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The Exchange Rate and its Fundamentals in a Complex World

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Abstract

We develop a nonlinear exchange rate model with heterogeneous agents. Some agents adopt a "fundamentalist" forecasting rule, while others use a "chartist" forecasting rule. We show that the model is capable of explaining the empirical puzzles relating to exchange rate movements. In particular, the model explains the "exchange rate determination" and PPP puzzles, the excess volatility, and fat tails in exchange rate returns.

1. Introduction

Since the start of the floating exchange rate regime in the early 1970s, an increasing number of empirical anomalies and puzzles have been uncovered that seem to contradict the existing exchange rate theories.

The first and foremost empirical puzzle has been called the "exchange rate determination" puzzle (see Lyons, 2001; Bacchetta and van Wincoop, 2003), i.e. the exchange rate appears to be disconnected from its underlying fundamentals most of the time. It was first analyzed by John Williamson (1985) who called it the "misalignment" problem. It is also referred to as the "disconnect" puzzle (Obstfeld and Rogoff, 2000). We will not use this term here because the disconnect puzzle refers to both a lack of influence of fundamentals on the exchange rate, and a lack of influence of the exchange rate on the real economy (e.g. trade balance). The exchange rate determination puzzle was also implicit in the celebrated Meese and Rogoff studies of the early 1980s documenting that there is no stable relationship between exchange rate movements and the *news* in the fundamental variables. Goodhart (1989), Goodhart and Figlioli (1991), and more recently Faust et al. (2003) found that most of the changes in the exchange rates occur when there is no observable news in the fundamental economic variables. This finding contradicted the theoretical models, which imply that the exchange rate can only move when there is news in the fundamentals.

Other empirical anomalies have been uncovered over the years. One is the puzzle of "excess volatility" of the exchange rate, i.e. the volatility of the exchange rate by far exceeds the volatility of the underlying economic variables. Baxter and Stockman (1989) and Flood and Rose (1995) found that while the movements from fixed to flexible exchange rates led to a dramatic increase in the volatility of the exchange rate, no such increase could be detected in the volatility of the underlying economic variables. This contradicted the "news" models that predicted that the volatility of the exchange

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rate can only increase when the variability of the underlying fundamental variables increases.

A third puzzle relates to PPP and is closely associated with the "exchange rate determination puzzle." Many researchers have found that the deviations from PPP are large and sustained (Rogoff, 1996; Cheung and Lai, 2000; Obstfeld and Rogoff, 2000). The half-life of the PPP deviations has been estimated to be of the order of four to five years. Some researchers have found even longer half-lives (Lothian and Taylor, 1996; Engel, 2000). Other researchers (e.g. Dumas, 1992) have stressed that the long time needed to adjust to PPP might be due to the existence of transaction costs. The transaction cost hypothesis implies a nonlinearity in the adjustment process. This hypothesis has been confirmed by the empirical evidence based on time-series analysis (see Michael et al., 1997; Kilian and Taylor, 2001).

A fourth puzzle is that the distribution of the exchange rate returns is not normal. Most of the empirical findings document that the exchange rate returns have fat tails (see Lux, 1998; de Vries, 2000; Lux and Marchesi, 2000). This evidence is difficult to rationalize in existing exchange rate models, since there is little evidence of fat tails in the fundamental variables that drive the exchange rate in these models.

Finally, it has recently been found (Engel and Morley, 2001; Cheung et al., 2002), that the speed of adjustment of the nominal exchange rate towards the equilibrium is actually smaller than the speed of adjustment of prices. This paradoxical result cannot easily be explained by the existing exchange rate models. For example, the Dornbusch model predicts that after the overshooting following an unanticipated shock, the speed of adjustment towards equilibrium is identical for the nominal exchange rate and for the price level.

It is obvious that in the face of these empirical anomalies the existing exchange rate models that have been developed mostly in the 1970s need to be re-evaluated. The empirical failure of these exchange rate models has led to new attempts to model the exchange rate. These attempts have led to three different modeling approaches. The first one uses the Obstfeld–Rogoff (1996) framework of dynamic utility optimization of a representative agent. The models that came out from this approach have a high content of intellectual excitement. However, they have not yet led to many testable propositions.

A second approach starts from the analysis of the microstructure of the foreign exchange market with heterogeneous agents (see Evans and Lyons, 1999; Lyons, 2001). This approach has led to new insights into the way information is aggregated and is important for the understanding of the very short-term behavior of the exchange rate. As a result, this approach is capable of explaining some of the empirical anomalies, e.g. the excess volatility. However, it is as yet unclear whether it can solve the other empirical anomalies discussed here.

In this paper we use the insights of the microstructure literature recognizing that heterogeneous agents have different beliefs about the behavior of the exchange rate. The difference in the beliefs of agents introduces nonlinear features in the dynamics of the exchange rate.[1] We develop a simple model of the exchange rate, which incorporates these nonlinear features and we analyze their implications for the dynamics of the exchange rate. It will be shown that our simple nonlinear model is capable of explaining the empirical puzzles about the exchange rate behavior.[2]

2. A Simple Nonlinear Exchange Rate Model

In this section we develop a simple nonlinear exchange rate model. We start by defining the fundamental exchange rate. This is the exchange rate that is consistent with

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equilibrium in the real part of the economy. In a very simple model this would be the PPP value of the exchange rate. In more elaborate models (e.g. the monetary model, or the Obstfeld–Rogoff new open economy macro model; Obstfeld and Rogoff, 1996) this fundamental exchange rate would be determined by the interaction of more variables than the price levels. We leave the modeling of the fundamental exchange rate outside the scope of this paper, and we will assume that the fundamental exchange rate behaves like a random walk without drift.[3] This implies

$$s_t^* = s_{t-1}^* + \varepsilon_t. \tag{1}$$

We now introduce the assumption that the agents have heterogeneous beliefs about the future exchange rate. Agents' beliefs can be classified depending on how they view the process by which the market price will creep towards the fundamental exchange rate s_t^*. We make the simplifying assumption that there are two types of agents in the foreign exchange market, which we will call *fundamentalists* and *chartists*.[4]

The *fundamentalists* compare the past market exchange rates with the fundamental rate and they forecast the future market rate to move towards the fundamental rate. In this sense they follow a *negative feedback* rule.[5] We will make the additional assumption that they expect the speed with which the market rate returns to the fundamental rate to be determined by the speed of adjustment in the goods market.

As pointed out earlier, there is an increasing amount of empirical evidence indicating that the speed of adjustment in the goods market follows a nonlinear dynamics, i.e. the speed with which prices adjust towards equilibrium depends positively on the size of the deviation from equilibrium. We will assume that this adjustment process is quadratic in nature.[6] Fundamentalists take this nonlinear dynamic adjustment into account in making their forecast. In addition, the fundamentalists take the existence of transactions costs in the goods market into account. This leads us to specify the following rule for the fundamentalists:

$$E_{f,t}(\Delta s_{t+1}) = -\theta |s_t - s_t^*|(s_t - s_t^*) \quad \text{if } |s_t - s_t^*| > C \tag{2a}$$

$$E_{f,t}(\Delta s_{t+1}) = 0 \qquad\qquad\qquad\qquad \text{if } |s_t - s_t^*| \leq C, \tag{2b}$$

where $E_{f,t}$ is the forecast made in period t by fundamentalists, C is the transaction cost in the goods market (assumed to be of the "iceberg" type), and $\theta > 0$.

Thus, when the size of the deviation from equilibrium is large the fundamentalists expect a faster speed of adjustment towards the fundamental rate than when the size of the deviation is small. The rationale for such an assumption is based on the existence of price rigidities (due, for example, to menu costs and pricing-to-market). It is well known from models of price rigidities that for small shocks the frequency of price changes is low (see Blanchard and Fischer, 1989). In contrast when shocks are large, prices adjust more frequently. Put differently, large shocks lead to a high speed of adjustment of goods prices. Note that we do not model the goods market explicitly but we assume that in order to form their expectations about the exchange rate, the fundamentalists take into account the dynamics of the goods market and the speed of adjustment of goods prices.

In addition, the forecasts made by the fundamentalists depend on the transactions cost in the goods market. Consider the first case, when the exchange rate deviation from its fundamental value is larger than the transaction costs, C, in the goods market. In this case the fundamentalists apply the forecasting rule specified in (2a). In the second case, when the exchange rate deviations from the fundamental value are

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smaller than the transaction costs, the fundamentalists know that arbitrage in the goods market does not apply. As a result, they expect the changes in the exchange rate to follow a white-noise process ε_t. The best they can do is to forecast no change.

Thus, the information set of the fundamentalists at time t consists of the market and the fundamental exchange rates at time t, the speed of adjustment in the goods market, and the transactions costs in the goods market.

The *chartists* are assumed to follow a *positive feedback* rule, i.e. they extrapolate past movements of the exchange rate into the future. Their forecast is written as:

$$E_{c,t}(\Delta s_{t+1}) = \beta \sum_{i=1}^{T} \alpha_i \Delta s_{t-i}, \tag{3}$$

where $E_{c,t}$ is the forecast made by chartists using information up to time t; Δs_t is the change in exchange rate.

