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NON-LINEARITIES IN THE RELATION BETWEEN THE EXCHANGE RATE AND ITS FUNDAMENTALS

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ABSTRACT

We develop a simple theoretical model in which chartists and fundamentalists interact. The model predicts the existence of different regimes, and thus non-linearities in the link between the exchange rate and its fundamentals. We test the model empirically by adopting a Markov-switching vector error correction model. The results suggest the presence of non-linear mean reversion in the nominal exchange rate process. The implications are that different sets of macroeconomic fundamentals act as driving forces of the exchange rates during different time periods. Copyright © 2008 John Wiley & Sons, Ltd.

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1. INTRODUCTION

One of the most pervasive pieces of econometric evidence we have about the exchange markets is that it appears to be difficult, if not impossible, to find a linear relation between the exchange rate and its underlying fundamentals that will remain stable over a sufficiently long period of time. Ever since the path breaking research of Meese and Rogoff (1983) who observed that the link between the exchange rate and its fundamentals is structurally unstable, the evidence about the unstable nature of linear econometric models of the exchange rates has continued to accumulate (see Cheung, 2005).[1] True there have been claims of empirical success in estimating linear models (see e.g. Mark, 1995), but each time the sample period was extended or when slight changes in specifications were made structural breaks in these linear models appeared (Faust *et al.*, 2003). As a result, it is fair to conclude that the relation between the exchange rate and its fundamentals is most probably a non-linear one.

This insight has led to a mushrooming of studies explicitly incorporating non-linearities between the exchange rate and its fundamentals. Many of these studies utilize the Multiple Regime Smooth Transition Autoregressive model to evaluate the non-linear relationship between the exchange rate and its fundamentals.[2] Introduced by van Dijk and Franses (1999), these models generalize the well-known STAR models.[3] Taylor and Peel (2000), for example, employ an exponential smooth transition autoregressive model to analyse the non-linear relationship between the exchange rate and its

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fundamentals in the US, UK and Germany. The results indicate that during periods of small deviations from fundamentals the exchange rate exhibits a unit-root. In contrast, the authors detect a rapid adjustment of the exchange rate towards its underlying fundamentals when there are large deviations from the equilibrium. Using the same methodology Taylor *et al.* (2001) find strong support for the non-linear representation of exchange rate deviations from PPP.

Other studies have applied Hamilton's (1989) Markov-switching model to the foreign exchange markets and found evidence of frequent shifts of regimes in the relation between the exchange rate and its fundamentals (Engel and Hamilton, 1990; De Grauwe and Vansteenkiste, 2001; Frömmel *et al.*, 2005). Sarno *et al.* (2004) used a Markov-switching vector equilibrium correction model (MS-VECM) in order to study the dynamic relationship between the exchange rate and fundamentals in six industrialized countries using data spanning over 100 years. Their results suggest that the speed at which exchange rate converges to the long-run fundamental equilibrium mostly depends on the nominal exchange rate regime in operation.

Thus, there is increasing evidence that the relation between the exchange rate and its fundamentals has important non-linear features. These explain why linear models are not robust in long sample periods.

In this paper we analyse two issues. First we study how this empirical evidence can be reconciled with the theory. In order to do so we propose a simple exchange rate model with heterogeneous agents. This model predicts that the relation between the exchange rate and its underlying fundamentals is subject to frequent regime shifts. Second, we analyse the nature of these regime shifts empirically using evidence of the dollar/DM (euro) exchange rate.

The remainder of the paper proceeds as follows. Section 2 proposes a theoretical model that incorporates non-linearities in the exchange rate dynamics. In Section 3 we estimate a linear vector error correction model and test for possible non-linearities in the residuals. Section 4 discusses the econometric methodology used in order to analyse the way in which monetary fundamentals affect the exchange rate depending on the state of the economy. In Section 5 we compare the out-of-sample forecast performance of three competing models of exchange rate determination. In Section 6 concluding remarks end the paper.

2. THEORETICAL FRAMEWORK

The empirical evidence suggests that the relationship between the exchange rate and the fundamentals is a non-linear one, characterized by frequent changes in the regimes linking the exchange rate to the fundamentals. Traditional linear rational expectations models cannot account for this except by introducing *exogenous* changes in regimes, i.e. by leaving these switches unexplained.

In this section we develop a very simple exchange rate model that exhibits endogenous switches in regimes. The model is based on De Grauwe and Grimaldi (2006). We start by defining the fundamental exchange rate. This is the exchange rate that is consistent with equilibrium in the real part of the economy. In a very simple model this could be the Purchasing Power Parity-value of the exchange rate. In more elaborate models (e.g. the monetary model or the Obstfeld–Rogoff new open economy macro model; Obstfeld and Rogoff, 1996) this fundamental exchange rate could be determined by the interaction of more variables than the price levels. Here, we assume that the fundamental exchange rate, e_t^*, is exogenous and that it behaves like a random walk without drift. This implies:

$$e_t^* = e_{t-1}^* + \epsilon_t \tag{1}$$

where ϵ_t is a white noise error term.

We now model the way agents make forecast about the future exchange rates. We assume that agents can use two types of simple forecasting rules. One type of forecasting rule will be called fundamentalist, and agents who use such a rule will be called fundamentalists for short. The second type of rule will be called chartist and the agents who use this rule will be labelled chartists. We will also use the term 'technical analysts' interchangeably.

The fundamentalists are assumed to know the fundamental exchange rate. They compare the present market exchange rate with the fundamental rate and they forecast the future market rate to move towards

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the fundamental rate. In this sense they follow a negative feedback rule. This leads us to specify the following rule for the fundamentalists:

$$E_{f,t}(\Delta e_{t+1}) = -\psi(e_t - e_t^*) \tag{2}$$

where $E_{f,t}$ is the forecast made in period t by the fundamentalists using information up to time t, e_t is the exchange rate in period t, Δe_t is the change in the exchange rate, and $\psi > 0$ measures the speed with which the fundamentalists expect the exchange rate to return to the fundamental one. This parameter is presumably related to the speed of adjustment of prices in the goods market, but we do not specify its precise link with this speed of adjustment.

The chartists are assumed to follow a positive feedback rule, i.e. they extrapolate past movements of the exchange rate into the future. We will use the simplest possible hypothesis here: we assume that chartists extrapolate only last period's exchange rate into the future. The chartists' forecast is written as

$$E_{c,t}(\Delta e_{t+1}) = \beta \Delta e_t \tag{3}$$

where $E_{c,t}$ is the forecast made by the chartists using information up to time t, and β is the coefficient expressing the degree with which chartists extrapolate the past change in the exchange rate; we assume that $0 < \beta < 1$ to ensure dynamic stability.[4]

Admittedly the assumption underlying (3) is very crude. Most chartists use more complicated rules. In De Grauwe and Grimaldi (2006) we analyse more sophisticated rules in which chartists use moving averages of past exchange rate movements and we show that the results of the model are not much affected by the use of more complex forecasting rules that take a longer series of past changes in the exchange rate into account.

The next step in our analysis is to specify how agents evaluate the usefulness of these two forecasting rules. The general idea that we will follow is that agents use one of the two rules, compare their profitability ex post and then decide whether to keep the rule or switch to the other one.

In order to implement this idea we use a fitness criterion in the spirit of Brock and Hommes (1997, 1998), which is based on discrete choice theory.[5] This means that the fractions of the total population of agents using chartist and fundamentalist rules are a function of the relative (risk adjusted) profitability of these rules. We specify this procedure as follows:

$$\begin{aligned} w_{f,t} &= \frac{\exp \gamma \pi'_{f,t}}{\exp \gamma \pi'_{f,t} + \exp \gamma \pi'_{c,t}} \\ w_{c,t} &= \frac{\exp \gamma \pi'_{c,t}}{\exp \gamma \pi'_{f,t} + \exp \gamma \pi'_{c,t}} \end{aligned} \tag{4}$$

where $w_{f,t}$ and $w_{c,t}$ are the fractions of the population who use fundamentalist, respectively, chartist forecasting rules. Obviously $w_{f,t} + w_{c,t} = 1$. The variables $\pi'_{f,t}$ and $\pi'_{c,t}$ are the (risk-adjusted) profits realized by the use of chartists' and fundamentalists' forecasting rule in period t, i.e. $\pi'_{f,t} = \pi_{f,t} - \mu\sigma^2_{f,t}$ and $\pi'_{c,t} = \pi_{c,t} - \mu\sigma^2_{c,t}$ and $\pi_{f,t}$ and $\pi_{c,t}$ are the profits made in forecasting, while $\sigma^2_{f,t}$ and $\sigma^2_{c,t}$ are variables expressing the risks chartists and fundamentalists incur when making forecasts. As a measure of this risk we will take the forecast errors. Finally μ is the coefficient of risk aversion.

Equation (4) can now be interpreted as follows. When the risk-adjusted profits of the technical traders' rule increase relative to the risk-adjusted profits of the fundamentalists' rule, then the share of agents who use technical trader rules in period t increases and vice versa. The parameter γ measures the intensity with which the technical traders and fundamentalists revise their forecasting rules. With an increasing γ agents react strongly to the relative profitability of the rules. In the limit when γ goes to infinity all agents choose the forecasting rule, which proves to be more profitable. When γ is equal to zero agents are insensitive to the relative profitability of the rules. In the latter case the fraction of technical traders and fundamentalists is constant and equal to 0.5. Thus, γ is a measure of inertia in the decision to switch to the more profitable rule.

