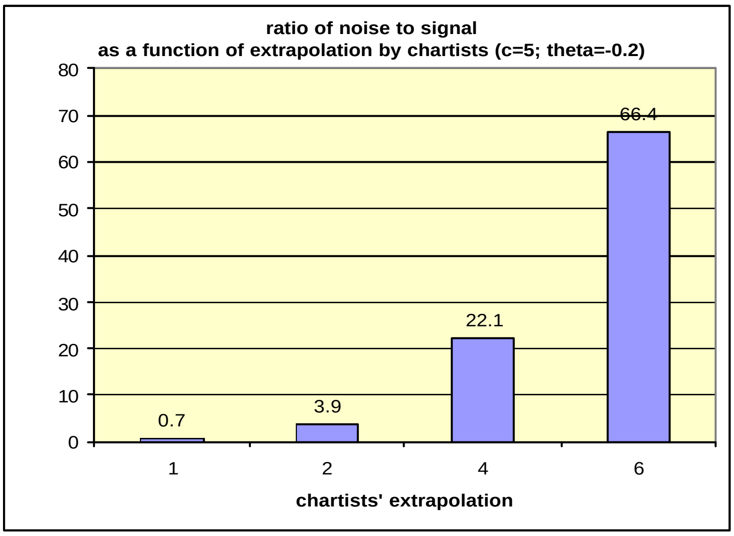


Figure 16

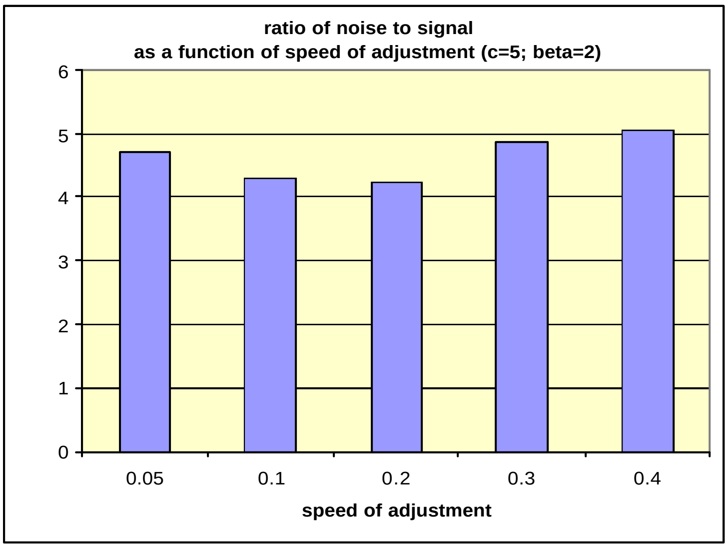
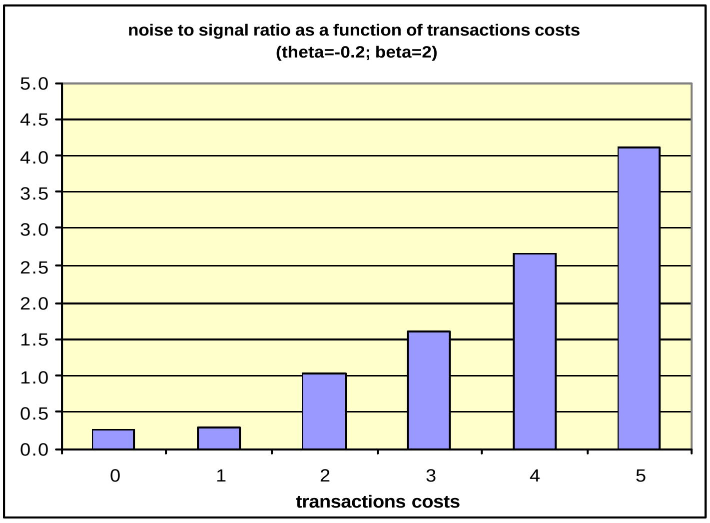


Figure 17



4.3 Small and large shocks and the dynamics of the exchange rate

In linear models the size of the shocks does not affect the nature of the dynamics. In non-linear models things are different. The size of the shocks matters. This is also the case in our exchange rate model. In order to illustrate this we simulated the model under two different assumptions about the variance of the shocks in the fundamental exchange rate. In the first case we assume low variance of these shocks, in the second case we assume a high variance (ten times higher). The results of our simulations are presented in figures 18-19. (The simulations shown here are representative for a wide range of parameter values).

