Preliminary draft

The exchange rate and its fundamentals. A new non-linear perspective.

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1. Introduction

One of the key predictions of the first generation models of the exchange rate developed in the 1970s is that movements of the exchange rate can only occur because of news (unanticipated) movements of the underlying fundamentals driving the exchange rate. This prediction has certainly not been borne out by the empirical evidence. There is now a large body of evidence making clear that most of the movements of the exchange rates cannot be associated with observable news in fundamentals.

This (and other) empirical failures of the first generation models have led researchers into different directions. The first one, exemplified by the work of Obstfeld-Rogoff(1995), consists in going back to the basics and to develop models where agents continuously solve dynamic utility programs. This literature has led to much intellectual excitement. At the same time, however, it has produced few testable propositions that can be falsified, so that it is not clear yet whether it has led to scientific progress.

In a second approach researchers have analysed the micro-structure of the foreign exchange market (see e.g. Lyons(), Evans&Lyons()). This has led to new insights about how the aggregation process of dispersed information affects the dynamics of the exchange rate.

A third approach consists in assuming that some agents are not fully rational (noise traders, chartists) and that they interact with rational agents (XXX). This together with the introduction of transactions costs introduces non-linearities into the model. This is the approach followed in this paper.

An implication of the first generation models is that changes in fundamental variables have predictable effects on the exchange rate. In this paper we challenge this view. We will show that in the context of a non-linear model the transmission process of changes in the fundamentals is obscured by the speculative dynamics generated endogenously by the model.

2. A Simple non-linear exchange rate model

In this section we analyse how different shocks in the equilibrium exchange rate affect the dynamics of the exchange rate. In order to do so, we develop a very simple non-linear exchange rate model. The main feature of this model is that some agents ("fundamentalists") take into account that the goods markets dynamics is non-linear. We analyse whether this simple model can generate a complex (chaotic) dynamics.

2.1 The model

We assume that there are two types of agents in the foreign exchange market, chartists and fundamentalists, using different information sets[1]. The chartists forecast the future exchange rate by extrapolating the past behaviour of the exchange rate. In this sense they use a *positive feedback rule*. The fundamentalists base their forecasts of the future exchange rate on a model of the economy. More particularly, they compute the equilibrium value of the exchange rate, e.g. the PPP value.

[1] This way of modelling the foreign exchange market was first proposed by Frankel and Froot (1988). It was further extended by De Long et al. (1990) and De Grauwe et al. (1993) and more recently Kilian and Taylor (2001). For evidence about the use of chartism see Allen and Taylor (1989).

When the exchange rate deviates from its equilibrium value then they forecasts that it returns to it in the future. In this sense they use a *negative feedback rule*.

We specify these two rules in the following way.

The chartists' forecast is written as:

$$E_{t-1}^{C}(\Delta e_t) = b \sum_{i=1}^{T} a_i \Delta e_{t-i} \qquad (1)$$

where E_{t-1}^{C} is the forecast made by chartists using information up to time t-1. Δe_t is the change in exchange rate. As can be seen, the chartists compute a moving average of the past exchange rate changes and they extrapolate this into the future exchange rate change. The degree of extrapolation is given by the parameter β.

The fundamentalists' behaviour can be represented as follows:

$$E_{t-1}^{F}(\Delta e_t) = q\left|e_{t-1} - e^*\right|(e_{t-1} - e^*) \qquad (2)$$

where E_{t-1}^{F} represents the forecast made by the fundamentalists at time t-1. e* is the equilibrium exchange rate. This equilibrium value might be the PPP equilibrium value but also the equilibrium value obtained by a more complex model. We do not model this equilibrium value but we assume that it is constant and normalised to zero[2]. We assume that the fundamentalists know that the adjustment process is non-linear in nature

[2] In De Grauwe & Grimaldi(2001) we assumed that the equilibrium exchange rate follows a random walk. The results are not affected by the new assumption of this paper.

and therefore they expect that the return to the equilibrium will follow the non-linear dynamics. We use a simple quadratic specification to model their forecasts. It implies that when deviations from PPP the speed with which the adjustment occurs is correspondingly fast. The underlying logic is that to undertake arbitrage some fixed cost must borne (e.g. buying a truck, chartering an airplane). Therefore, as arbitrage profits increase more agents are willing to undertake such investments, so that the resources used in arbitrage activities increase. Recently, new empirical evidence has been produced indicating that these non-linearities in the dynamics towards PPP are important.