We now go into the problem of defining with more precision the profits and the risk associated with it. We define the profits as the one-period earnings of investing \$1 in the foreign asset. More formally,

$$\pi_{i,t} = [e_t(1 + r^*) - e_{t-1}(1 + r)]\,\mathrm{sgn}[(1 + r^*)E^i_{t-1}(e_t) - (1 + r)e_{t-1}] \tag{5}$$

where

$$\mathrm{sgn}[x] = \begin{cases} 1 \quad \text{for } x > 0 \\ 0 \qquad \text{for } x = 0 \text{ and } i = c,f \\ -1 \text{ for } x < 0 \end{cases}$$

Thus, when agents forecasted an increase in the exchange rate and this increase is realized, their per unit profit is equal to the observed increase in the exchange rate (corrected for the interest differential). If instead the exchange rate declines, they make a per unit loss, which equals this decline (because in this case they have bought foreign assets that have declined in price).

Finally we specify the risk variables in the following way. As mentioned earlier, we define the risk associated with forecasting to be the forecast error. In the logic of the short-run memory hypothesis used in this section we assume that agents just look at last period's forecast error. Thus, we have

$$\sigma^2_{i,t} = [E^i_{t-1}(e_t) - e_t]^2 \tag{6}$$

We now aggregate the forecasts of chartists and fundamentalists to obtain the aggregate market forecast. The market forecast of the exchange rate change can be written as a weighted average of the expectations of chartists and fundamentalists, i.e.

$$E_t\Delta e_{t+1} = -w_{f,t}\psi(e_t - e_t^*) + w_{c,t}\beta\Delta e_t \tag{7}$$

where $w_{f,t}$ and $w_{c,t}$ are defined in (4).

The realized change in the market exchange rate in period $t+1$ equals the market forecast made at time t plus some white noise errors, ϵ_{t+1} occurring in period $t+1$ (i.e. the news that could not be predicted at time t). We obtain:

$$\Delta e_{t+1} = -w_{f,t}\psi(e_t - e_t^*) + w_{c,t}\beta\Delta e_t + \epsilon_{t+1} \tag{8}$$

We now have all the equations of the model, and we can start analysing its characteristics.

The non-linear structure of our model does not allow for a simple analytical solution. As a result we have to use numerical simulation methods. One drawback of this approach is that we cannot easily derive general conclusions. We will compensate for this drawback by presenting sensitivity analyses of the numerical solutions. The simulations we perform are stochastic. Stochastic shocks occur in the model because the fundamental exchange rate is driven by a random walk (see equation (1)) and because there is noise in the process determining the market exchange rate (see equation (8)). We will assume that the noise process in these equations is normally distributed with mean equal to 0, and standard deviation equal to 0.1.

We present two examples of stochastic simulations that are quite typical for the kind of dynamics predicted by our model (see Figure 1).[6] The two upper parts of Figure 1 present the simulated market and fundamental exchange rates obtained in two different simulation runs, using the same parameter configurations. The two lower parts present the corresponding shares of the chartists.

The most striking features of these simulations are the following. First, it appears that the exchange rate is very often disconnected from the fundamental exchange rate. This means that the market exchange rate follows movements that are dissociated from the fundamental rate.

This is especially obvious in the first simulation run (left panels), where we find that the exchange rate is disconnected from the fundamental most of the time. In the right hand panel there are many periods of disconnection, but these are less frequent. This leads to a second feature of these exchange rate movements.

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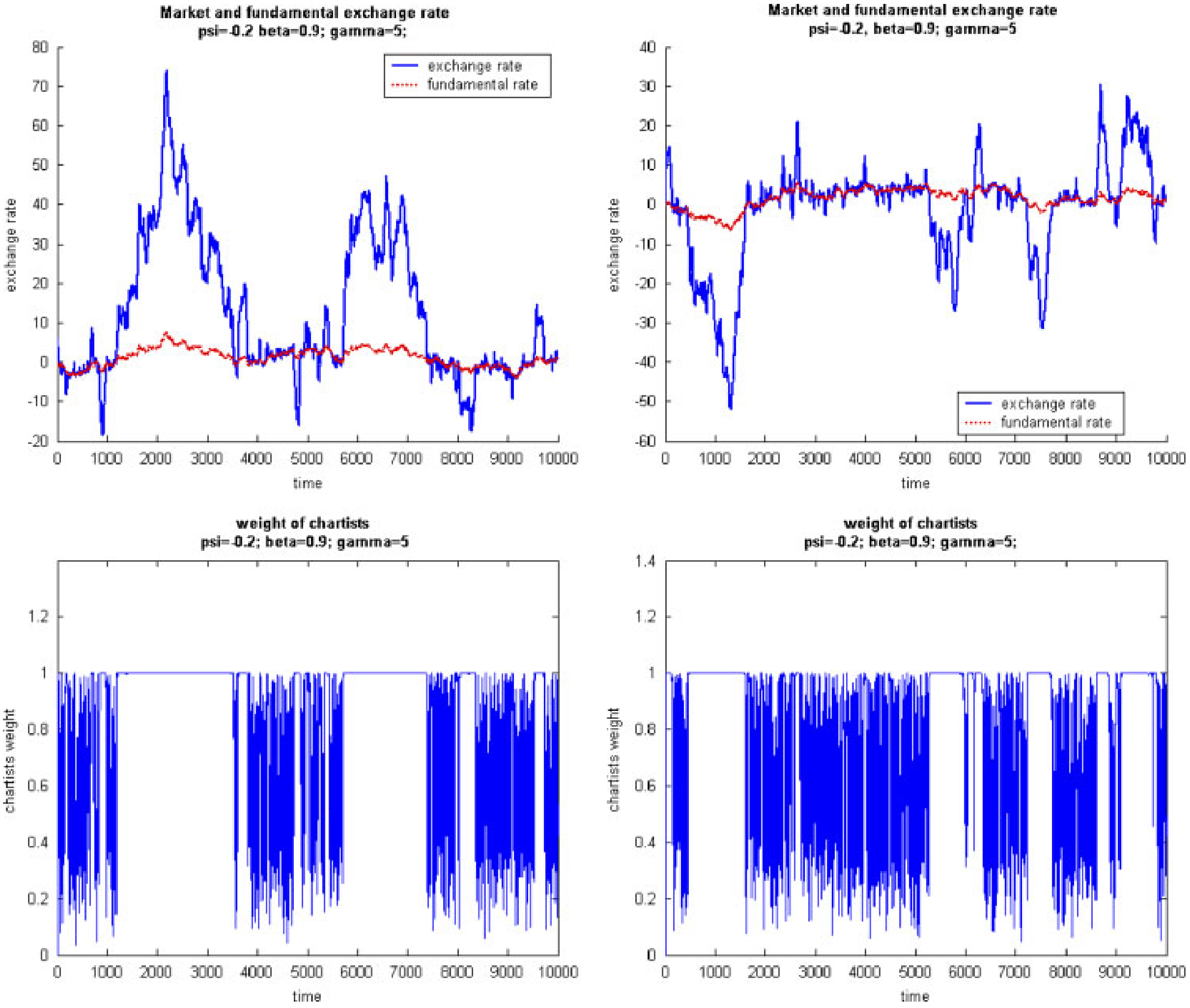

Figure 1. Stochastic simulations of the model.

There appear to be two regimes. In one regime the exchange rate follows the fundamental exchange rate quite closely. These ‘fundamental regimes’ alternate with regimes in which the fundamental does not seem to play a role in determining the exchange rate. We will call these ‘non-fundamental regimes’. The nature of the latter can be seen in the lower panels of Figure 1. Non-fundamental regimes are characterized by situations in which the chartists’ weights are very close to 1. In contrast, fundamental regimes are those during which the chartists weights are below 1 and fluctuating significantly. These two regimes appear to correspond to two types of equilibria. Thus, a fundamental regime seems to occur when the exchange rate stays within the basin of attraction of a fundamental equilibrium. In such a regime the exchange rate movements stay very close to the fundamental exchange rate. Conversely, a non-fundamental regime seems to occur when the exchange rate moves within the basins of attraction around bubble equilibria.

We also note from Figure 1 that fundamental and non-fundamental regimes alternate in unpredictable ways. The left hand panels show a simulation during which non-fundamental regimes tend to dominate, while the right hand panels show a simulation during which fundamental regimes are more frequent. The two simulations, however, were run with exactly the same parameters. The only difference is the underlying stochastic of the fundamental exchange rate.

As mentioned earlier the numerical solutions are sensitive to the parameter values chosen. We illustrate this sensitivity by presenting simulations assuming different parameter values. Figure 2 shows the results of stochastic simulations of the model for different values of γ.