As can be seen, the chartists compute a moving average of the past exchange rate changes and they extrapolate this into the future exchange rate change. The degree of extrapolation is given by the parameter β. Note that, in contrast to the fundamentalists, they do not take into account information concerning the fundamental exchange rate. The information set of the chartists consists of the present and past market exchange rates only. In this sense they can be considered to be pure *noise traders*.[7]

Our choice to introduce chartists' rules of forecasting is based on empirical evidence. The evidence that chartism is used widely to make forecasts is overwhelming (see Taylor and Allen, 1992; Cheung and Chinn, 2000). Therefore, we give a role to chartists in our model, even if these agents do not behave according to the rational-expectations paradigm. Judgments about whether this is a good modeling approach should be based on the empirical success of the model. It remains important, however, to check if the model is internally consistent. In particular, the chartists' forecasting rule must be shown to be profitable within the confines of the model. If these rules turn out to be unprofitable, they will not continue to be used. They can then also not be maintained in the model. We will therefore check whether these rules are indeed profitable (see section 5).

The market's expectations about the future exchange rate are a weighted average of the expectations of chartists and fundamentalists, i.e.

$$E_t \Delta s_{t+1} = -1(|s_t - s_t^*| > C) n_{ft} \theta |s_t - s_t^*| (s_t - s_t^*) + n_{ct} \beta \sum_{i=1}^{T} a_i \Delta s_{t-i}, \tag{4}$$

where $1(|s_t - s_t^*| > C)$ is the indicator function equal to 1 if the statement is true and 0 otherwise, and where n_{ft} and n_{ct} are the weights of the fundamentalists and chartists, respectively.

The realized market rate in period $t + 1$ equals the market forecast made at time t plus some white-noise error (i.e. the news that could not be predicted at time t).

$$\Delta s_{t+1} = -1(|s_t - s_t^*| > C) n_{ft} \theta |s_t - s_t^*| (s_t - s_t^*) + n_{ct} \beta \sum_{i=1}^{T} a_i \Delta s_{t-i} + \varepsilon_{t+1}. \tag{5}$$

In the following we will assume that the weights n_{ft} and n_{ct} are constant. We set them equal to 0.5, not because we think this is realistic but to see how far the simplest possible model goes in explaining the exchange rate dynamics. In our future research we will allow the weights given to fundamentalists and chartists to react endogenously to the profitability of these forecasting rules.

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Note that in our model fundamentalists and chartists do not have rational expectations. Our approach contrasts with a recent paper by Bacchetta and van Wincoop (2003) who develop a model like ours, with two types of agents, i.e. one type that uses information about fundamentals and another type that uses information about nonfundamentals. Both types of agents have rational expectations. The implication of rational expectations in models with heterogeneous agents is that it creates "infinite regress," i.e. the exchange rate depends on the expectations of other agents' expectations, which depends on the expectations of the expectations of other agents' expectations, and so on, *ad infinitum*. This leads to intractable mathematical problems except under very restrictive simplifying assumptions. Although this approach is intellectually satisfying, it is unclear that it is a good representation of what agents do in the exchange market. It requires these agents to solve a mathematical problem to which mathematicians have as yet been unable to give a general solution. This seems to us as imposing too large an informational burden on individual agents. Our approach contrasts with this rational-expectations approach in that agents use simple rules, the "fitness" of which is then controlled *ex post* by checking their profitability.[8]

3. The Model in the Absence of News

The nonlinear structure of our model does not allow us to derive analytic solutions. Therefore we provide results with simulation techniques using plausible values of the parameters. We will also analyze how sensitive the results are with respect to these parameter values.

In this section we look at the deterministic part of the model, i.e. we eliminate all stochastic variables (news). In the next section we will introduce the stochastic variables (news).

We simulated the model represented by equations (1) to (5) using different combinations of parameters. Our main result is that this simple model is capable of generating very complex exchange rate behavior. In the Appendix we produce a table where we present the nature of the solution for different combinations of parameters. It can be seen that for some combinations we obtain a fixed-point solution, for other combinations we have periodic solutions, and others give chaotic solutions. In fact we find that the exchange rate follows a chaotic pattern for a relatively broad range of parameter values. We show some examples of chaotic dynamics in Figures 1 and 2. Figure 1 presents results when we assume transactions costs ($C = 5$) and Figure 2 shows results in the absence of transactions costs ($C = 0$). In panel (a) of Figures 1 and 2 we show the strange attractors in the phase space. In panel (b) we show the results of performing a sensitivity analysis, which consisted in increasing slightly (0.01) the size of the shocks in the initial exchange rate. Note that we have normalized the equilibrium value of the exchange rate to be equal to zero.

The strange-attractor panels in both Figures 1 and 2 show that our model has a potential of creating a chaotic structure, i.e. for certain combinations of parameters the exchange rate follows a chaotic path designed by the shape of the strange attractor. The tests of sensitivity to initial conditions confirm the intrinsic chaotic nature of the model. Note that chaos is more likely to occur for relatively large values of the extrapolation parameter (β) used by chartists. In fact, for all $\beta < 2$ the model does not produce chaotic dynamics. The intuition of this result is that when chartists are very aggressive in extrapolating past exchange rate changes, they introduce a strongly destabilizing force in the neighborhood of the equilibrium. This leads to a departure from

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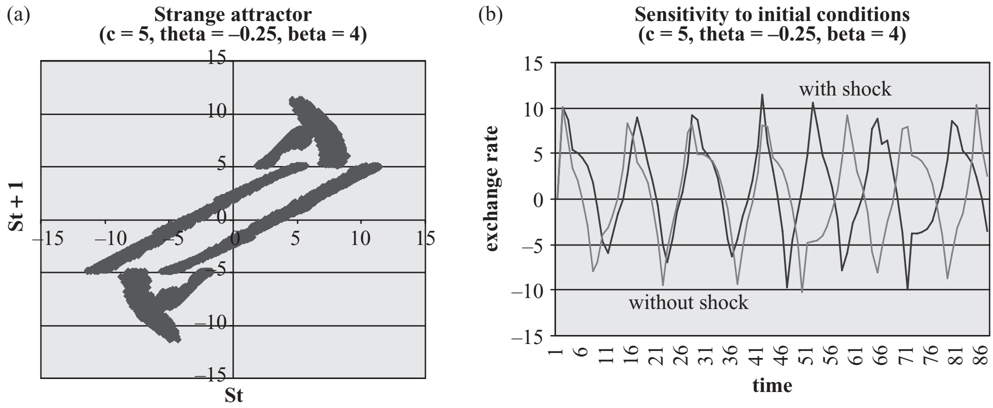

Figure 1.

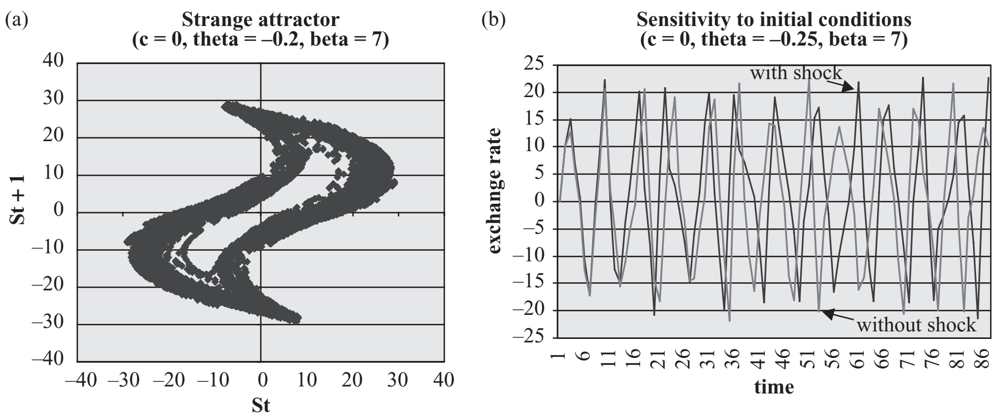

Figure 2.

equilibrium until the mean reverting force exerted by fundamentalists is strong enough to bring the exchange rate back to its fundamental value. When β is large, these two opposing forces are strong enough to create a chaotic dynamics.

Although there is little empirical evidence about the size of the extrapolation parameter β, values exceeding 2 are unlikely to be observed in the forecasting rules used by chartists. A value of β exceeding 2 implies that chartists systematically believe that a change of the exchange rate in the past of x% will be followed by a future change of $2 \times x$%, which is quite implausible. In a later section, when we calibrate the model, we will show in an indirect way that such large extrapolation parameters are unrealistic. It will turn out that the model does not produce a realistic exchange rate dynamics when β is large.