Figure 18: low variance of equilibrium exchange rate

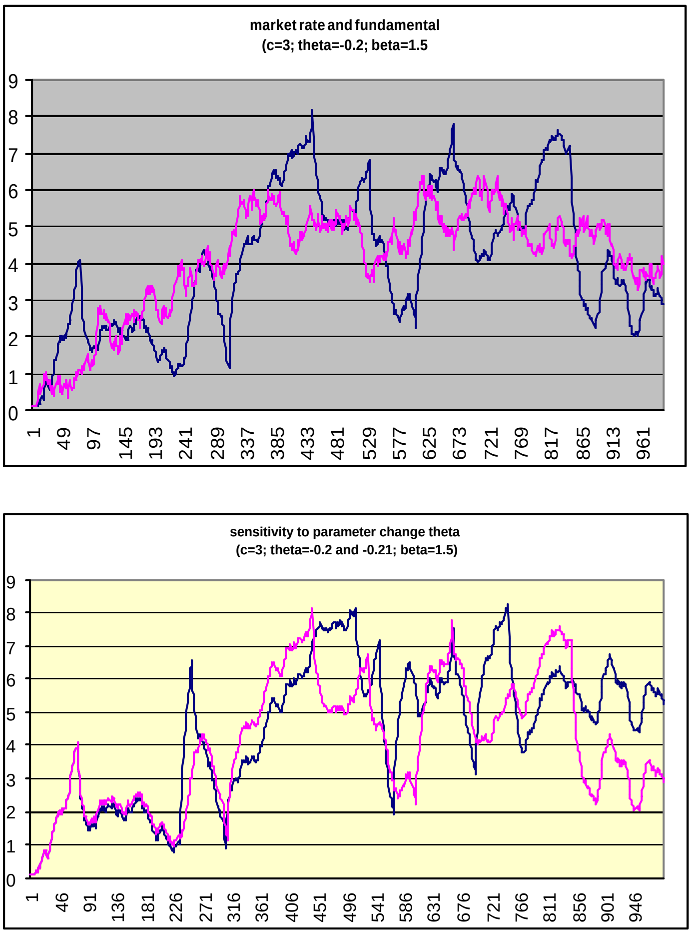
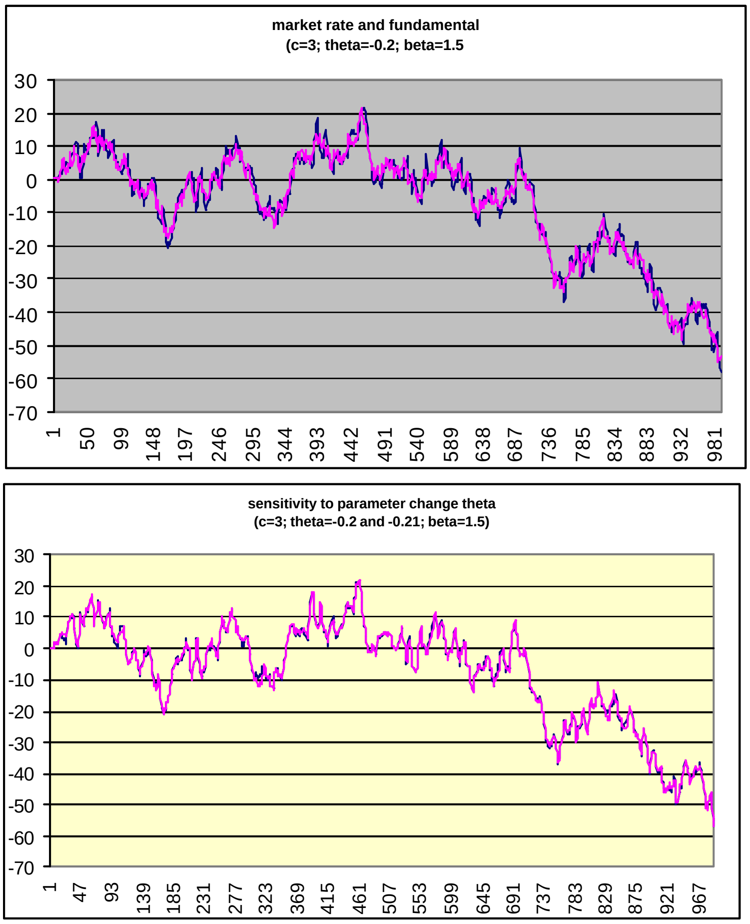