It will be remembered that γ measures the sensitivity of the switching rule to risk adjusted profits. Thus, when γ is high agents react strongly to changing profitabilities of the forecasting rules they have been using. Conversely when γ is small they do not let their forecasting rules depend much on these relative profitabilities. The results shown in Figure 2 are quite remarkable. We find that when γ is large, the exchange rate tends to deviate strongly from the fundamental value most of the time. Thus, when γ is high the exchange rate seems to be attracted most of the time by non-fundamental equilibria. Conversely, when agents are not very sensitive to relative profitabilities (low γ) the exchange rate follows the fundamental rate closely, suggesting that it is then attracted by the fundamental equilibrium most of the time. This result may seem surprising. Its interpretation is the following. Stochastic shocks in one direction can make extrapolative forecasting (chartism) profitable for a while. When switching is intense (large γ) the ensuing increased profitability attracts a lot of new chartists. This will reinforce the extrapolative dynamics,

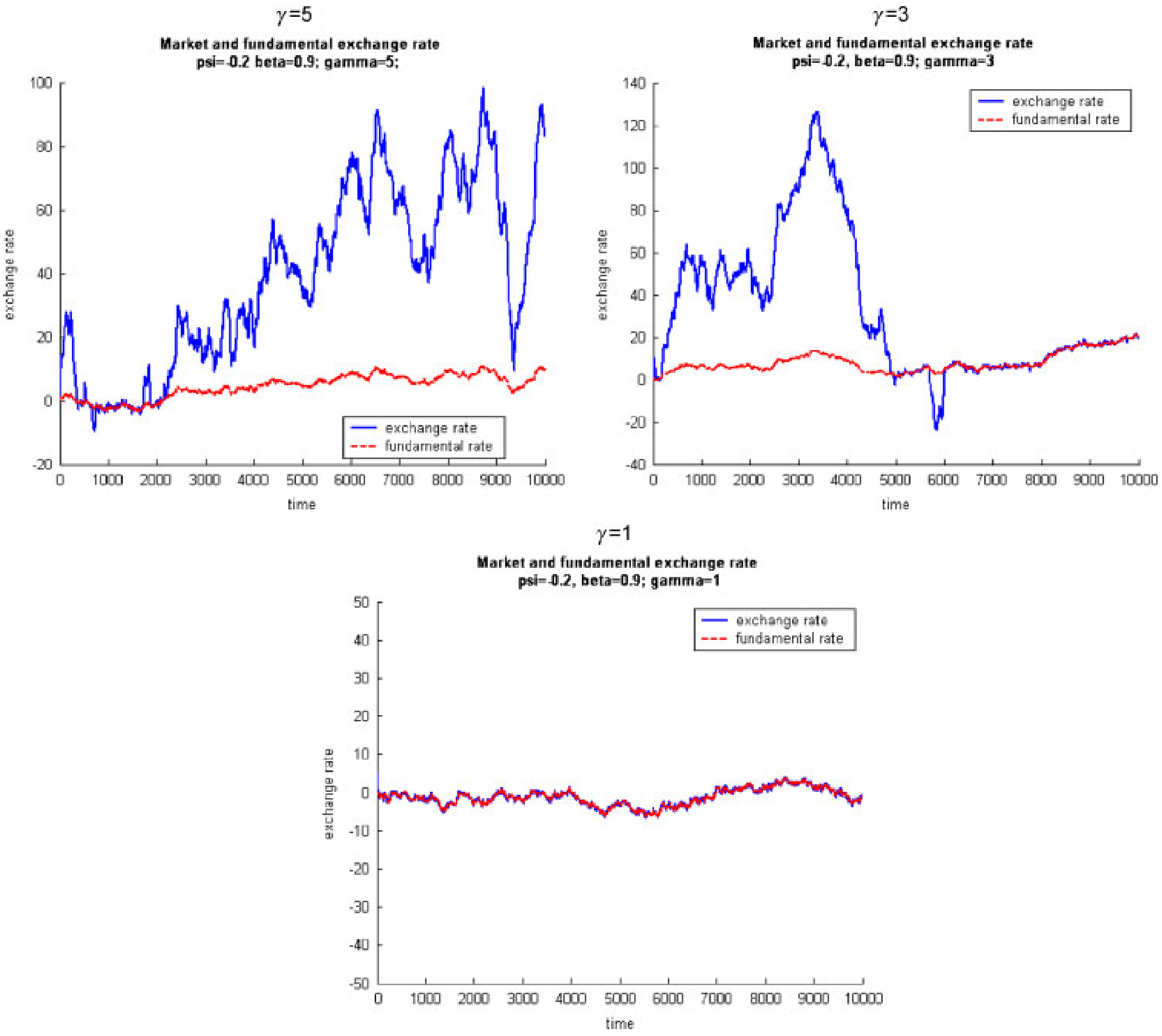

Figure 2. Model sensitivity analysis.

increasing its profitability and again attracting new chartists. Chartism becomes profitable in a self-fulfilling way attracting most agents to become chartists. This 'bandwagon' effect will be weak when switching is not intense (low γ).

Thus, the model shows that when agents react intensely to relative profitabilities of different forecasting rules, there will be frequent switches between fundamental and non-fundamental regimes. This also implies that there are periods during which news in the fundamentals have no or very little impact on the exchange rate. As a result, the relation between the exchange rate and the fundamentals is an unstable one. In the next sections we test this proposition empirically.

3. EMPIRICAL TESTING

The purpose of our empirical testing strategy is twofold. First, we want to find out whether there is evidence for regime switches. Second we want to explore the nature of these regimes.

In this section we perform some diagnostic tests. These aim at finding out whether there is evidence of non-linearities in the exchange rate data. In the next sections (Section 4) we then turn to testing for regime switches.

In order to test for non-linearity in the exchange rate dynamics we proceed as follows. First, we estimate a linear VECM with the maximum likelihood technique. Then, we check the non-linearity of the residuals by employing a battery of standard tests. The sample period, for both the euro area and the United States, goes from 1979 : 1 to 2004 : 4. The data used in the empirical analysis for the USA are seasonally adjusted quarterly observations and were drawn from DataStream, which, in turn, takes the data from OECD Main Economic Indicator Database. The aggregate variables for the Euro Area, instead, come from the data set used by Fagan *et al.* (2001) to construct the area-wide model for the euro area. As the last data set ends in 1998 : 4 we extend the time series by adding the data reported in the ECB monthly bulletin. Finally, the pre-EMU exchange rate is approximated by a synthetic Euro/US dollar rate.[7]

3.1. A linear vector error correction model

We assume that the equilibrium value of the euro-dollar exchange rate is determined by a set of economic fundamentals. The study concentrates on three of such fundamentals: the relative GDP, the relative inflation rate and the interest rate differential. In order to characterize the long-run dynamic adjustments, we use the following vector error correction model (VECM):

$$\begin{aligned} \Delta x_t &= c + \sum_{i=1}^{k-1} \Gamma_i \Delta x_{t-i} + \Pi x_{t-k} + \epsilon_t \\ x_t &= [y_t \;\; \pi_t \;\; i_t \;\; e_t]' \end{aligned} \tag{9}$$

In the above model, y_t is the GDP differential, measured as the difference between the EU and USA real GDP; π_t represents the inflation rate differential[8]; i_t is the short term interest rate differential, and e_t is the euro-dollar exchange rate.

The residuals from the cointegrating vector, lagged once, act as the error correction term.[9] This term captures the disequilibrium adjustment of each variable towards its long-run value. The parameter of the error correction terms in each individual equation indicates the speed of adjustment of this variable back to its long-run value. A significant error correction term implies long-run causality from the explanatory variables to the dependent variables. The matrix Π is usually decomposed as

$$\Pi = \alpha\beta' \tag{10}$$

where α and β are $n \times r$ matrices, n is the number of variables and r is the number of cointegrating relationships, containing the adjustment coefficient and the cointegrating vector, respectively; Δ is the first

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difference operator. In this form all terms are stationary, that is integrated of order zero, denoted $I(0)$. The system can be written as

$$\begin{bmatrix} \Delta y_t \\ \Delta \pi_t \\ \Delta i_t \\ \Delta e_t \end{bmatrix} = \Gamma(L) \begin{bmatrix} \Delta y_{t-1} \\ \Delta \pi_{t-1} \\ \Delta i_{t-1} \\ \Delta e_{t-1} \end{bmatrix} + \begin{bmatrix} \alpha_{11} \\ \alpha_{21} \\ \alpha_{31} \\ \alpha_{41} \end{bmatrix} [\beta_{11}\ \beta_{12}\ \beta_{13}\ \beta_{14}] \begin{bmatrix} y_{t-1} \\ \pi_{t-1} \\ i_{t-1} \\ e_{t-1} \end{bmatrix} + \begin{bmatrix} u_t^y \\ u_t^\pi \\ u_t^i \\ u_t^e \end{bmatrix} \tag{11}$$

We estimated this model by using the maximum likelihood procedure developed by Johansen (1988, 1991). The results are reported in Table 1.

For the inflation rate differential, the interest rate differential and for the exchange rate equations the adjustment coefficients (EC) are significantly different from zero, meaning these variables adjust to restore the long-run equilibrium. By contrast, in the GDP equation the error correction term is not significant. As α_{11} is not statistically different from zero, the GDP differential is said to be long-run weakly exogenous with respect to the long-run equilibrium.

The absolute value of α gives information about the number of quarters needed to restore the long-term equilibrium. Specifically, for values of α close to unity, adjustment is very fast, with the disequilibrium being totally eliminated within one quarter. For $0<\alpha<1$ the dynamic adjustment path will be monotonically convergent.

In our case, the estimated error–correction coefficients in the inflation and interest rate equation, i.e. the speed of adjustment to the long-run equilibrium, are quite low (0.021 and 0.014, respectively). After almost 8 years, 50% of the disequilibrium gap created by the shock has been closed by the adjustment in inflation rate.[10] The interest rate effect on the long-term equilibrium is even slower. In fact, it takes more than 12 years for the interest rate to close the 50% of the disequilibrium. Only the exchange rate has a short-time effect on the long-term equilibrium. According to our estimates, the exchange rate closes the 50% of the gap in almost 2 years.