We assume that the market forecast is an average between the forecasts made by chartists and fundamentalists assuming equal weights,i.e. :

$$E_{t-1}(\Delta e_t) = 0.5E^{C}_{t-1}(\Delta e_t) + 0.5E^{F}_{t-1}(\Delta e_t) \qquad (3)$$

where $E_{t-1}(\Delta e_t)$ is the market expectation of the changes in the exchange rate.

It is important to note here that the non-linearity in the fundamentalists' rule ensures that the weights the fundamentalists and chartists exert in the market are in fact variable. In particular, as can be seen from equation (2), the weight of the fundamentalists' expectations increases as the market rate deviates from the equilibrium rate. Conversely, when the market rate and the equilibrium rate are close to each other the forecasts made by the fundamentalists will have little impact on the market forecasts. The latter are then dominated by what the chartists expect.

Assuming market efficiency, the change in the market exchange rate from t-1 to t is equal to the change forecasted in t-1 plus a white noise error, i.e.

$$\Delta e_t = E_{t-1}(\Delta e_t) + e_t \tag{4}$$

We introduce transactions costs as a second source of non-linearity. Recently (see e.g. Obstfeld & Rogoff(2000)) the importance of transactions costs has been stressed to understand some long-standing puzzles in international macro-economics. Therefore, we introduce transaction costs in the model represented by equations (1)-(4). If there are transaction costs the fundamentalists will take this information into account. Thus, if the exchange rate is within the transaction costs band the fundamentalists will behave differently than if the exchange rate moves out of the transaction costs band. Consider the first case, when the exchange rate deviation from its equilibrium value is larger than the transaction costs C. Then the fundamentalists follow the same forecasting rule as in equation (2). More formally, when $|e_{t-i} - e^*| > C$ holds, then equation (2) applies. In the second case the exchange rate deviations from the equilibrium value are smaller than the transaction costs. Then the fundamentalists know that arbitrage in the goods market does not apply. As a result, they expect the changes in the exchange rate to follow the white noise process ε_t. The best they can do is to forecast no change. More formally, when $|e_{t-1} - e^*| < C$, then $E^F_{t-1}(\Delta e_t) = 0$.

The solution of this model was presented in a previous paper (De Grauwe and Grimaldi (2001)). We showed that the solution exhibits complexity and for certain parameter values it leads to chaotic dynamics.

2.2 Shocks in the equilibrium exchange rate

In this section we analyse how shocks in the equilibrium exchange rate affect the dynamics of the exchange rate. We first analyse a permanent shock in the level of the equilibrium exchange rate, which we set equal to 10. The effect of this shock is to shift the strange attractor upwards. We show this in figure 1. It can be seen that as a result of the shock the new strange attractor is displaced upwards while the shape of the attractor is unchanged. However, in a non-linear dynamic environment this does not imply that the new market exchange rate will just increase by the size of the shock in the equilibrium rate after which it then follows the same fluctuations, as it would have done without the shock. Put differently, the shock in equilibrium rate has a double effect: it shifts the strange attractor and, because of the sensitivity to initial conditions, it will put the new market rate in a different starting position in the new strange attractor compared to the position it had in the old strange attractor. This makes the effect of the shock in the equilibrium rate on the market rate unpredictable.

In the next step we analyse the complexity of this effect on the market exchange rate. We measure the impact of the shock by computing the difference between the market rate we obtained with shock and the market rate we obtained without shock. We present the results in figure 2 for a set of parameters that we consider being the standard case. In the next section we present a sensitivity analysis to check to what extent the results are affected by the choice of different parameter values.