4. The Model with Random Shocks (News)

In this section we investigate the solution of the model when random shocks in the equilibrium exchange rate occur. We will not restrict the analysis to the cases where the deterministic part of the model produces a chaotic dynamics. In fact all the results

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we obtain do not rely on the existence of deterministic chaos. Thus, our results have a general character.

The question we analyze in this section is how well our model mimics the empirical anomalies and puzzles identified in the introduction. We start with the "exchange rate determination puzzle" and we analyze how the market exchange rate behaves relative to the fundamental exchange rate in our model.

The Exchange Rate Determination Puzzle

In Figure 3 we show the two variables, for a combination of parameters that does not produce deterministic chaos. (Our results hold equally well for a large set of parameter values including those that produce deterministic chaos. In the next subsection we analyze the sensitivity of the exchange rate determination puzzle with respect to transactions costs.)

We observe that the market rate can deviate from the fundamental value substantially and in a persistent way. Moreover, it appears that the exchange rate movements are often disconnected from the movements of the underlying fundamental. In fact, they often move in opposite directions.

We show the nature of the misalignment phenomenon by applying a cointegration analysis to the exchange rate and its fundamental using the same parameter values as in Figure 3 (i.e. $C = 5$, $\beta = 1.2$, $\theta = -0.2$) for a simulated sample of 8000 periods. We found that there is a cointegration relationship between the exchange rate and its fundamental.[9] Note that in our setting there is only one fundamental variable. This implies that no bias from omitted variables can occur.

In the next step we specify an EC model in the following way:

$$\Delta s_t = \mu(s_{t-1} - \gamma s^*_{t-1}) + \lambda_1 \Delta s_{t-1} + \lambda_2 \Delta s_{t-2} + \varphi_1 \Delta s^*_{t-1} + \varphi_2 \Delta s^*_{t-2}. \tag{6}$$

The first term on the right-hand side is the error-correction term. The result of estimating this equation is presented in Table 1.

We find that the error-correction coefficient (μ) is very low. This suggests that the mean reversion towards the equilibrium exchange rate takes a very long time. In

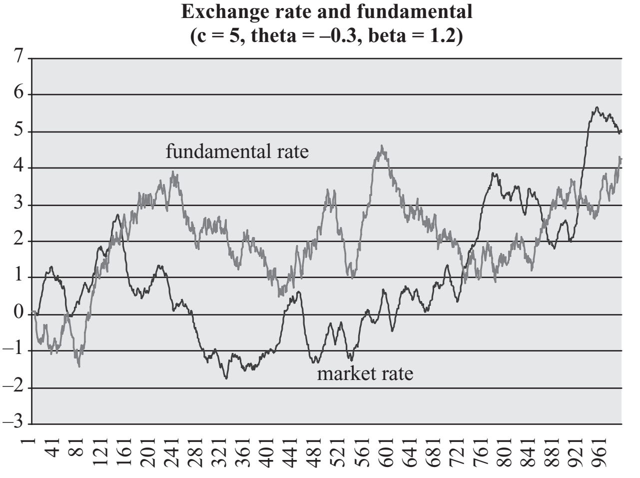

Figure 3.

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Table 1. Parameter Estimates of VEC Model (Equation (6))

Error-correction term		Δs_{t-i}		Δs^*_{t-i}		
μ	γ	λ_1	λ_2	φ_1	φ_2	R^2
–0.004 (–5.2)	1.17 (5.78)	0.26 (24.25)	0.19 (16.7)	0.04 (2.36)	0.02 (1.21)	0.14

Note: the sample consists of 8000 periods.

particular, less than half of 1% of the adjustment takes place each period. It should be noted that in the simulations we have assumed a speed of adjustment in the goods market equal to –0.2. This implies that during each period the adjustment in the goods market is 20%. Thus the nominal exchange rate is considerably slower to adjust towards its equilibrium than what is implied by the speed of adjustment in the goods market. This slow adjustment of the nominal exchange rate is due the chartist extrapolation behavior; we will come back to this issue. From Table 1, we also note that the changes in fundamentals have a small impact on the change in exchange rate.[10] In contrast, the past changes in the exchange rate play a significant role.

We also performed a cointegration analysis for shorter sample periods (1000 periods). We find that in some sample periods the exchange rate and its fundamental are cointegrated; in other sample periods we do not find cointegration. This is in line with the empirical evidence indicating that in some periods the exchange rate seems to be disconnected from its fundamentals while in other periods it tightly follows the fundamentals.[11]

Thus, the model is able to generate an empirical regularity (the "exchange rate determination" puzzle) that has also been observed in reality. We can summarize the features of this puzzle as follows. First, over the very long run the exchange rate and its fundamentals are cointegrated. However, the speed with which the exchange rate reverts to its equilibrium value is very slow. Secondly, in the short run the exchange rate and its fundamentals are "disconnected," i.e. they do not appear to be cointegrated. Our model closely mimics these empirical regularities.

Closely related to the "exchange rate determination" puzzle is the empirical evidence provided by Meese and Rogoff (1983) in their celebrated empirical tests of exchange rate models. The Meese–Rogoff analysis implies that the existing exchange rate models do not forecast the frequent structural breaks in the link between the exchange rate and the fundamentals. We show that our model is capable of reproducing the Meese–Rogoff result. We implemented this idea by introducing a small change in the speed of adjustment in the goods market, θ, i.e. this parameter was changed from –0.2 to –0.21. All the other parameters were kept unchanged. We then simulated the exchange rate with the small error in θ, and compared this with the exchange rate obtained with the "true" θ (–0.2). We show the result in Figure 4. These simulations can be interpreted as follows. The simulation using $\theta = -0.2$ generates the exchange rate produced by the true model. The second simulation can be considered to be the exchange rate, which is produced by a researcher who has estimated the model and made a small measurement error. This exchange rate can also be interpreted as the exchange rate forecast using the estimated model containing a small measurement error. We observe that the small error leads after some time to a very different

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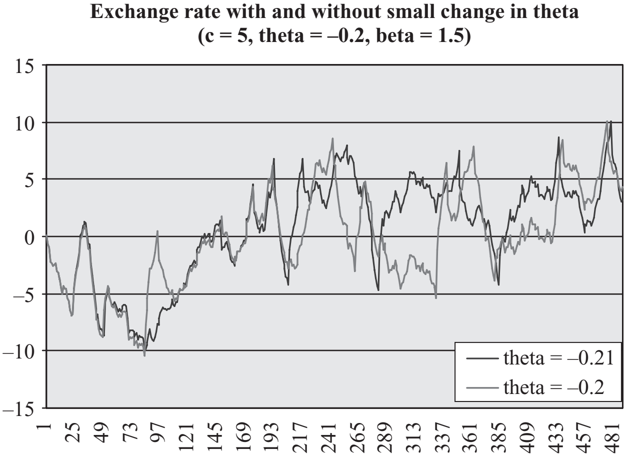

Figure 4.

Table 2. Measures of Forecast Errors Using an "Econometric" Model and the Random Walk (One-period-ahead forecasts)

	Root mean squared errors		*Mean absolute deviation*	
	Model	*Random walk*	*Model*	*Random walk*
Sample 1–100	3.20 (21.57)	0.78	1.57 (29.06)	0.75
Sample 101–200	2.31 (15.57)	0.81	1.29 (19.68)	0.77
Sample 201–300	2.78 (22.66)	0.80	1.50 (30.15)	0.76

Note: the numbers in parentheses are the *t*-statistics testing for the differences of the means.

time-path of the exchange rate, producing the appearance of large structural breaks. This result is related to the "sensitivity to initial conditions" of nonlinear models. It should be noted, however, that we obtain this result even though we have a parameter combination that does not produce chaos in the deterministic part of the model.

In the next step we computed the one-period-ahead forecast error made by this researcher who uses her almost perfect estimate of the true model, and we compared this forecast error with the forecast error using the random walk. As measures of forecast errors, we computed both the mean-root-squared error and the mean of the absolute deviations. The latter is a better measure of the forecast error when the exchange rate distribution is not Gaussian but shows fat tails.[12]

We find that the random walk forecast outperforms the forecast made by a researcher who made a small measurement error estimating the "true" model.[13] We present some examples in Table 2. The remarkable aspect of these results is that such a small measurement error is capable of producing large structural breaks and poor forecasting performance. It is therefore no surprise that when real-life econometrics is

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Table 3. Measures of Forecast Errors Using an "Econometric" Model and the Random Walk: (Five-period- and 10-period-ahead forecasts)

	Root mean squared errors		*Mean absolute deviation*	
	Model	*Random walk*	*Model*	*Random walk*
Five periods				
Sample 1–100	3.20	2.38	1.26	2.03
	(8.94)		(–3.11)	
Sample 101–200	2.31	2.55	2.98	2.54
	(–2.86)		(1.73)	
Sample 201–300	2.78	2.46	1.37	1.74
	(4.03)		(–2.67)	
Ten periods				
Sample 1–100	3.20	3.49	1.26	3.04
	(–2.85)		(–6.02)	
Sample 101–200	2.31	3.66	2.98	3.73
	(–12.00)		(–2.37)	
Sample 201–300	2.78	3.63	1.37	2.53
	(–8.47)		(–5.62)	

Note: the numbers in parentheses are the *t*-statistics testing for the differences of the means.

used to estimate models, these models perform so poorly in forecasting exercises when compared to the random walk forecast.