Table 1. ML estimates of the linear VECM (2)

	Δy_t	$\Delta \pi_t$	Δi_t	Δe_t
Constant	−0.091	−0.002	0.153	−0.220
	[0.07]	[0.09]	[0.10]	[0.48]
Δy_{t-1}	0.265	−0.216	0.263	−0.576
	[0.10]	[0.12]	[0.13]	[0.17]
Δy_{t-2}	0.308	0.130	0.391	−0.454
	[0.10]	[0.12]	[0.13]	[0.65]
$\Delta \pi_{t-1}$	−0.098	−0.176	−0.054	0.199
	[0.07]	[0.09]	[0.10]	[0.09]
$\Delta \pi_{t-2}$	0.030	−0.380	0.108	1.105
	[0.07]	[0.09]	[0.10]	[0.49]
Δi_{t-1}	0.084	0.101	0.007	−0.431
	[0.08]	[0.10]	[0.10]	[0.15]
Δi_{t-2}	−0.394	0.067	−0.451	−0.035
	[0.08]	[0.10]	[0.11]	[0.52]
Δe_{t-1}	0.000	0.017	0.038	0.322
	[0.02]	[0.02]	[0.02]	[0.10]
Δe_{t-2}	0.005	0.039	−0.006	0.081
	[0.02]	[0.02]	[0.02]	[0.11]
EC_{t-1}	−0.001	−0.021	−0.014	0.080
	[0.005]	[0.006]	[0.007]	[0.033]

The results obtained in the analysis corroborate the so-called exchange rate disconnect puzzle. This puzzle, coined by Obstfeld and Rogoff (2001), states that there is a lack of relationship between exchange rates and macroeconomic fundamentals over short to medium horizons.

3.2. *Testing for non-linearity*

Next we check for non-linearity of the residuals by using three of the most popular tests. We apply the BDS test, Tsay's test and Ramsey's Reset test, the so-called V23 test to the residuals of each equation in the VECM system, i.e. the output gap differential, the inflation rate differential, the interest rate differential and the exchange rate.

The null hypothesis for these tests is that the residual generating process is linear. Tables 2 and 3 show the results.

The tables report, for each equation from the VECM, the *p*-values under the null hypothesis that the corresponding residual is a serially *i.i.d.* process. Table 2 also reports the bootstrapped *p*-values for the BDS test statistic. All tests reject the null hypothesis of a linear generating mechanism for the residuals of the selected variables.

In principle the results provided by these tests might be biased if the variable being tested present a non-constant variance. As suggested in Lee *et al.* (1993), Hurn (2004) and Becker and Hurn (2006), when time-series are heteroskedastic, which is often the case in exchange rate modelling, usual tests employed to detect non-linearity might produce misleading results. In particular, these tests tend to over reject the null of linearity. In order to control for the size distortion eventually arising from non-constant variance in selected time-series we compute a different version of the Ramsey test based on the heteroskedasticity-consistent regression as suggested in Becker and Hurn (2006). The results are shown in the last column of Table 3. The analysis altogether suggests the presence of non-linearities in the residuals. This evidence corroborates the decision of estimating the model in non-linear form.

Table 2. BDS test statistics

Dimension	$\varepsilon = 0.5\sigma$		$\varepsilon = 1.0\sigma$		$\varepsilon = 1.5\sigma$	
	Asymptotic	Bootstrap	Asymptotic	Bootstrap	Asymptotic	Bootstrap
GDP differential						
2	6.5E−01	6.6E−01	4.2E−01	3.8E−01	4.8E−01	4.6E−01
3	6.0E−01	6.0E−01	1.9E−01	2.1E−01	1.7E−01	2.0E−01
4	6.0E−02	2.2E−01	2.8E−02	6.2E−02	2.0E−02	4.3E−02
5	4.6E−02	2.6E−01	1.1E−02	4.4E−02	2.8E−03	1.3E−02
6	3.1E−02	3.0E−01	1.5E−02	5.4E−02	9.2E−04	9.2E−03
Inflation differential						
2	4.4E−04	1.6E−02	1.5E−01	1.9E−01	1.8E−02	5.2E−02
3	6.5E−03	6.6E−02	6.1E−02	1.1E−01	2.7E−02	5.8E−02
4	2.4E−05	3.0E−02	1.9E−02	5.2E−02	2.1E−02	5.3E−02
5	3.7E−06	4.3E−02	2.6E−03	2.8E−02	6.5E−03	3.3E−02
6	1.3E−05	9.8E−02	1.0E−04	1.4E−02	3.8E−03	2.8E−02
Interest rate differential						
2	3.7E−08	0.0E+00	8.1E−07	0E+00	2.3E−02	7.0E−02
3	1.0E−15	0.0E+00	1.0E−11	0E+00	9.8E−05	2.8E−03
4	3.0E−30	0.0E+00	9.2E−16	0E+00	3.4E−06	8.0E−04
5	9.3E−47	0.0E+00	3.5E−21	0E+00	3.4E−08	0.0E+00
6	1.4E−69	8.0E−04	2.7E−27	0E+00	7.7E−10	0.0E+00
Exchange rate						
2	3.0E−03	2.9E−03	5.3E−03	6.4E−03	5.9E−03	7.1E−03
3	4.9E−04	1.9E−03	8.7E−03	7.8E−03	1.1E−02	1.1E−02
4	6.0E−04	9.7E−04	2.2E−03	7.1E−03	1.3E−03	1.4E−02
5	5.9E−04	4.3E−04	2.3E−03	5.7E−03	9.4E−05	1.5E−02
6	3.2E−04	1.7E−04	3.3E−05	4.2E−03	1.2E−02	1.4E−02

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Table 3. p-Values for Tsay, V23 and V23hc tests

	Tsay	V23	V23 hc
GDP differential	−3.0E−02	−7.0E−02	−5.0E−02
Inflation differential	−5.9E−03	−6.9E−03	−3.2E−03
Interest rate differential	−4.9E−02	−4.6E−02	−2.4E−02
Exchange rate	−1.9E−02	−9.0E−03	−1.4E−02

4. MODELLING NON-LINEARITY: AN MS-VECM

We account for non-linearity in the selected variables by estimating a multivariate Markov-switching model. In the MS-VECM framework, the shocks to each variable in the model are allowed to influence the transition probabilities of moving from one phase to another.

The asymmetry of the effects is captured by allowing for state-dependent parameters where the latent state variable follows a Markov-switching process. The idea behind this class of models is that the parameters underlying the data generating process of the observed time series vector y_t depend upon the unobservable regime variable s_t, which represents the probability of being in a different state of the world.

This variable s_t is governed by a discrete state of a Markov stochastic process, which is defined by the following transition probabilities:

$$p_{ij} = \Pr(s_{t+1} = j | s_t = i)$$

$$P = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1m} \\ p_{21} & p_{22} & \cdots & p_{2m} \\ \vdots & \vdots & \ddots & \cdots \\ p_{m1} & p_{m2} & \cdots & p_{mm} \end{bmatrix}$$

where p_{ij} is the probability that state i is followed by state j and P is the corresponding transition matrix. The idea is that the relation between the exchange rate and fundamentals is time-varying but constant conditional on the stochastic and unobservable regime variable.

Within this framework we can address two different questions. The first relates to the extent of currency misalignments. The second concerns the identification of the driving forces that govern the adjustment of the exchange rate and fundamentals towards their long-term equilibrium.

In order to identify the appropriate characterization of the Markov-switching model we apply a standard bottom-up procedure as suggested in Krolzig (1997). According to this procedure the most accurate representation of a Markov model might be obtained by first restricting the effects of regime shifts on a particular set of parameters and then testing the model against possible alternatives.

In particular, we test whether the intercept term, the variance–covariance matrix and the autoregressive component of the VECM are regime-dependent by employing likelihood ratio (LR) tests suggested by Krolzig (1997, pp. 135–136) and implemented by Sarno *et al.* (2004). More formally, these tests are calculated as $2(\ln L^* - \ln L) \sim \chi^2_k$, where k represents the number of restrictions imposed, and L^* and L indicate the unconstrained and the constrained maximum likelihood, respectively.

Table 4 reports three LR test statistics with the associated p-values. We also test whether the model with three regimes is equivalent to the model with two regimes (fourth row) by employing the upper bound LR test of Davies (1977, 1987).

All tests suggest that a Markov-switching-intercept-autoregressive-heteroskedasticity VECM model is the most accurate characterization of the relationship between exchange rate and fundamentals. Moreover, as the null of two regimes can be significantly rejected (see Table 4, last row) the number of regimes is then fixed at three.

Table 4. Bottom-up identification procedure: Model *i* vs. Model *j*

Model *i*	Model *j*	Test	*p*-Values
MSIAH(3)-VECM(*p*)	MSIA(3)-VECM(*p*)	221.135	[0]
	MSAH(3)-VECM(*p*)	332.4772	[0]
	MSI(3)-VECM(*p*)	168.9078	[0]
MSIAH(3)-VECM(*p*)	MSIAH(2)-VECM(*p*)	440.47	[3.5E−78]

4.1. Analysing non-linear exchange rate dynamics

As in the linear case, we assume that the equilibrium value of the euro-dollar exchange rate is determined by a set of economic fundamentals, i.e. the relative GDP, the relative inflation rate and the interest rate differential.

Starting from the equilibrium exchange rate retrieved by computing a cointegrating vector we can provide a measure of the possible misalignment by comparing this equilibrium value with the actual exchange rate.