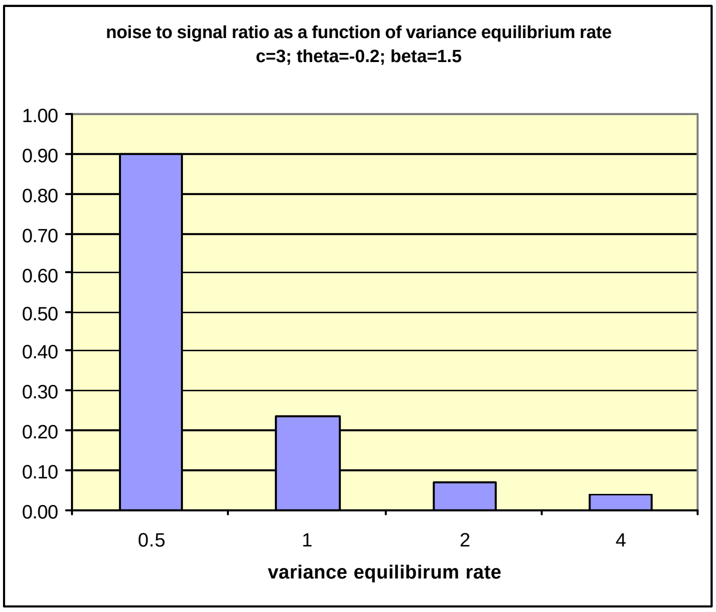
Figure 19: high variance of equilibrium exchange rate



Two conclusions follow from a comparison of the low and high variance cases. First, in the low variance case we observe sustained deviations from the equilibrium exchange rate. This is not the case when the equilibrium exchange rate is subject to large shocks. Second, the sensitivity to small changes in parameters is clearly visible when the variance of the exchange rate is low. When this variance is high, no such sensitivity can be observed. It is important to stress that the transactions cost band is the same in both cases. Thus, when the shocks are small relative to the given band of transactions costs the movements of the exchange rate show more complexity than when the shocks are large.

This feature is also evident from a comparison of the noise to signal ratio for different variances of the equilibrium exchange rate. We show this in figure 20. We observe that when the variance of the equilibrium exchange rate is low a large part of the volatility of the exchange rate is produced by the noise from the non-linear dynamics. For high variance the noise is very small, implying that the exchange rate follows the fundamental rate very closely[5].

Figure 20



The intuition of this result is that when the fundamental shocks are small the exchange rate regularly switches from the dynamics inherent in the band to the one prevalent outside the band. This non-linearity produces a lot of noise and complexity in the dynamics of the exchange rate. When

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[5] It should be stressed that the total variability of the exchange rate in the high variance scenario is much larger than the total variability of the exchange rate in the low variance scenario. The point is that in the high variance scenario almost all of the variability of the exchange rate is explained by the (much higher) variability of the fundamental. This is not the case in the low variance scenario where a large part of the variability of the exchange rate cannot be related to the variability of the underlying fundamental.

the shocks are large relative to transactions cost band the dynamics outside the band mostly prevails, leading to a tighter link between the exchange rate and the fundamental. This feature has also been found to hold empirically (See De Grauwe an Vansteenkiste(2001)). These results of our model are also consistent with the evidence discussed in section 2, suggesting that the link between inflation differentials and exchange rate changes of low inflation countries is weak, if non-existent.