Another interesting aspect of these results is that the poor performance of model-based forecasts is not due to the fact that there are regime switches. Economists have been tempted to interpret the Meese–Rogoff results as reflecting changes in a regime that led to parameter instability. The problem with that explanation is that one has to assume too many regime switches to explain the parameter instability. There are just not enough regime switches around. Our model shows that one can have the appearance of frequent structural changes without regime switches. These structural changes arise from the nonlinearities in the model.

We extended the analysis for *n*-period-ahead forecasts. Meese and Rogoff found that when forecasts are made over longer periods, the forecasts based on econometric models become more useful. We find the same result in our model. We computed the five-period- and 10-period-ahead forecasts using the "econometric model" and using the random walk. The results are presented in Table 3. We observe that when the forecasting horizon increases to 10 periods the model-forecasting becomes better than the random walk. It should be stressed, however, that as in Meese and Rogoff we load the dice in favor of the model forecaster by assuming that she has perfect knowledge of the fundamental variable 10 periods in the future. No such strong informational advantage is assumed when using the random walk forecast.

The Exchange Rate Determination Puzzle: Sensitivity Analysis

In this section we investigate the sensitivity of the results relating to the "exchange rate determination" puzzle to changes in the value of the *transaction costs band*. In particular, in Figure 5 we compare the movements of the market exchange rate under

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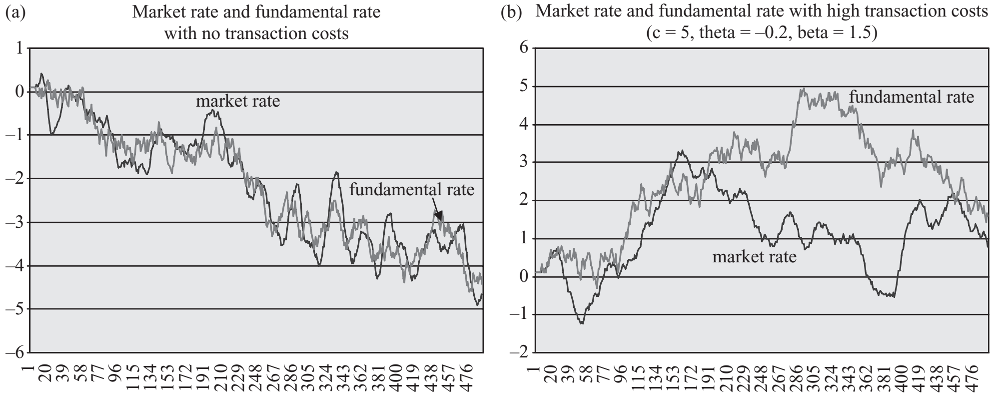

Figure 5.

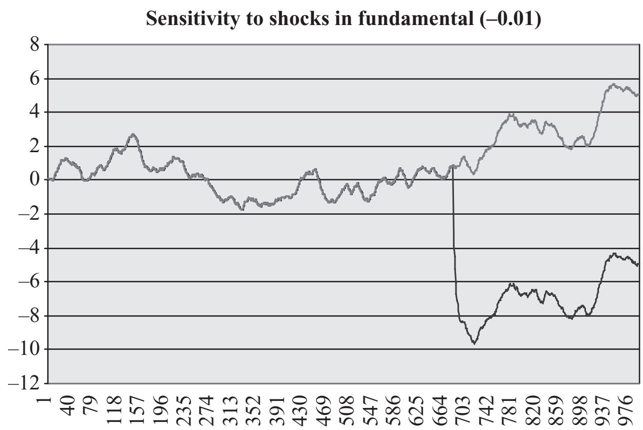

Figure 6.

two assumptions about *transaction costs*. In panel (a) we assume that transaction costs are zero and in panel (b) we assume them to be high, i.e. equal to 5.

The contrast between the two panels is striking. When transaction costs are zero, the market exchange rate does not deviate substantially for a long period of time from its fundamental value. In contrast, when transaction costs are present the deviations of the market exchange rate from its equilibrium value are large and persistent. Thus, transactions costs in the goods markets are important in explaining the exchange rate determination puzzle (see Obstfeld and Rogoff, 2000).

Transaction costs have also other important implications for the dynamics of the exchange rate. We show this in Figure 6, where we introduce a negative and permanent shock (–0.01) in the fundamental exchange rate. Thus, over time the “new” fundamental exchange rate progressively but slowly departs from the “old” one.

Figure 6 allows us to see how this accumulating small change in the equilibrium value of the exchange rate, which occurs in period 1, leads to a large jump in the exchange rate many periods later. This change has the appearance of a structural break in spite of the fact that the change in the fundamental exchange rate is very small and

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continuous. This feature is much related to the existence of transaction costs, which implies that the effect of the accumulated changes in the fundamental exchange rate will be visible only when it overcomes the transaction costs band.

"Excess Volatility"

The model discussed in the previous sections is driven by exogenous news in the fundamentals and by the noise produced by the nonlinear speculative dynamics embedded in the model. As a result, the nonlinear dynamics is capable of producing "excess volatility" in the exchange rate, i.e. volatility that exceeds the volatility of the underlying fundamental. In this section we analyze the sources of this excess volatility. We do this by computing the noise-to-signal ratio in the simulated exchange rate. We derive this noise-to-signal ratio as follows:

$$\mathrm{var}(s) = \mathrm{var}(f) + \mathrm{var}(n), \tag{7}$$

where var(s) is the variance of the simulated exchange rate, var(f) is the variance of the fundamental, and var(n) is the residual variance (noise) produced by the nonlinear speculative dynamics which is uncorrelated with var(f). Rewriting (7) we obtain

$$\frac{\mathrm{var}(n)}{\mathrm{var}(f)} = \frac{\mathrm{var}(s)}{\mathrm{var}(f)} - 1. \tag{8}$$

The ratio var(n)/var(f) can be interpreted as the noise-to-signal ratio. It gives a measure of how large the noise produced by the nonlinear dynamics is with respect to the exogenous volatility of the fundamental exchange rate. We simulate this noise-to-signal ratio for different values of the parameters of the model. We show the results in Figures 7–9. We find that the noise-to-signal ratio is very sensitive to the chartists' extrapolation parameter and to transactions costs. When chartists extrapolate more, the noise-to-signal ratio increases. This implies that when the chartists increase the degree to which they extrapolate past exchange rate changes the volatility of the

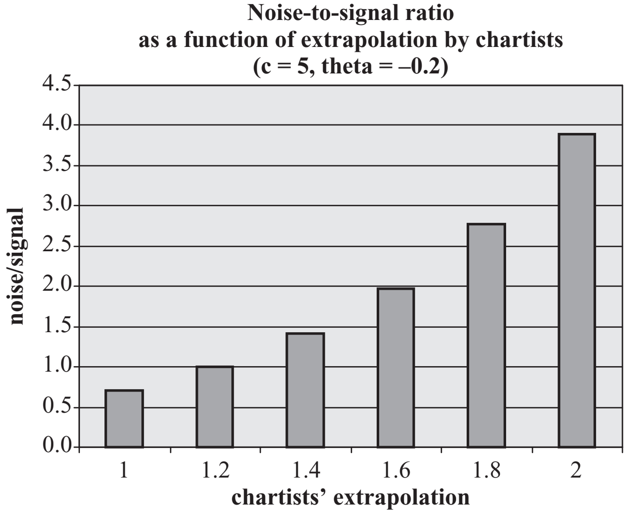

Figure 7.

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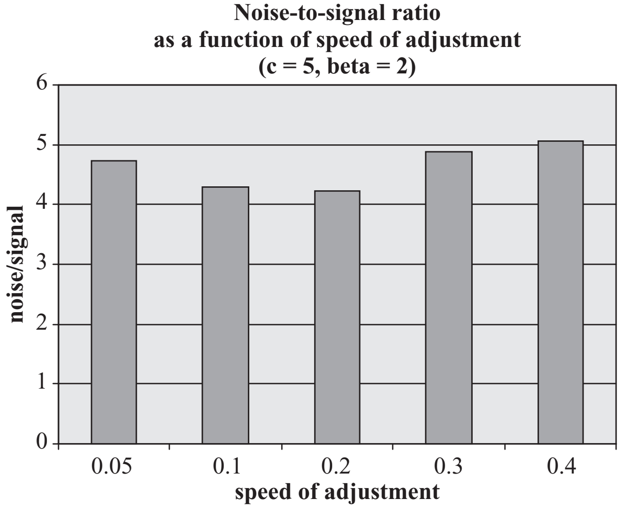

Figure 8.