The model we estimate takes the following form:

$$\Delta y_t = c(s_t) + \sum_{i=1}^{k-1} \Gamma_i(s_t)\Delta y_{t-i} + \alpha(s_t)\beta' y_{t-k} + \epsilon_t \tag{12}$$

This specification allows for regime shifts in the intercept, the autoregressive coefficients, the speed of adjustment component in the cointegration matrix and the variance–covariance matrix. In our analysis, the regime-dependent cointegrating vector provides information about the adjustment process through which the long-term relationship between the exchange rate and fundamentals evolves during different periods of time. Table 5 shows the maximum likelihood estimates of the above model.

The LR linearity test significantly rejects the linearity hypothesis even when considering the upper bound suggested by Davies (1987). Table 5 allows us to evaluate the difference in the regime-dependent speed of adjustment parameters.

In the first regime, a shock to the long-run equilibrium is essentially closed by GDP differential and interest rate differential. As in the exchange rate and inflation equations the adjustment coefficient is not significant, only the GDP and the interest rate participate in the adjustment needed to restore the long-run equilibrium. More precisely, 50% of a shock to the long-term equilibrium is corrected by the output differential and by the interest rate differential in almost 2 years.

The second regime identifies periods of time where only the exchange rate significantly drives the long-term equilibrium. In fact, although the error correction coefficients are significant in all equations, only the exchange rate seems to bear the burden of the adjustment, with half-lives of reversion to equilibrium taking place in almost 1 year.

In the third regime, a deviation from the long-run equilibrium is corrected by changes in inflation rate differential and interest rate differential. In the second regime only these two variables act as a driving force for the exchange rate equilibrium.

The proportion of the disequilibrium that is corrected by the interest rate and inflation after one year and a half is near to 40% and 60%, respectively. This means that the estimated speed of adjustment for this regime is higher than that estimated by using the linear model. Within this framework, the exchange rate is driven by fundamentals also in the short and medium term. As a consequence, the disconnect puzzle does not emerge as strongly. The evidence of episodic instability involving different sets of macroeconomic fundamentals during different time periods may explain why empirical studies have found so frequently a disconnection between macroeconomic fundamentals and the exchange rate.

Tables 6 and 7 describe the properties of the estimated regimes and the matrix of transition probability, respectively.

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Table 5. ML estimates of the MSIAH(3)-VECM(2)

Regime 1											
	Const.	Δy_{t-1}	Δy_{t-2}	$\Delta \pi_{t-1}$	$\Delta \pi_{t-2}$	Δi_{t-1}	Δi_{t-2}	Δe_{t-1}	Δe_{t-2}	EC_{t-1}	σ^2 (Reg.1)
Δy_t	−0.31	0.67	0.10	−0.32	0.05	0.10	−0.37	−0.01	−0.01	−0.08	0.3
	[0.20]	[0.16]	[0.17]	[0.18]	[0.19]	[0.16]	[0.16]	[0.02]	[0.02]	[0.01]	
$\Delta \pi_t$	−0.33	−0.17	0.11	0.95	−0.25	0.05	−0.14	−0.03	0.02	0.01	0.2
	[0.17]	[0.14]	[0.14]	[0.15]	[0.16]	[0.13]	[0.13]	[0.01]	[0.01]	[0.01]	
Δi_t	−0.69	0.09	0.00	0.19	0.16	0.47	0.22	0.01	−0.04	−0.09	0.1
	[0.11]	[0.09]	[0.09]	[0.10]	[0.10]	[0.08]	[0.08]	[0.01]	[0.01]	[0.01]	
Δe_t	−2.23	−0.32	0.54	1.24	−0.98	1.85	−0.76	0.60	0.12	0.14	12.9
	[0.49]	[1.19]	[1.26]	[1.28]	[1.37]	[1.12]	[1.12]	[0.12]	[0.11]	[0.19]	
Regime 2											
	Const.	Δy_{t-1}	Δy_{t-2}	$\Delta \pi_{t-1}$	$\Delta \pi_{t-2}$	Δi_{t-1}	Δi_{t-2}	Δe_{t-1}	Δe_{t-2}	EC_{t-1}	σ^2 (Reg. 2)
Δy_t	−0.40	0.77	−0.23	−0.12	−0.37	0.14	−0.23	−0.05	0.01	−0.03	0.52
	[0.17]	[0.14]	[0.14]	[0.21]	[0.23]	[0.12]	[0.12]	[0.03]	[0.02]	[0.01]	
$\Delta \pi_t$	−0.23	−0.37	0.33	0.61	−0.06	−0.06	0.14	−0.01	0.02	−0.02	0.37
	[0.15]	[0.12]	[0.11]	[0.17]	[0.19]	[0.10]	[0.10]	[0.02]	[0.02]	[0.01]	
Δi_t	0.11	−0.05	0.32	−0.90	0.75	0.88	−0.24	−0.02	0.05	−0.02	0.67
	[0.20]	[0.16]	[0.15]	[0.23]	[0.26]	[0.14]	[0.14]	[0.03]	[0.03]	[0.01]	
Δe_t	−0.09	−2.37	0.93	2.75	−4.55	−2.08	0.82	0.75	−0.20	−0.14	6.08
	[1.07]	[0.85]	[0.82]	[1.25]	[1.41]	[0.75]	[0.73]	[0.16]	[0.14]	[0.06]	
Regime 3											
	Const.	Δy_{t-1}	Δy_{t-2}	$\Delta \pi_{t-1}$	$\Delta \pi_{t-2}$	Δi_{t-1}	Δi_{t-2}	Δe_{t-1}	Δe_{t-2}	EC_{t-1}	σ^2 (Reg. 3)
Δy_t	2.13	0.32	−0.01	−0.08	−0.80	−0.25	−0.38	−0.11	0.06	0.02	0.01
	[0.09]	[0.05]	[0.04]	[0.06]	[0.08]	[0.03]	[0.02]	[0.01]	[0.01]	[0.01]	
$\Delta \pi_t$	0.34	−0.13	0.09	0.69	−0.15	0.06	0.24	−0.02	−0.01	−0.12	0.03
	[0.18]	[0.10]	[0.07]	[0.13]	[0.15]	[0.06]	[0.04]	[0.01]	[0.01]	[0.01]	
Δi_t	2.62	0.23	0.38	0.08	0.06	−0.35	−0.33	0.02	0.08	−0.07	0.13
	[0.36]	[0.19]	[0.14]	[0.25]	[0.30]	[0.13]	[0.08]	[0.02]	[0.02]	[0.02]	
Δe_t	0.65	−0.57	1.33	−2.45	1.09	2.08	−1.35	0.75	−0.43	0.41	19.08
	[0.15]	[1.30]	[0.97]	[1.72]	[2.04]	[0.86]	[0.57]	[0.15]	[0.16]	[0.24]	
Log-likelihood	−475.9	Linear system	−661.1								
AIC criterion	12.5	Linear system	14.1								
HQ criterion	14.1	Linear system	14.6								
SC criterion	16.6	Linear system	15.4								
LR linearity test:	370.5 $\chi(100)$ = [0.0000] **										
	$\chi(106)$ = [0.0000] ** DAVIES = [0.0000] **										

A single asterisk (*) indicates significance at 5%, and a double asterisk (**) means a 1% significance level.

Table 6. Regime properties

	nObs	Prob.	Duration
Regime 1	31	0.3326	6.61
Regime 2	27	0.3504	10.16
Regime 3	31	0.317	4.84

The regimes are estimated to be quite persistent. The expected duration of Regimes 1 and 3 is 6.61 quarters and 4.84 quarters, respectively. Regime 2 is expected to last for 10.16 quarters.

The main characteristics of the estimated regimes are summarized in Figure 3 and Table 8. In Figure 3 the periods where output and interest rate drive the long-term equilibrium (Regime 1) coincide with the

Table 7. Matrix of transition probabilities

	Regime 1	Regime 2	Regime 3
Regime 1	0.85	0.00	0.15
Regime 2	0.05	0.90	0.04
Regime 3	0.10	0.11	0.79

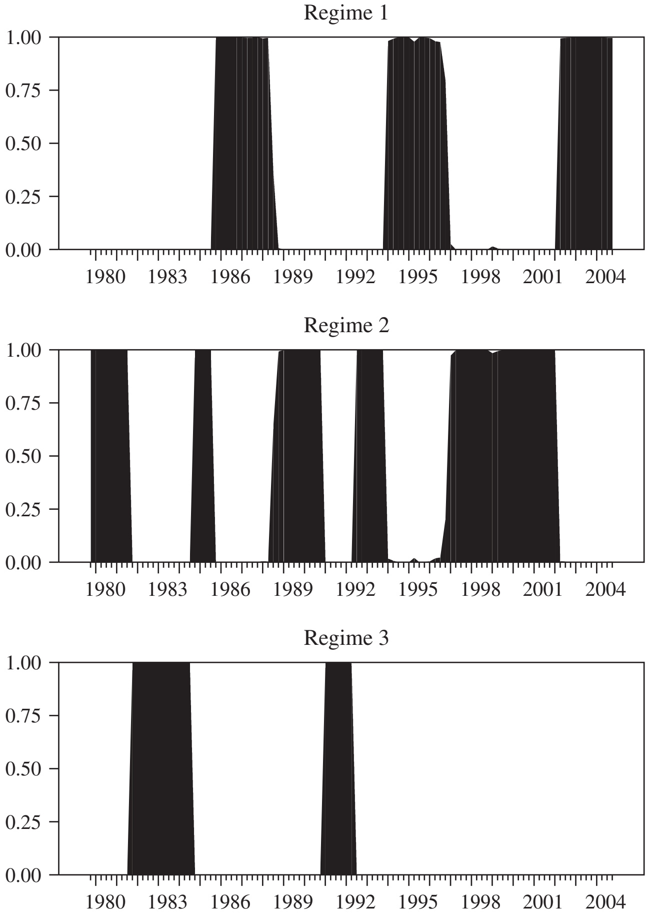

Figure 3. Smoothed regime probabilities.