5. Conclusion

In this paper we first studied the nature of the price dynamics leading to PPP. Using a cross-section of approximately 100 countries we found that the dynamics of adjustment towards PPP is inherently non-linear. These empirical results are in line with other recent econometric studies of the PPP-dynamics (see e.g. Kilian and Taylor(2001), Peel and Taylor(2000), Polan(2001)) and go a long way towards explaining the so-called PPP-puzzle.

We then analysed the implications of the existence of a non-linear dynamics in the goods market. In order to do so, we developed a model of the exchange rate with chartists and fundamentalists, where the chartists apply a *positive feedback rule* and the fundamentalists a *negative feedback rule*.

Our main results can be summarised as follows. First, the simple non-linear structure of the model is capable of generating a very complex exchange rate dynamics. We found that for plausible parameter values this complex dynamics can be chaotic. This implies that small shocks in the equilibrium exchange rate lead to very different time-paths of the exchange rate. Second, our model is capable of explaining some empirical regularities. One empirical regularity is that the market exchange rate can deviate substantially and for relatively long periods of time from its fundamental value (misalignments). We showed that such misalignments are a natural outcome of the non-linear dynamics in our simple model. There is no need to invoke exogenous events and special factors to explain why exchange rates deviate from their fundamental values. It should also be noted that our model generates these misalignments even in the absence of deterministic chaos. In other words we do not need to invoke chaos to explain misalignments.

Another empirical regularity observed in exchange rate economics is the frequent occurrence of "regime shifts", i.e. structural breaks in the relation between the exchange rate and the fundamentals. This phenomenon has first been noted in the celebrated studies of Meese and Rogoff(1982)[6]. It is now customary to explain these structural breaks by changes in the policy regime. Our model provides an alternative explanation. The non-linear dynamics embedded in the model produces endogenous regime shifts that change the link between the exchange rate and its fundamentals. These structural breaks can be triggered by very small changes in parameters, or by small errors in the estimates of these parameters by agents who forecast the future exchange rate. Thus, in a non-linear world, structural breaks in the link between the exchange rate and its fundamentals occur naturally even when no changes occur in the policy regime.

Third, we found that our simple non-linear dynamic model can generate "excess volatility" in the exchange rate. The size of this excess volatility

[6] For more recent evidence see De Grauwe and Vansteenkiste(2001).

crucially depends on the degree of extrapolation applied by chartists and on the size of the transactions cost band.

Finally we found that the size of shocks to the underlying fundamental exchange rate matters for the dynamics of the exchange rate. More specifically we found that when these shocks are small relative to the size of the transactions cost band, the phenomena just described will tend to be prevalent. That is, in regimes of low shocks relative to the transactions cost band, the exchange rate movements are complex, and can even be chaotic. In such a regime exchange rates deviate substantially from the underlying fundamentals and frequent structural breaks in the link between the fundamentals and the exchange rate are observed. The latter occur in the absence of changes in the policy regime.

Some implications of these findings are the following. The exchange rates of the major currencies are subject to relatively small shocks in the underlying fundamentals (e.g. inflation differentials are almost zero). Compared to these shocks the transactions costs can be said to be relatively large (see Obstfeld & Rogoff(2000) on this), i.e. a large part of goods and services are non-traded (or difficult to trade) because the cost of shipping them across borders is quite high. Thus the regime confronted by the exchange rates of the major industrialised countries comes close to the regime we have identified to be the one producing complexity, speculative noise, and structural breaks between exchange rates and underlying fundamentals. Put differently, the movements of the exchange rates of the industrialised countries are likely to be clouded by a non-linear speculative dynamics that makes it difficult if not impossible to explain this or that movement of these exchange rates. In contrast the exchange rates of high inflation countries experience large shocks in the fundamentals. As a result, the movements of the exchange rates of these countries can be explained much better by movements in underlying fundamentals (e.g. inflation differentials).