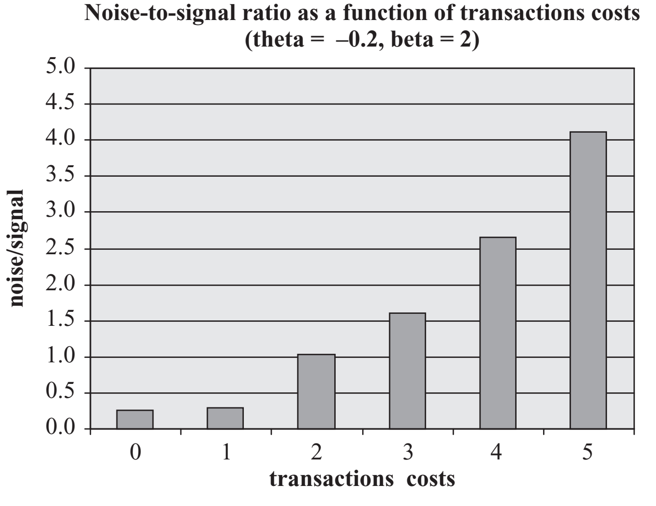

Figure 9.

exchange rate, which is unrelated to the fundamental volatility, increases. Thus the signal about the fundamental that one can extract from the exchange rate movements becomes more clouded when chartists extrapolate more. A similar conclusion holds with respect to the influence of the transactions costs on the noise-to-signal ratio. With increasing transactions costs the signal that one can extract about the fundamental becomes more clouded. Finally, an increase in the parameter θ, which measures the speed of adjustment in the goods market, has little effect on the noise-to-signal ratio.

Fat Tails

It is well known that the exchange rate changes do not follow a normal distribution. Instead it has been observed that the distribution of exchange rate changes has more

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Table 4. Kurtosis Index

Kurtosis of simulated exchange rate returns					
	Beta				
Theta	*–0.05*	*–0.1*	*–0.2*	*–0.3*	*–0.4*
0.5	3.14	3.88	9.46	21.62	31.50
1	3.10	3.79	8.20	16.06	17.92
1.5	3.01	3.24	6.22	9.63	8.38
2	2.95	2.87	3.84	5.25	6.08
2.5	1.65	2.45	2.62	3.85	4.98
3	1.38	1.57	2.24	3.03	4.77

Table 5. Measure of Fat Tails (the Hill index)

		Median Hill index (5 samples, 2000 observations)		
Parameter values	*Kurtosis*	*2.5% tail*	*5% tail*	*10% tail*
$C = 5$, beta = 0.5, theta = –0.2	8.48	2.42 (2.10–2.61)	2.66 (2.50–3.15)	2.93 (2.89–3.83)
$C = 5$, beta = 1, theta = –0.2	8.1	2.14 (2.08–2.37)	2.35 (2.21–2.45)	2.65 (2.57–2.73)
$C = 5$, beta = 1, theta = –0.3	15.17	1.54 (1.34–3.89)	1.62 (1.46–2.00)	2.14 (1.90–2.55)
$C = 5$, beta = 1.5, theta = –0.3	9.27	7.36 (5.57–7.67)	1.82 (1.58–3.13)	1.70 (1.62–1.80)

density around the mean than the normal and exhibits fatter tails than the normal (see de Vries, 2000). Put differently, the exchange rate returns typically have a kurtosis exceeding 3 and a measure of fat tails (Hill index) ranging typically between 2 and 5. It implies that most of the time the exchange rate movements are relatively small but that occasionally periods of turbulence occur with relatively large exchange rate changes. However, it has been also detected that the kurtosis is reduced under time aggregation. This phenomenon has been observed for most exchange rates. We checked whether this is also the case with the simulated exchange rate changes in our model.

The model was simulated using normally distributed random disturbances (with mean = 0 and standard deviation = 1). We computed the kurtosis and the Hill index of the simulated exchange rate returns. We computed the Hill index for five different samples of 2000 observations. In addition, we considered three different cut-off points of the tails (2.5%, 5%, 10%). We show the results of the kurtosis in Table 4 and of the Hill index in Table 5. We find that for a broad range of parameter values the kurtosis exceeds 3 and the Hill index indicates the presence of fat tails. Finally, we check if the kurtosis of our simulated exchange rate returns declines under time aggregation. In order to do so we chose different time aggregation periods and we computed the kurtosis of the time-aggregated exchange rate returns. We found that the kurtosis declines under time aggregation. In Table 6 we show the results for some sets of parameter values.

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Table 6. Kurtosis under Time-Aggregation

Parameter values		*1-period returns*	*10-period returns*	*25-period returns*	*50-period returns*
$C = 5$, theta = –0.2, beta = 1	skewness	0.05	0.03	–0.20	–0.20
	kurtosis	8.18	3.17	2.87	2.48
$C = 5$, theta = –0.2, beta = 1.5	skewness	0.07	–0.02	0.09	–0.27
	kurtosis	6.14	3.03	2.50	2.56
$C = 5$, theta = –0.1, beta = 1.9	skewness	–0.02	–0.02	–0.03	–0.02
	kurtosis	1.97	2.80	2.32	2.51
$C = 5$, theta = –0.3, beta = 1	skewness	0.13	–0.05	–0.07	–0.04
	kurtosis	15.16	3.40	2.80	2.68
$C = 5$, theta = –0.1, beta = 1.5	skewness	–0.08	–0.06	0.01	–0.23
	kurtosis	9.35	2.72	2.72	2.85
$C = 5$, theta = –0.1, beta = 1.9	skewness	–0.04	0.00	–0.01	0.00
	kurtosis	5.79	2.28	2.75	2.51

Another empirical regularity of the distribution of the exchange rate returns pertains to the measure of skewness, i.e. the degree of asymmetry of a distribution around its mean.[14] It has been observed that most exchange rates present a (very) small degree of (positive or negative) skewness (see Lux, 1998). Our simulated exchange rate returns also mirror this feature. The results are shown in Table 6. The previous results suggest that the nonlinear dynamics of the model transforms normally distributed noise in the exchange rate into exchange rate movements with tails that are significantly fatter than the normal distribution and with more density around the mean. Thus our model mimics an important empirical regularity, i.e. that exchange rate movements are characterized by tranquil periods (occurring most of the time) and turbulent periods (occurring infrequently).

Speeds of Adjustment of Nominal Exchange Rates and of Goods Prices

One of the most perplexing empirical discoveries made recently by Engel and Morley (2001) and by Cheung et al. (2002) is that the PPP puzzle (i.e. the slow rate of convergence towards PPP) is mostly due to a slow rate of convergence of the nominal exchange rate and not to a slow rate of convergence of prices in the goods market. This seems to counter another empirical observation, i.e. that the nominal exchange rate is much more volatile than goods prices. Our model is capable of mimicking this result and to reconcile these two apparently conflicting empirical results.[15] The crucial ingredient in the explanation is the role played by the chartists in the model. We show that as the weight of the chartists increases, the rate of convergence of the nominal exchange rate declines despite the fact that the short-term volatility increases. In order to show this, we first estimated error-correction models on the simulated exchange rates assuming different values of the weight of the chartists in the market.[16] The error-correction coefficient is a measure of the speed with which the nominal exchange rate returns to equilibrium. We present the error-correction coefficients in Table 7. These coefficients turned out to be highly significant. Table 7 shows that as the weight of the chartists increases, the error-correction coefficients decline significantly. At the same time the volatility of the exchange rate increases when the weight of the chartists increases.

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Table 7. Rate of Convergence, Volatility, and Weight of Chartists

Weight of chartists	*0.1*	*0.3*	*0.5*	*0.7*	*0.9*
Error-correction coefficient	–0.038	–0.031	–0.024	–0.019	–0.013
	(–12.93)	(–12.29)	(–12.32)	(–13.75)	(–19.30)
Variance exchange rate	2.01	2.04	2.08	2.18	2.72

Note: numbers in parentheses are *t*-statistics.

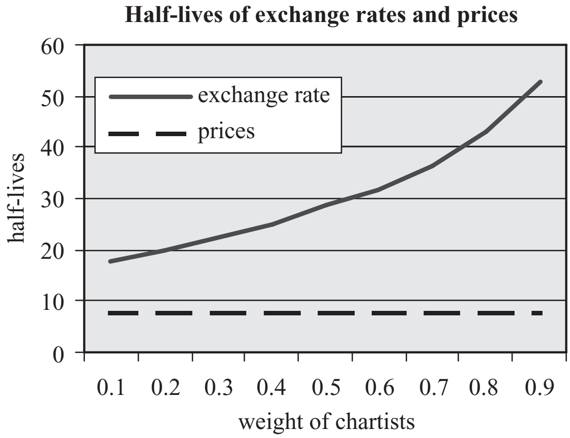

Figure 10.
Note: the vertical line represents the number of periods in our model. These can conveniently be thought of as months.