Table 8. Regime classification

Regime 1	Regime 2	Regime 3
1985:1–1988:2 [0.9791]	1979:4–1981:3 [0.9980]	1981:4–1984:3 [0.9089]
1994:1–1996:4 [0.9415]	1984:4–1985:3 [0.9043]	1991:1–1992:2 [0.9534]
2002:2–2004:4 [0.9543]	1988:3–1990:4 [0.9636]	
	1992:3–1993:4 [0.9236]	
	1997:1–2002:1 [0.9353]	

periods where there has been an appreciation of the exchange rate. On the contrary, when inflation and interest rate largely influence the long-term equilibrium (Regime 3) the exchange rate depreciates. Finally, the second half of the 1980s, the early 90s and the years 1997–2002 are associated with the increasing importance of exchange rate in determining the long-run equilibrium.

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Table 9. Comparing non-linear model with data

	Data	Model	$\lvert R^{1/2} \rvert$
Sample mean	−0.099213	−0.0874	0.15
Variance	0.0274	0.041	0.24

The evidence suggests that during euro-depreciation episode (e.g. early 1980s and early 1990s), the emphasis is on differences in the rates of inflation between countries and the interest rate differentials that might have caused them. On the contrary, during euro-appreciation episodes (e.g. 1985–1986, 1993–1995, 2002–2004), inflation rates came down and converged across countries but exchange rate movements remained large. This leads to more emphasis being placed on GDP differential and factors that affect the real economy. Finally, when the exchange rate is close to its fundamental value (e.g. 1999–2001), non-monetary factors largely affect exchange rates. During these periods exchange rate movements do not depend on economic fundamentals but instead on self-fulfilling beliefs and expectations.

These results can be explained in the context of our theoretical model. This predicts that the exchange market is characterized by the existence of different regimes. In one regime the exchange rate is closely linked to fundamentals; in another one the exchange rate is disconnected from the fundamentals. In the latter regime news in the fundamentals has no or only a weak impact on the exchange rate. The exchange rate switches from one regime to the other. We broadly find the predictions of the model in the data.

Finally, we employ different tests to assess the goodness-of-fit of the estimated non-linear model. The first class of test we use is the parametric encompassing tests. Practically, we evaluate the ability of the estimated model to match the mean and the variance of the in-sample exchange rate data.

Table 9 compares the first two moments of the empirical and estimated exchange rate distribution by using the conservative test proposed in Breunig *et al.* (2003).

The results suggest that the selected model is cable of producing an adequate representation of the data. We can conclude that the estimated non-linear model significantly replicates important features of the in-sample data.

5. OUT-OF-SAMPLE FORECASTING PROPERTIES

In the previous section, we argued that even if the relationship between the exchange rate and its fundamentals is found to be time-varying, it is possible to identify periods of time where the relationship becomes stable. An interesting question is then whether it is possible to improve the out-of-sample forecasting performance by using models that incorporate non-linear mean reversion. In order to do it, we perform a standard forecast estimation and evaluation strategy. In particular, we first estimate a set of competing models over some periods and construct out-of-sample forecasts. Then we compare these forecasts with the actual exchange rate outcome.

The aim of this section is to test whether large deviation from fundamentals creates a tendency for exchange rate predictability to emerge. To this end, we examine evidence on how the ability of alternative models to forecast the exchange rate might change over time.

The empirical analysis is based on three alternative models which we, now, describe in detail. The first one (RW) consists of a driftless random walk model. As stressed above, the random walk remains a useful benchmark against which exchange rate models are judged. The model is as follows:

$$e_t = e_{t-1} + \epsilon_t \tag{13}$$

The second one is the four variable vector error correction model (VECM) described in equation (9). The last one consists of a Markov switching VECM (MS-VECM).

Once each different type of model has been estimated, the question arises as to how their performance might best be compared. There are a number of different ways in which the forecast accuracy of competing models can be assessed. In this paper we use out-of-sample prediction errors and consider standard statistical measures of forecast accuracy as well as a new test for equal predictive accuracy in nested models.

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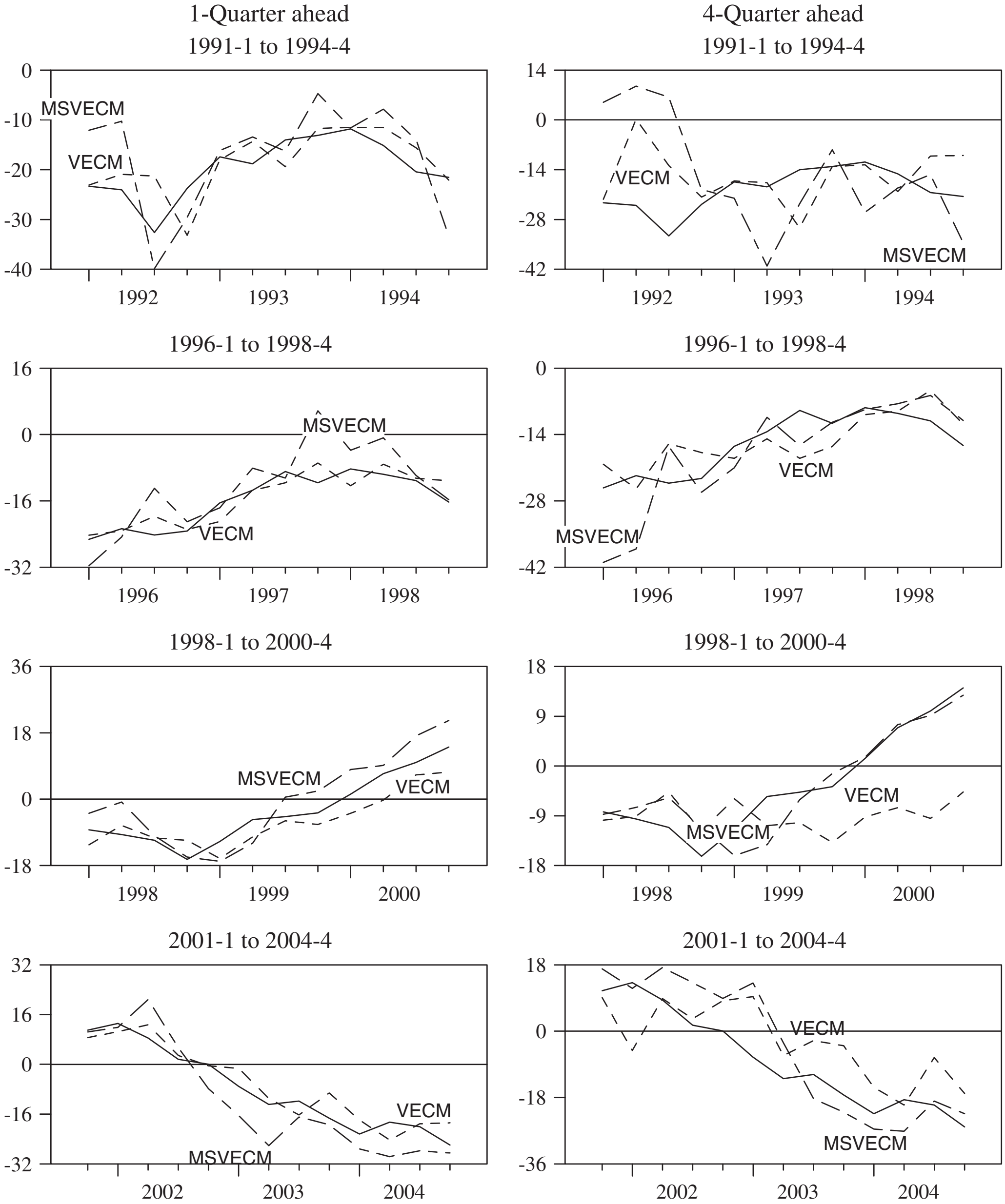

Figure 4. Out-of-sample point forecasts: MSVECM vs. linear VECM.

5.1. Forecasting procedure

The three models explained above are estimated on a sub-sample of the historical data. Then the out-of sample forecast of the competing models for alternative periods are evaluated. The forecast accuracy is measured by computing rolling forecasts. The estimation period goes from 1978:1–1989:4, while the forecast period goes from 1990:1 to 2004:4. This means that the first sequence of 1–4-quarter ahead forecast is generated starting from 1990:1. Then, the starting date of the forecast period is rolled forward one period, and another sequence of forecasts is generated. This loop is repeated until we have 60 × 1-quarter forecasts, down to 57 × 4-quarter forecasts.

Figures 4 and 5 characterize four periods of the euro-dollar history.