First we observe that the effect of the shock in the equilibrium rate has a highly complex nature. It appears that the shock creates noise around the new equilibrium rate. This is also visible from the strange attractor of the effect of the shock on the exchange rate. We show this strange attractor in figure 3. It can be seen that the effect of the shock on the market rate has a chaotic nature itself. This means that we cannot predict the effect of the shock in any given point in time. Nonetheless, we can make statistical predictions about this effect. However, in order to find a statistically significant effect of the shock we need a sufficiently large number of observations. This can be seen from figure 2 where the old equilibrium rate, which is equal to zero, falls more than half of the time within the complex noise range around the new equilibrium rate. Consequently, it is difficult to discern the effect of the permanent shock on the market rate. However, when the size of the shock in the equilibrium rate increases the statistical significance of the effect of the shock will be detected more easily (see Figure 4). We observe that the old equilibrium rate is outside the noise range around the new equilibrium rate most of the time. Thus we will discern the effect of the shock more easily than when the shock was small.

Figure 1

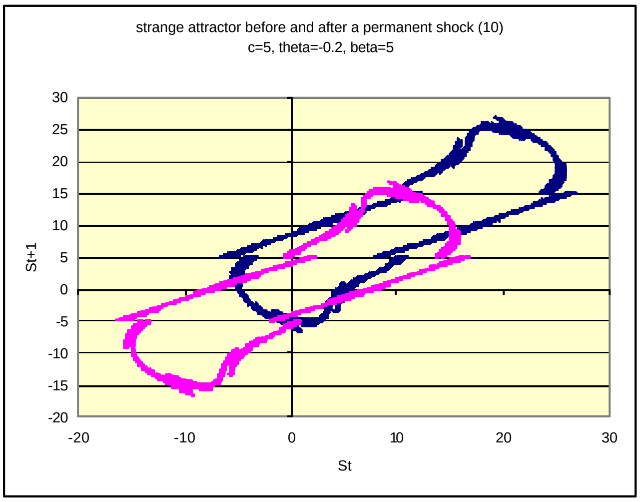


Figure 2

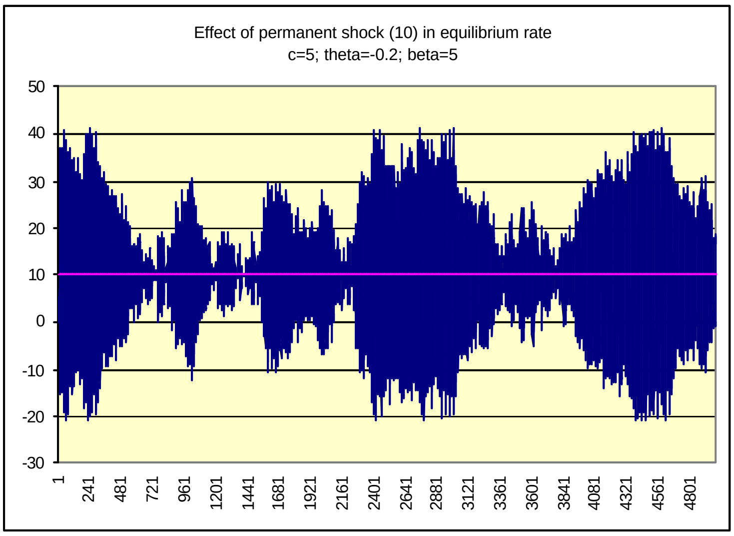


Figure 3

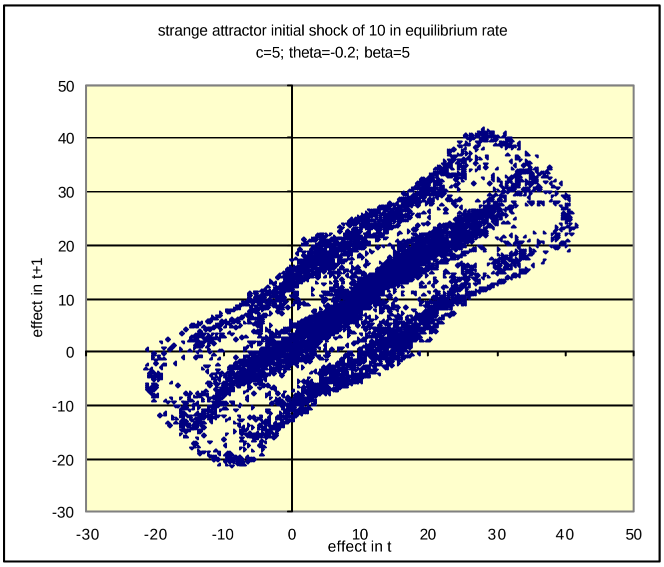
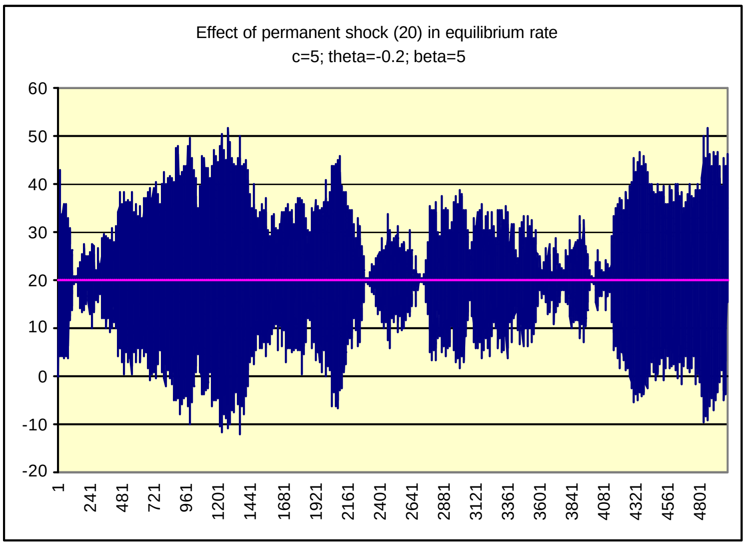


Figure 4



2.3 Sensitivity analysis

In this section we analyse how the exchange rate dynamics is affected by changing the values of the parameter in the model. We start with the transaction costs. We show that when transaction costs are very equal to zero the effect of the shock on the market rate converges to a fixed point. In this case the effect of the shock is fully predictable. However, we only obtain this result when the extrapolation parameter is sufficiently low (beta <2.3), for a given value of the speed of adjustment (theta equal to –0.05).