The next step in the analysis was to use these error-correction coefficients to compute the half-lives of the nominal exchange rate. We present the results in Figure 11. We find that the half-lives of the nominal exchange rate increase significantly with the weight of the chartists. In other words, when the chartists become more important in the market, it takes the nominal exchange rate more time to return to its equilibrium value. We also show the half-life of goods prices in Figure 10. This was obtained by assuming that there are no chartists in the market. In that case, the speed of adjustment of the exchange rate is determined uniquely by the fundamentalists. The latter use the speed of adjustment of goods prices to make their forecasts. Thus, in this case the speed of adjustment of the nominal exchange rate is determined exclusively by the speed of adjustment in the goods market. We observe that the half-life of goods prices is lower than the half-lives of the exchange rate. Put differently, the speed of adjustment in the goods market is faster than the speed of adjustment in the nominal exchange rate. This was also observed empirically by Engel and Morley (2001) and by Cheung et al. (2002).

The intuition of this result can be better understood by analyzing the time path of the exchange rate for two different weights of the chartists. This is done in Figure 11. We see that when the chartists have a high weight the exchange rate is pushed away from the equilibrium exchange rate (shown by the horizontal line, because we assume the equilibrium exchange rate to be constant) for sustained periods of time. In con-

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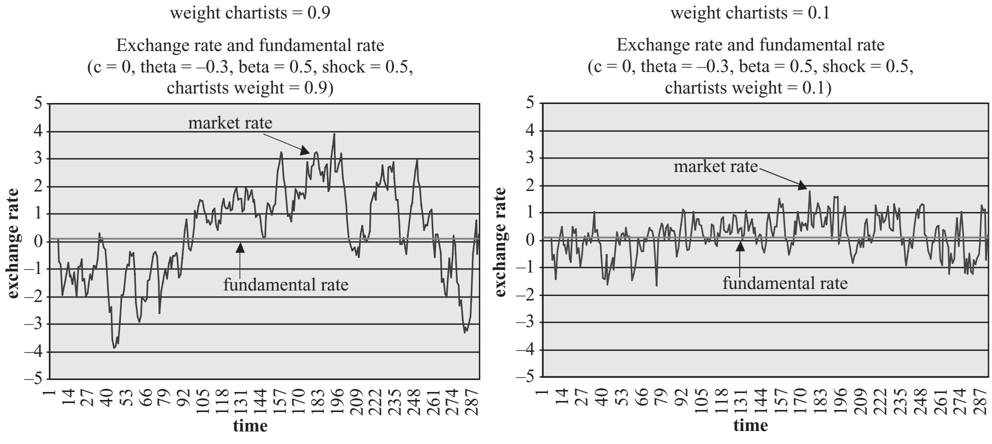

Figure 11.

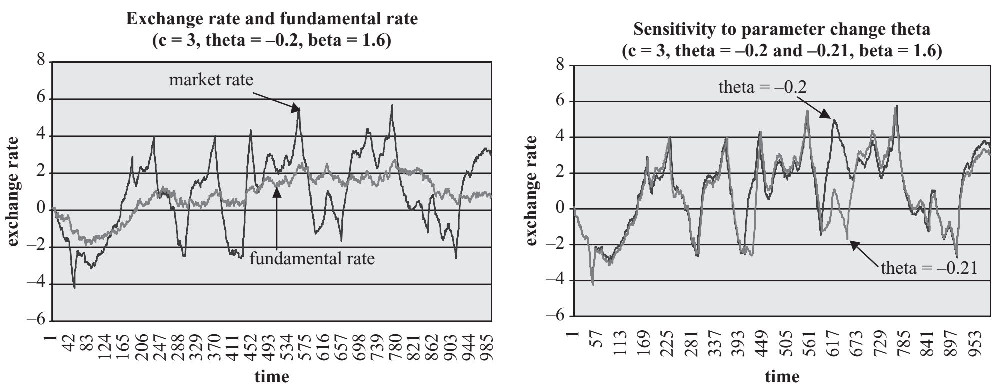

Figure 12. Low Variance of Fundamental Exchange Rate

trast, when the chartists have a low weight the exchange rate is quickly forced back to equilibrium by the action of the fundamentalists. We also observe that when the chartists have a high weight the volatility of the exchange rate is high. Thus, the presence of the chartists has a double effect: it increases the noise in the exchange rate and it also reduces the mean reversion in the exchange rate.

5. Small and Large Shocks and the Dynamics of the Exchange Rate

In linear models the size of the shocks does not affect the nature of the dynamics. In nonlinear models things are different. The size of the shocks matters. This is also the case in our exchange rate model. In order to illustrate this, we simulated the model under two different assumptions about the variance of the shocks in the fundamental exchange rate. In the first case we assume low variance of these shocks; in the second case we assume a high variance (10 times higher). The results of our simulations are presented in Figures 12 and 13. (The simulations shown here are representative for a wide range of parameter values.)

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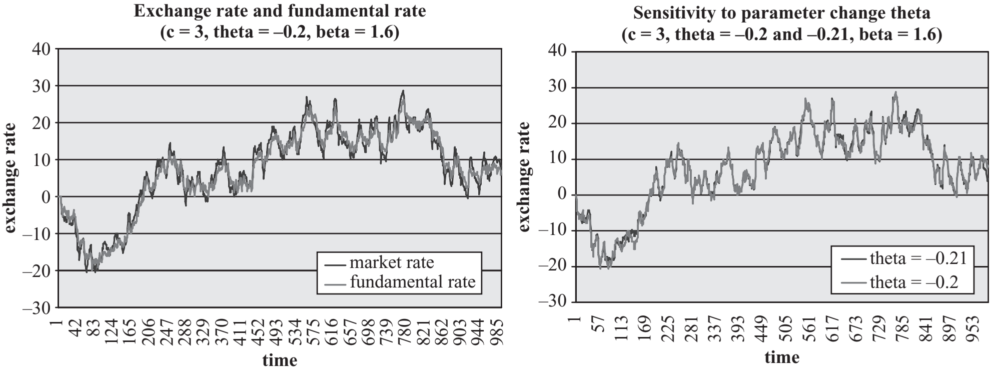

Figure 13. High Variance of Fundamental Exchange Rate

Table 8. Parameter Estimates VEC Model with High Variance of Shocks (equation (11))

Error-correction term		Δs_{t-i}		Δs^*_{t-i}		
μ	γ	λ_1	λ_2	φ_1	φ_2	R^2
–0.15 (–26.6)	1.00 (209.68)	0.16 (14.29)	0.13 (11.5)	0.17 (10.51)	0.06 (4.25)	0.15

Note: sample 8000 periods.

Two conclusions follow from a comparison of the low and high variance cases. First, in the low variance case we observe sustained deviations from the fundamental exchange rate; this is not the case when the fundamental exchange rate is subject to large shocks (compare the left-hand panels of Figures 12 and 13). Secondly, the sensitivity to small changes in parameters is clearly visible when the variance of the exchange rate is low (see the right-hand panel of Figure 12). When this variance is high, no such sensitivity can be observed (see the right-hand panel of Figure 13). It is important to stress that the transactions cost band is the same in both cases. Thus, when the shocks are small relative to the given band of transactions costs the movements of the exchange rate show more complexity than when the shocks are large.

The previous results are confirmed by a cointegration analysis like the one we performed in section 4 (see Table 1) where this analysis refers to a low variance environment. We show the results for the high variance regime in Table 8. These results contrast with those obtained in Table 1. First, the error-correction coefficient is much larger in the high variance regime of Table 8 than in the low variance regime of Table 1. In particular, 15% of the adjustment is realized each period in the high variance regime in contrast with only 0.4% in the low variance case. Secondly, the impact of past changes in fundamentals is significantly higher in the high variance than in the low variance case.

As in the low variance case we also performed a cointegration analysis over shorter sample periods. The results contrast with the low variance case. For sample periods of

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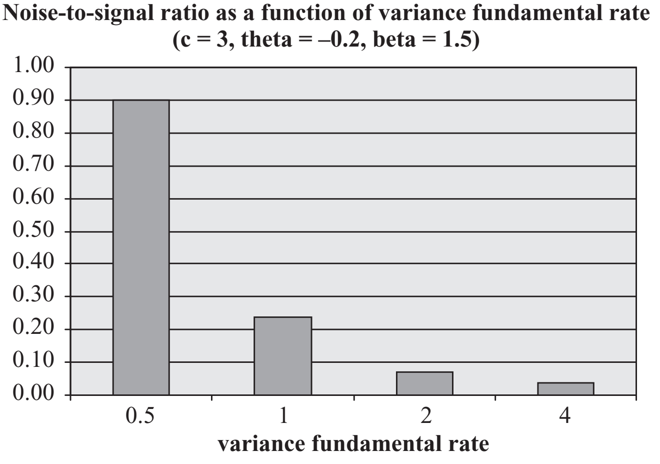

Figure 14.

1000 we find that the exchange rate and its fundamentals are cointegrated, while we do not find cointegration in the low variance case.