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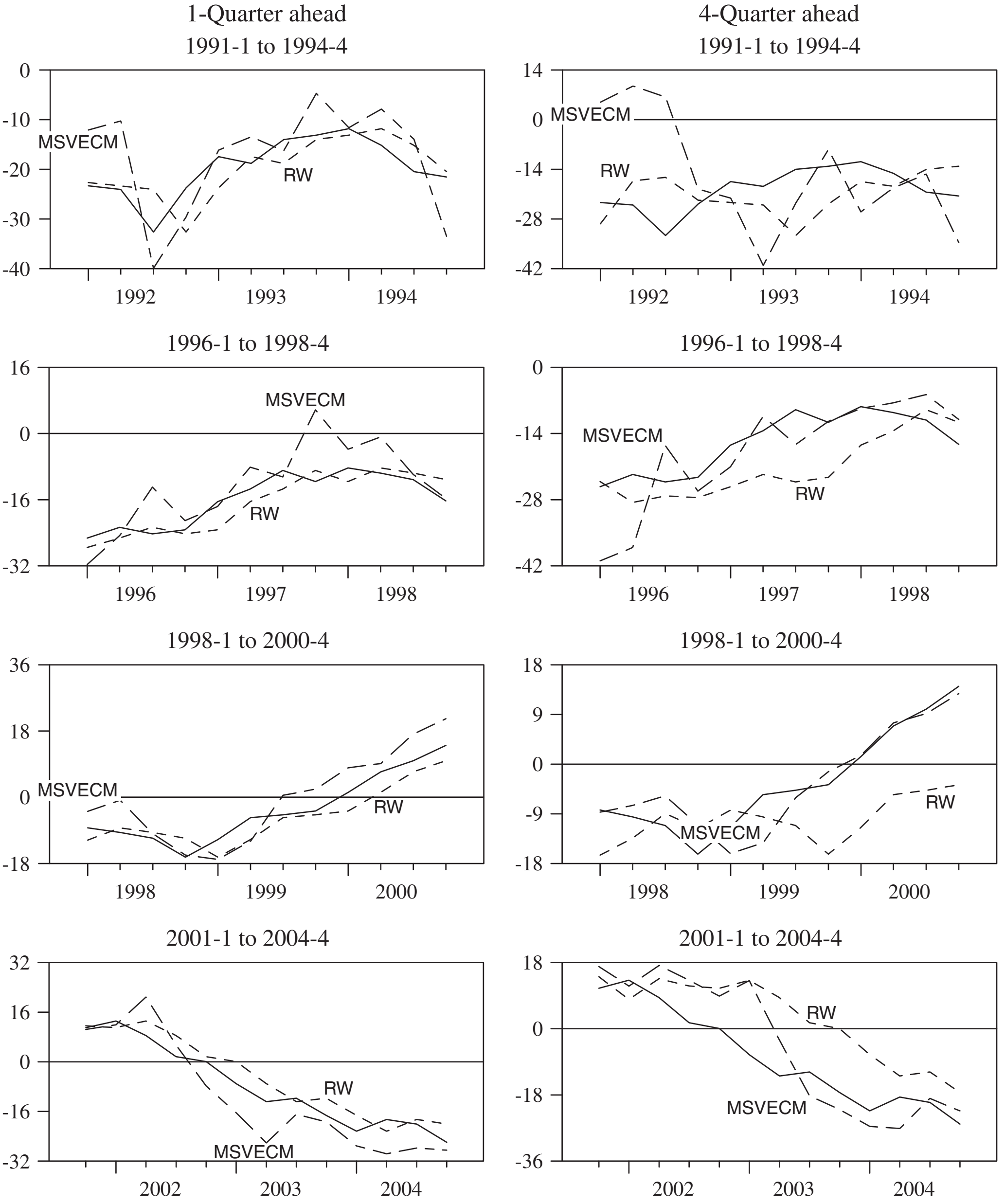


Figure 5. Out-of-sample point forecasts: MSVECM vs. naïve random walk.

The four periods are 1992:1–1994:4, 1996:1–1998:4, 1998:1–2000:4 and 2001:4–2004:4. Each period is analysed in terms of the ability of the linear VECM; the MSVECM and the RW to forecast the actual exchange rate. For each period the figures show the 1- and 4-quarter ahead forecasts (the first and second column, respectively).

Figures 4 and 5 provide a graphical summary of the performances of the three models described above over particular time periods in forecasting the euro-dollar exchange rate. Visual inspection seems to suggest that MS-VECM does not consistently outperform, in terms of forecast errors, the linear VECM and the RW. In fact, the evidence emerging from Figures 4 and 5 does not provided a consistent and unambiguous picture concerning the forecast ability of the selected models. The forecast ability of alternative models seems to vary across the sub-sample. The evidence emerging from these figures corroborates the hypothesis of having more than one states operating during the sample period.

However, in order to assess the performance of the alternative models we have to analyse the forecast accuracy through a set of statistical measure.

5.2. Assessing forecast accuracy

We first use standard quantitative procedures involving the forecast errors. Precisely, the forecast error can be defined as $e_{t+k} = x_{t+k} - \hat{x}_{t+k}$, where $k \geqslant 1$ and $\hat{x}_{t+k}$ represents the k-step ahead forecast. Three widely used measures of forecast accuracy are the mean error (ME), the mean absolute error (MAE) and the root mean square error (RMSE) of a model. We can calculate them as follows:

$$\mathrm{ME} = \frac{1}{n}\sum_{i=1}^{n} e_{t+k+i}, \quad \mathrm{MAE} = \frac{1}{n}\sum_{i=1}^{n} |e_{t+k+i}| \quad \text{and} \quad \mathrm{RMSE} = \left(\frac{1}{n}\sum_{i=1}^{n} e_{t+k+i}^{2}\right)^{1/2} \tag{14}$$

The comparison of forecasting performance based on these measures is summarized in Table 10.

The table contains the ME, the MAE and the RMSE from five estimated models, covering two forecasting horizons: 1- and 4-step-ahead forecast (where $k = 1{,}4$ denotes the forecast step). This table also reports the relative ranking in terms of forecast error.

The first period, occurring in the early 1990s, embraces the European monetary system crises. During these years, the actual exchange rate appears to be better approximated by the non-linear VECM. The second period covers the mid-1990s, while the third period, ranging from the late 1990s to the early 2000s, embraces the launch of the euro. In these years the RW forecasts appear to be much closer to the actual

Table 10. Comparing forecast accuracy

1991(1)–1994(4)	ME		MAE		RMSE		Rank	
k	1	4	1	4	1	4	1	4
RW	0.86	1.06	5.08	10.40	6.43	14.63	3	3
VECM(2)	−0.85	−1.32	4.71	7.84	5.88	11.12	2	2
MS(3)-VECM(2)	0.61	1.69	4.04	7.92	5.33	9.09	1	1
1996(1)–1998(4)	ME		MAE		RMSE		Rank	
k	1	4	1	4	1	4	1	4
RW	0.94	5.18	2.98	6.51	3.42	7.66	2	3
VECM(2)	−0.59	−0.57	2.56	4.46	3.18	5.24	1	1
MS(3)-VECM(2)	−3.34	1.89	5.20	5.84	7.12	7.63	3	2
1998(1)–2000(4)	ME		MAE		RMSE		Rank	
k	1	4	1	4	1	4	1	4
RW	2.15	6.90	3.48	8.57	3.93	9.95	1	2
VECM(2)	2.66	5.72	4.02	8.55	4.46	10.44	2	3
MS(3)-VECM(2)	−3.16	0.12	5.13	2.77	5.68	3.76	3	1
2001(1)–2004(4)	ME		MAE		RMSE		Rank	
k	1	4	1	4	1	4	1	4
RW	−2.70	−7.46	4.79	9.98	5.09	11.54	2	3
VECM(2)	−0.88	−1.86	4.23	8.72	5.66	10.17	3	2
MS(3)-VECM(2)	−2.50	−3.96	4.09	6.07	4.63	8.79	1	1
1991(1)–2004(1)	ME		MAE		RMSE		Rank	
k	1	4	1	4	1	4	1	4
RW	0.12	1.39	3.66	12.99	4.47	17.39	1	2
VECM(2)	0.17	1.58	4.88	13.49	6.05	18.39	2	3
MS(3)-VECM(2)	0.30	0.60	5.34	8.76	6.78	10.16	3	1

exchange rate. Finally, the fourth period goes from the early 2000s to the end of the sample. Contrary to the results of previous time periods, the forecast ability of the MS-VECM seems to be higher than the one of the random walk.

Overall, non-linear mean-reversion models outperform random walk models when the deviation from long-term equilibrium is large.

The above measures provide a quantitative estimate of the forecasting ability of a specific model, allowing different models to be ranked, but does not provide a formal statistical indication of whether one model is significantly better than another. We also explicitly test the null hypothesis of no difference in the accuracy of the two competing forecasts by using forecast encompassing tests.

We use an approximately normal test for equal predictive accuracy in nested models as described in a recent paper by Clark and West (2007). Models are tested bilaterally. For each model, the null hypothesis of equal accuracy (equal MSPE) is checked against the alternative that the larger model has a smaller MSPE. The null is tested by examining the difference between the MSPE of Model 1 (the restricted one) and that of Model 2 (the larger one), i.e. MSPE^1-(MSPE^2-‘adj.’), where the ‘adj.’ term ‘adjust for the upward bias in MSPE produced by estimation of parameters that are zero under the null’. Specifically, starting from the adjusted difference between the MSPE of two models:

$$\hat{f}_{t+k} = (x_{t+k} - \hat{x}_{1t,t+k})^2 - [(x_{t+k} - \hat{x}_{2t,t+k})^2 - (\hat{x}_{1t,t+k} - \hat{x}_{2t,t+k})^2]$$

the test consists of regressing $\hat{f}_{t+k}$ on a constant and computing the usual t-statistic for a zero coefficient.