When transaction costs are larger than zero, we find that the effect of the shock on the market rate has a complex nature so that the effect of the shock is not predictable. This can be seen in figure 5 where we show the time pattern and the strange attractor of the effect of the shock on the market rate when transaction costs are equal to zero. We do this for increasing values of the extrapolation parameter. We observe how the complexity of the effect of the same shock increases dramatically when the chartists extrapolate more.

In figure 6 we show the results when transaction costs are positive (transaction costs equal to 5). In this case we find that whatever the extrapolation parameter by chartists, the effect of the shock in the market rate has a complex nature. For example, with a chartist's extrapolation parameter equal to one we now find that the effect of the shock jumps up and down around the new equilibrium rate in an unpredictable way. This contrasts with the case of absence of transaction costs where the effect of the shock is fully predictable. Note that the degree of complexity increases with the degree of extrapolation by chartists. Thus, for example, when the extrapolation parameter is equal to 3 we obtain an extraordinarily complex feature implying that the effect of the shock in the equilibrium rate is unpredictable.

It should be noted that the noise generated in the effect of the shock is not related to the exogenous noise represented in the model by ε_t. The reason is that the effect of the shock in the equilibrium rate is computed by subtracting the market rate without the shock from the market rate with the shock. Thereby the exogenous noise is eliminated. Thus the noise obtained in our simulations is endogenous and is produced by the non-linear dynamics.