These results confirm what we observed from Figures 12 and 13, i.e. that in a regime of high variance of shocks the exchange rate is linked much tighter to the fundamentals, and that the speed of adjustment towards the fundamental is significantly higher than in low variance regimes.

The differences between a low and high variance environment also become evident from a comparison of the noise-to-signal ratio for different variances of the fundamental exchange rate. We show this in Figure 14. When the variance of the fundamental exchange rate is low, a large part of the volatility of the exchange rate is produced by the noise from the nonlinear dynamics. For high variance the noise is very small, implying that the exchange rate follows the fundamental rate very closely.[17]

The intuition of this result is that when the fundamental shocks are small the exchange rate regularly switches from the dynamics inherent in the band to the one prevalent outside the band. This nonlinearity produces a lot of noise and complexity in the dynamics of the exchange rate. When the shocks are large relative to the transactions cost band the dynamics outside the band mostly prevails, leading to a tighter link between the exchange rate and the fundamental. This feature has also been found to hold empirically. In particular, it has been found that the PPP relationship holds much tighter in high inflation countries than in low inflation countries (see De Grauwe and Grimaldi, 2001). Put differently, in high inflation countries the link between the exchange rate and one of its most important fundamentals is tighter than in low inflation countries. It may be argued that this is not yet evidence for the model because the latter predicts that it is a high *variance* in the fundamental that produces the tight link. However, there is also strong empirical evidence that the variance of inflation increases with its level (see Okun, 1971; Fischer, 1982). This strong correlation between the level and the variance of inflation is illustrated in Figure 15, which shows the average rate of inflation and its standard deviation in a cross-section of 81 countries during 1971–99.

6. Is Chartism Profitable?

In this section we analyze how profitable forecasting based on chartism is in relation with fundamentalism. This analysis is important because particular forecasting rules

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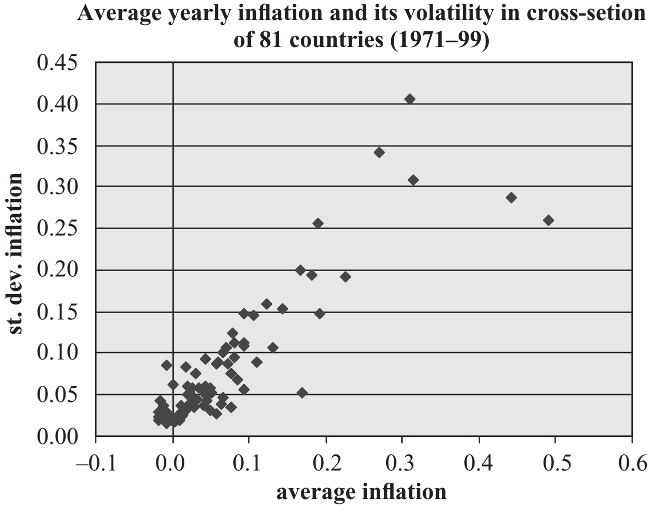

Figure 15.

will only survive if they are profitable. If chartism turns out to be unprofitable, fewer and fewer agents will use this technique, and it will disappear. In that case chartists should have no role in our model.

In order to analyze this issue we simulated the model and asked the question how the profit and loss accounts of chartists and fundamentalists evolve over time. We assumed that each of them started with an initial capital of €1. When they expect the exchange rate to increase (decrease) they buy (sell), and hold for one period. They repeat this operation each period.

We calculated the net present value of these profits and losses using a discount rate of 4%. Results are shown in Figure 16 where the present values of profits and losses are related to different values of beta.

We observe the following. First, the cumulative profits of both chartists and fundamentalists are positive.[18] Secondly, for values of beta lower than 4 the chartists make profits higher than the fundamentalists. However, for high values of beta the chartists' rule loses its profitability and the fundamentalists' rule becomes much more profitable. This implies that we are unlikely to observe chartists using large extrapolation parameter values in their forecasting.

The next step was to analyze profits and losses under two different stochastic regimes. The first one has a low variance of noise (the same as in previous simulations). The second regime has a variance 10 times higher. Results are shown in Figures 17 and 18. We see that chartism becomes less profitable in a regime of high variance, while fundamentalism then becomes more profitable. It is worthwhile noting that this result is consistent with the results obtained in the previous section, where we showed that in a high variance regime the link between fundamentals and the market exchange rate is tighter than in the low variance regime. Thus, it is not surprising that in a high variance regime the fundamentalists' forecasting rule is relatively profitable. This result also implies that we should observe more chartists in the low variance currency markets than in the high variance markets. This prediction is confirmed by recent empirical evidence of Cheung et al. (2002).

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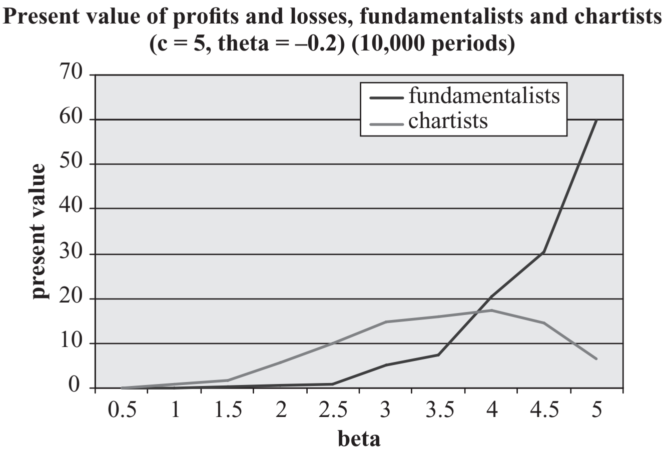

Figure 16.

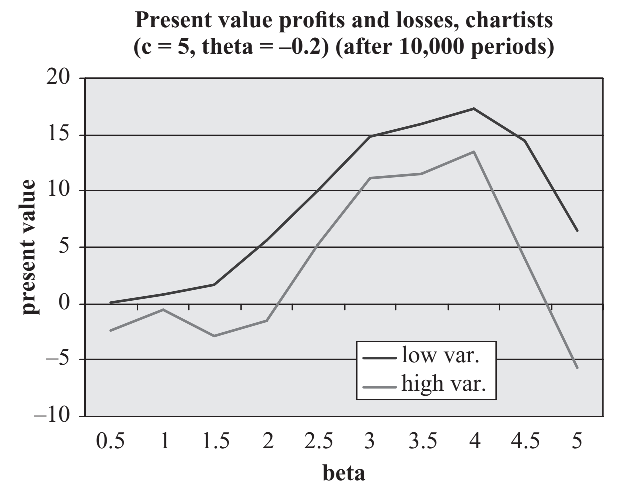

Figure 17.

7. Conclusion

In this paper we analyzed the workings of a simple nonlinear exchange rate model in which agents hold different beliefs about the underlying model. We distinguished between chartists and fundamentalists, where the chartists apply a *positive feedback rule* and the fundamentalists a *negative feedback rule*. The nonlinearities in the model originate from transactions costs and from the existence of nonlinear adjustment dynamics in the goods market.

Our main results can be summarized as follows. First, the simple nonlinear structure of the model is capable of generating a very complex exchange rate dynamics. We found that for some parameter values this complex dynamics can be chaotic. This implies that small shocks in the equilibrium exchange rate lead to very different time-paths of the exchange rate.

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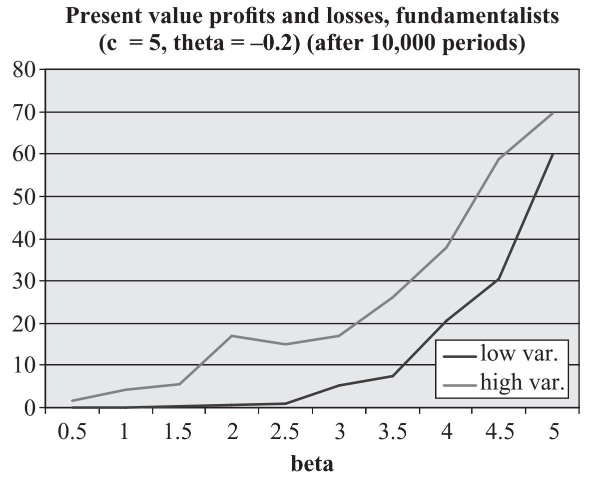

Figure 18.

Secondly, our model is capable of explaining most empirical puzzles associated with exchange rate movements. The first puzzle is that the market exchange rate can deviate substantially and for relatively long periods of time from its fundamental value (“exchange rate determination puzzle”). We showed that such disconnections are a natural outcome of the nonlinear dynamics in our simple model. There is no need to invoke exogenous events and special factors to explain why exchange rates deviate from their fundamental values. It should also be noted that our model generates these misalignments even in the absence of deterministic chaos. In other words, we do not need to invoke chaos to explain the misalignment between the exchange rate and its fundamentals.