The results of our paper make it easier to understand why it will remain difficult, if not impossible to find (fundamental) logic in the movements of the exchange rates of major currencies. However, our inability to understand why, say, the dollar moved up against the euro during the 1999-2000 does not prevent analysts from developing exotic theories explaining these movements. Probably this has to do with the fact that the human mind abhors the emptiness created by its inability to understand. It is no surprise therefore that new explanations based on fundamentals are created, and will continue to be created for each and every new turn of the exchange rate[7].

[7] In another paper we have suggested that when agents are very uncertain about the factors that drive the exchange rate, they will be very much influenced by "framing". In such a situation the movements of the exchange rate create frames, i.e. an upward movement of the exchange rate leads agents to go on a search of positive news about the currency, discarding the negative news (De Grauwe(2000)).

Appendix 1: Sensitivity of dynamics to parameter values

theta / beta	-0.1	-0.2	-0.3	-0.4	-0.5
1.9	FPM	FPM	FPM	FPM	U
2	FPC	FPC	FPC	FPC	U
2.1	C	C	C	C	U
2.2	C	C	C	C	U
2.3	C	C	C	C	U
2.4	C	C	C	C	U
2.5	C	C	C	C	U
2.6	C	C	C	C	U
2.7	C	C	C	C	U
2.8	C	C	C	C	U
2.9	C	C	C	C	U
3	C	C	C	C	U
3.1	C	C	C	C	U
3.2	C	C	C	C	U
3.3	C	C	C	C	U
3.4	C	C	C	C	U
3.5	P (12)	P (12)	P (12)	P (12)	U
3.6	C	C	C	C	U
3.7	C	C	C	C	U
3.8	C	C	C	C	U
3.9	C	C	C	C	U
4	C	C	C	C	U
4.1	C	C	C	C	U
4.2	C	C	C	C	U
4.3	C	C	C	C	U
4.4	C	C	C	C	U
4.5	P (10)	P (10)	P (10)	P (10)	U
4.6	C	C	C	C	U
4.7	C	C	C	C	U
4.8	C	C	C	C	U
4.9	C	C	C	C	U

Note: C = chaos
P (N) = N-period cycles
U = unstable
FPM = fixed point reached monotonically
FPC = fixed point reached cyclically

Table continued

theta beta	-0.1	-0.2	-0.3	-0.4	-0.5
5	C	C	C	C	C
5.1	C	C	C	C	C
5.2	C	C	C	C	C
5.3	C	C	C	C	C
5.4	P (8)	P (9)	P (9)	P (9)	P (9)
5.5	P (9)	P (8)	P (8)	P (9)	P (9)
5.6	C	C	C	C	C
5.7	P (23)	P (26)	P (26)	P (26)	P (26)
5.8	P (34)	P (17)	P (17)	P (17)	P (17)
5.9	P (42)	P (42)	P (42)	P (42)	P (42)
6	P (24)	P (8)	P (12)	P (8)	P (8)
6.1	P (8)	P (8)	P (8)	P (8)	P (8)
6.2	P (18)	P (17)	P (17)	P (17)	P (17)
6.3	P (34)	P (34)	P (34)	P (34)	P (34)
6.4	C	C	C	U	C
6.5	C	C	C	U	C
6.6	C	C	C	U	C
6.7	C	C	C	U	C
6.8	C	C	C	U	C
6.9	C	C	C	U	P (16)
7	C	C	C	U	C
7.1	U	U	U	U	U

Note: C = chaos

P (N) = N-period cycles

U = unstable

FPM = fixed point reached monotonically

FPC = fixed point reached cyclically