We analyse bilateral comparison over several subsample periods and report this information in Table 11. A p-value lower than 0.05 means that we reject the null of equal accuracy in favor of Model 2 (the larger one). In our case, RW represents the restricted model while VECM and MS-VECM are the larger models.

Over our forecasting sample, we find that RW does not consistently out-perform competing models. More precisely, the non-linear specification significantly improves forecast accuracy during periods when the deviation between exchange rate and fundamentals is large (1991:1–1994:4 and 2001:1–2004:4). On the contrary, when the exchange rate is close to its equilibrium value it tends to be better approximated by a naïve random walk.

Moreover, when considering the whole forecasting period (1991:1–2005:2), we find that RW tend to outperform linear VECM at short forecast horizons. On the opposite, non-linear models with more elaborate mean-reverting components dominate at both 1- and 4-steps ahead.

Table 11. Clark and West test of equal accuracy: RW vs. competing models

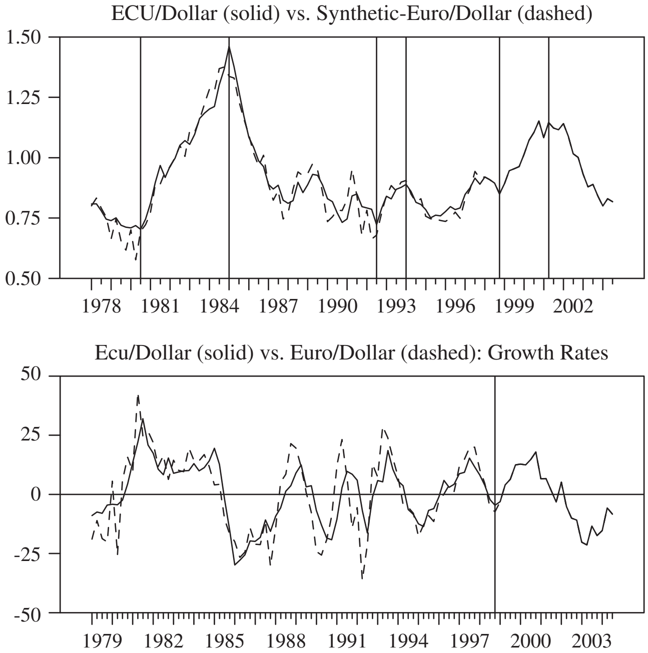
	$k=1$		$k=4$	
	Coeff.	p-Value	Coeff.	p-Value
1991q1–1994q4				
VECM(2)	4.224	[0.37]	−45.611	[0.30]
MS(3)-VECM(2)	−0.326	[0.49]	155.730	[0.05]
1996q1–1998q4				
VECM(2)	1.958	[0.37]	88.728	[0.00]
MS(3)-VECM(2)	8.162	[0.20]	17.360	[0.30]
1998q1–2000q4				
VECM(2)	−4.196	[0.10]	−57.642	[0.06]
MS(3)-VECM(2)	22.992	[0.03]	68.281	[0.11]
2001q1–2004q4				
VECM(2)	10.355	[0.19]	30.349	[0.20]
MS(3)-VECM(2)	25.494	[0.06]	123.008	[0.01]
1991q1–2005q2				
VECM(2)	10.881	[0.14]	58.640	[0.06]
MS(3)-VECM(2)	6.846	[0.02]	60.771	[0.01]

6. CONCLUSIONS

In this paper we investigated whether the dynamic interaction between the exchange rate and its fundamentals is time-varying. We first developed a simple theoretical model of the exchange rate in which chartists and fundamentalists interact. This model predicts that exchange rate movements will be characterized by different regimes, which we called fundamental and non-fundamental regimes. When in a fundamental regime the exchange rate stays close to the fundamental. There are also non-fundamental regimes in which the exchange rate is disconnected from the fundamentals. The existence of different regimes creates a non-linearity in the link between the exchange rate and its fundamentals. This also implies that in some regimes the news in the fundamentals has little or no impact on the exchange rate, while in others it strongly affects the exchange rate.

In the empirical part of the paper we analysed the nature of these non-linearities by specifying and estimating a Markov switching model. This model aims at identifying the driving forces that govern the adjustment of the exchange rate and fundamentals toward their long-term equilibrium. We found that the relationship between the exchange rate and its fundamentals is episodically unstable. This implies that the switching nature of the exchange rate process is inconsistent with a linear representation of the relation between the exchange rate and its fundamentals. The evidence of episodic instability involving different sets of macroeconomic fundamentals during different time periods may explain why empirical studies have found so frequently a disconnection between macroeconomic fundamentals and the exchange rate.

Finally, we examine the predictive power of various models. In out-of-sample forecasting tests, mean-reversion models (both linear and non-linear) are compared with random walks. A naïve constant change forecast remains a benchmark against which exchange rate models are judged. A set of forecast evaluation techniques were employed to judge the relative performance of three competing models of the exchange rate determination. We find that the non-linear specification significantly improves forecast accuracy during periods when the deviation between exchange rate and fundamentals is large. Conversely, when the exchange rate is close to its equilibrium value it tends to be better approximated by a naïve random walk.



ECU/Dollar (solid) vs. Synthetic-Euro/Dollar (dashed)

Ecu/Dollar (solid) vs. Euro/Dollar (dashed): Growth Rates

Figure A1. The euro/dollar exchange rate.

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APPENDIX: A SYNTHETIC EURO-DOLLAR EXCHANGE RATE

In the empirical analysis we investigate the relationship between the euro-dollar exchange rate and its fundamentals for the period 1979 : 1–2004 : 4. The length of the sample period raises the question of what variable should be used to proxy the euro exchange rate during the pre-EMU period. Some studies, like la Cour and MacDonald (2000), consider the European Currency Unit (ECU). However, the use of the ECU is not completely appropriate.

In fact, on the one hand the ECU currency basket includes countries, such as Denmark and the United Kingdom, which have not introduced the new currency in 1999; on the other hand, it does not take into account countries, such as Austria and Finland, which joined the EMU since 1999.

For these reasons we construct a synthetic euro-dollar exchange rate for the pre-EMU era. The value of the euro is derived from a weighted average of the exchange rates of the EMU member countries. As Greece joined the Eurozone on 01 January 2001, the drachma is not considered in the aggregation. The weights used in the aggregation are the GDP weights (1995) at PPP exchange rates proposed by Fagan *et al.* (2001). The exchange rate series of the individual economies, defined as national currency per US dollar, are taken from the International Financial Statistics (IFS) of IMF.

Figure A1 depicts the ECU/dollar and the synthetic euro/dollar. All variables are shown in levels as well as in first differences. The figure suggests that the two measures of the pre-EMU euro exchange rate do not show large discrepancies. In fact, the external value of the euro (according to both definitions) depreciates against the US dollar during the periods 1980 : 3–1985 : 1, 1992 : 3–1994 : 1 and 1998 : 4–2001 : 3 (the vertical gridlines in the upper panel of Figure A1 highlight these periods). On the contrary, during the second half of the eighties, the mid-nineties and the period 2001:4–2004:3 the euro appreciates against the dollar.

However, some differences might be noted concerning the time path of the two series during the ERM crises. During the early nineties, the level of the synthetic-aggregate euro was somewhat below that of the ECU. This evidence mostly reflects the greater stability of the national exchange rates comprised in the ECU with respect to the currencies of the EMU participating economies.

NOTES

1. See also Baillie and Selover (1987), McNown and Wallace (1994) and Baillie and Pecchenino (1991).
2. See for example, Michael *et al.* (1997), Sarno (2000), Sarantis (1999), Taylor and Peel (2000) and Baum *et al.* (2001).
3. Smooth Transition Autoregressive (STAR) models were originally introduced by Terasvirta and Anderson (1992). Their statistical properties are studied in Luukkonen *et al.* (1988), Luukkonen and Terasvirta (1991), Granger and Terasvirta (1993), Eitrheim and Terasvirta (1996).
4. For more information on technical analysis and chartism see James (2003).
5. This specification is often applied in discrete choice models. For an application in the markets for differentiated goods, see Anderson *et al.* (1992). There are other ways to specify a rule that governs the selection of forecasting strategies. One was proposed by Kirman (1993). Another one was formulated by Lux and Marchesi (1999). See also Pilbeam (1995) on the profitability of traders in the foreign exchange market providing evidence about the specification used here.
6. In De Grauwe and Grimaldi (2005) an extensive sensitivity analysis is performed indicating that our results are robust for a wide spectrum of parameter values.
7. Details on the aggregation methodology for the Pre-EMU exchange rate time series are given in the Appendix.
8. The inflation rate in each country is calculated as the percentage change in the annual CPI inflation rate, i.e. $100(\log \mathrm{CPI}_t - \log \mathrm{CPI}_{t-4})$.
9. This means that the deviation of the nominal exchange rate from its fundamental value can be written as follows: $\mathrm{dev}_t = e_t - (y_t + \pi_t + i_t)$.
10. In order to obtain the number of quarters (τ) required to dissipate $x\%$ of a shock we use the following formula: $(1-\alpha)^\tau = (1-x\%)$, where α is the absolute value of the estimated speed adjustment parameter.

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