Another empirical puzzle observed in exchange rate economics is the frequent occurrence of “regime shifts,” i.e. structural breaks in the relation between the exchange rate and the fundamentals. This phenomenon was first noted in the celebrated studies of Meese and Rogoff (1983). It is now customary to explain these structural breaks by changes in the policy regime. Our model provides an alternative explanation. The nonlinear dynamics embedded in the model produces endogenous regime shifts that change the link between the exchange rate and its fundamentals. These structural breaks can be triggered by very small changes in parameters or by small errors in the estimates of these parameters by agents who forecast the future exchange rate. Thus, in a nonlinear world, structural breaks in the link between the exchange rate and its fundamentals occur naturally even when no changes occur in the policy regime.

Thirdly, we found that our simple nonlinear dynamic model can generate “excess volatility” in the exchange rate. The size of this excess volatility crucially depends on the degree of extrapolation applied by chartists and on the size of the transactions cost band.

Fourthly, the model is also capable of generating another empirical regularity, i.e. the existence of fat tails in the distribution of the exchange rate returns. Thus the model mimics an empirical phenomenon that has been widely observed in exchange rate economics, i.e. that tranquil periods (which occur most of the time) alternate with turbulent periods (which occur infrequently).

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Fifthly, we found that the size of shocks to the underlying fundamental exchange rate matters for the dynamics of the exchange rate. More specifically, we found that when these shocks are small relative to the size of the transactions cost band, the phenomena just described will tend to be prevalent. That is, in regimes of low shocks relative to the transactions cost band, the exchange rate movements are complex, and can even be chaotic. In such a regime exchange rates deviate substantially from the underlying fundamentals, and frequent structural breaks in the link between the fundamentals and the exchange rate are observed. The latter occur in the absence of changes in the policy regime.

Sixthly, our model explains another empirical puzzle that was uncovered recently by Engel and Morley (2001) and Cheung et al. (2002). This is that the speed of adjustment of the exchange rate towards PPP is slower than the speed of adjustment of goods prices.

Finally, we checked whether the forecasting rules used by chartists and fundamentalists are profitable. We found that for a broad range of parameter values both rules are profitable.

Some implications of these findings are the following. The exchange rates of the major currencies are subject to relatively small shocks in the underlying fundamentals (e.g. inflation differentials are almost zero). Compared to these shocks the transactions costs can be said to be relatively large (see Obstfeld and Rogoff, 2000, on this), i.e. a large part of goods and services are nontraded (or difficult to trade) because the cost of shipping them across borders is quite high. Thus, the regime confronted by the exchange rates of the major industrialized countries comes close to the regime we have identified to be the one producing complexity, speculative noise, and structural breaks between exchange rates and underlying fundamentals. Put differently, the movements of the exchange rates of the industrialized countries are likely to be clouded by a nonlinear speculative dynamics that makes it difficult, if not impossible, to explain this or that movement of these exchange rates. In contrast, the exchange rates of high inflation countries experience large shocks in the fundamentals. As a result, the movements of the exchange rates of these countries can be explained much better by movements in underlying fundamentals (e.g. inflation differentials).

The results of our paper make it easier to understand why it will remain difficult, if not impossible, to find (fundamental) logic in the movements of the exchange rates of major currencies that are subject to relatively low nominal disturbances. However, our inability to understand why, say, the dollar moved up against the euro during 1999–2000 does not prevent analysts from developing exotic theories explaining these movements. Probably this has to do with the fact that the human mind abhors the emptiness created by its inability to understand. It is no surprise, therefore, that new explanations based on fundamentals are created, and will continue to be created for each and every new turn of the exchange rate.

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Appendix: Sensitivity of Dynamics to Parameter Values

	Beta				
Theta	*–0.1*	*–0.2*	*–0.3*	*–0.4*	*–0.5*
1.9	FPM	FPM	FPM	FPM	U
2	FPC	FPC	FPC	FPC	U
2.1	C	C	C	C	U
2.2	C	C	C	C	U
2.3	C	C	C	C	U
2.4	C	C	C	C	U
2.5	C	C	C	C	U
2.6	C	C	C	C	U
2.7	C	C	C	C	U
2.8	C	C	C	C	U
2.9	C	C	C	C	U
3	C	C	C	C	U
3.1	C	C	C	C	U
3.2	C	C	C	C	U
3.3	C	C	C	C	U
3.4	C	C	C	C	U
3.5	P (12)	P (12)	P (12)	P (12)	U
3.6	C	C	C	C	U
3.7	C	C	C	C	U
3.8	C	C	C	C	U
3.9	C	C	C	C	U
4	C	C	C	C	U
4.1	C	C	C	C	U
4.2	C	C	C	C	U
4.3	C	C	C	C	U
4.4	C	C	C	C	U
4.5	P (10)	P (10)	P (10)	P (10)	U
4.6	C	C	C	C	U
4.7	C	C	C	C	U
4.8	C	C	C	C	U
4.9	C	C	C	C	U
5	C	C	C	C	C
5.1	C	C	C	C	C
5.2	C	C	C	C	C
5.3	C	C	C	C	C
5.4	P (8)	P (9)	P (9)	P (9)	P (9)
5.5	P (9)	P (8)	P (8)	P (9)	P (9)
5.6	C	C	C	C	C
5.7	P (23)	P (26)	P (26)	P (26)	P (26)
5.8	P (34)	P (17)	P (17)	P (17)	P (17)
5.9	P (42)	P (42)	P (42)	P (42)	P (42)
6	P (24)	P (8)	P (12)	P (8)	P (8)
6.1	P (8)	P (8)	P (8)	P (8)	P (8)
6.2	P (18)	P (17)	P (17)	P (17)	P (17)
6.3	P (34)	P (34)	P (34)	P (34)	P (34)
6.4	C	C	C	U	C
6.5	C	C	C	U	C
6.6	C	C	C	U	C
6.7	C	C	C	U	C
6.8	C	C	C	U	C
6.9	C	C	C	U	P (16)
7.	C	C	C	U	C
7.1	U	U	U	U	U

Notes: C = chaos; P (*N*) = *N*-period cycles; U = unstable; FPM = fixed point reached monotonically; FPC = fixed point reached cyclically.

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Notes

1. It should be noted that the heterogeneity of agents' expectations has been recognized as being important to explain the dynamics of asset prices, including the exchange rate; see Frankel and Froot (1986); De Long et al. (1990); Brock and Hommes (1997); Lux and Marchesi (2000).
2. One important empirical puzzle, the "forward discount bias," will not be analyzed in this paper, not because we think this puzzle is unimportant, but because, as will be clear from the next section, our model is not structured to analyze the forward discount puzzle.
3. Introducing a drift does not change the nature of the model, nor its results. We also experimented with an AR(1) process for the fundamental rate. This did not affect our results.
4. This way of modeling the foreign exchange market was first proposed by Frankel and Froot (1988). It was further extended by De Long et al. (1990) and De Grauwe et al. (1993) and more

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recently by Kilian and Taylor (2001). For evidence about the use of chartism see Taylor and Allen (1992). See also Kurz (1994) and Kurz and Motolese (2000).
5. Note that this is also the approach taken in the Dornbusch (1976) model.
6. See Kilian and Taylor (2001) and Taylor et al. (2001); see also De Grauwe and Grimaldi (2001) in which we showed that a quadratic specification fits the data rather well.
7. See De Long et al. (1990).
8. By stressing the use of simple rules, our approach comes close to that of behavioral finance (Shleiffer, 2000).
9. We used the Johansen cointegration procedure (see Johansen, 1991). We assumed that there is no deterministic trend in the data. However, we do allow the intercept different from zero.
10. Note that this result is consistent with the empirical evidence on the effect of fundamental shocks on the exchange rate. See, for example, Eichenbaum and Evans (1995) who analyze the effects of monetary policy shocks on the exchange rate; see also Faust et al. (2003).
11. See Obstfeld and Rogoff (2000). In De Grauwe and Vansteenkiste (2001) we present additional empirical evidence.
12. This approach was also followed by Meese and Rogoff (1983).
13. Note that, as in Meese and Rogoff, the researcher using a model to make forecasts has more information available than the random walk forecaster. The former has perfect knowledge of the one-period-ahead fundamental while the latter does not.
14. Positive skewness indicates a distribution with an asymmetric tail extending toward more positive values. Negative skewness indicates a distribution with an asymmetric tail extending toward more negative values.
15. This was already hinted at in section 4.
16. See section 4 for the procedure that was followed.
17. It should be stressed that the total variability of the exchange rate in the high variance scenario is much larger than the total variability of the exchange rate in the low variance scenario. The point is that in the high variance scenario almost all of the variability of the exchange rate is explained by the (much higher) variability of the fundamental. This is not the case in the low variance scenario where a large part of the variability of the exchange rate cannot be related to the variability of the underlying fundamental.
18. We assume implicitly that the total gains (losses) of these forecasters must have a counterpart loss (gain) borne by other agents (e.g. exporters and importers) outside the model. We assume that the gains and losses of the forecasters are small in a macroeconomic sense so that spread over many agents in the rest of the economy they do not affect the latters' behavior.

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