Figure 5

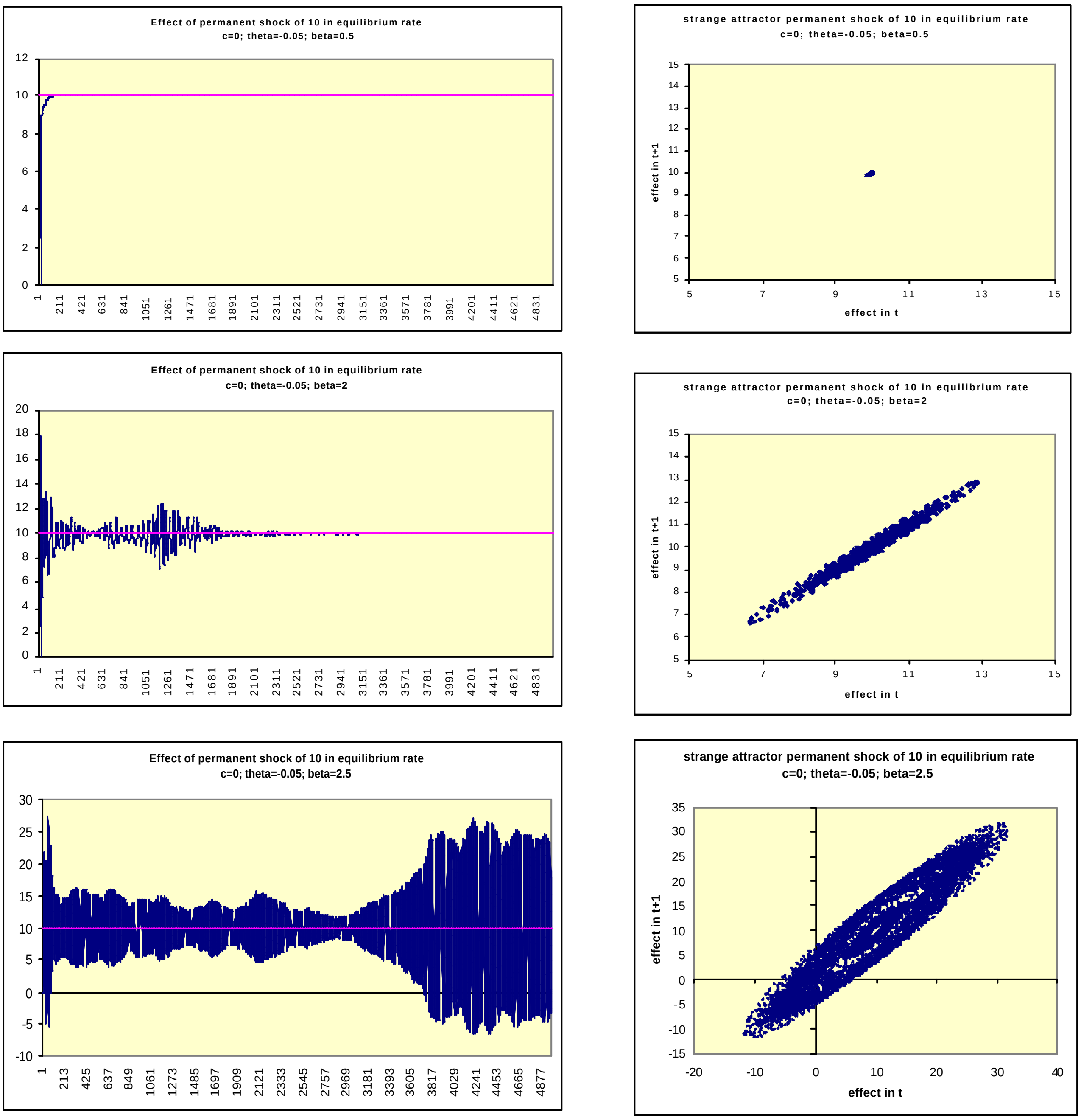
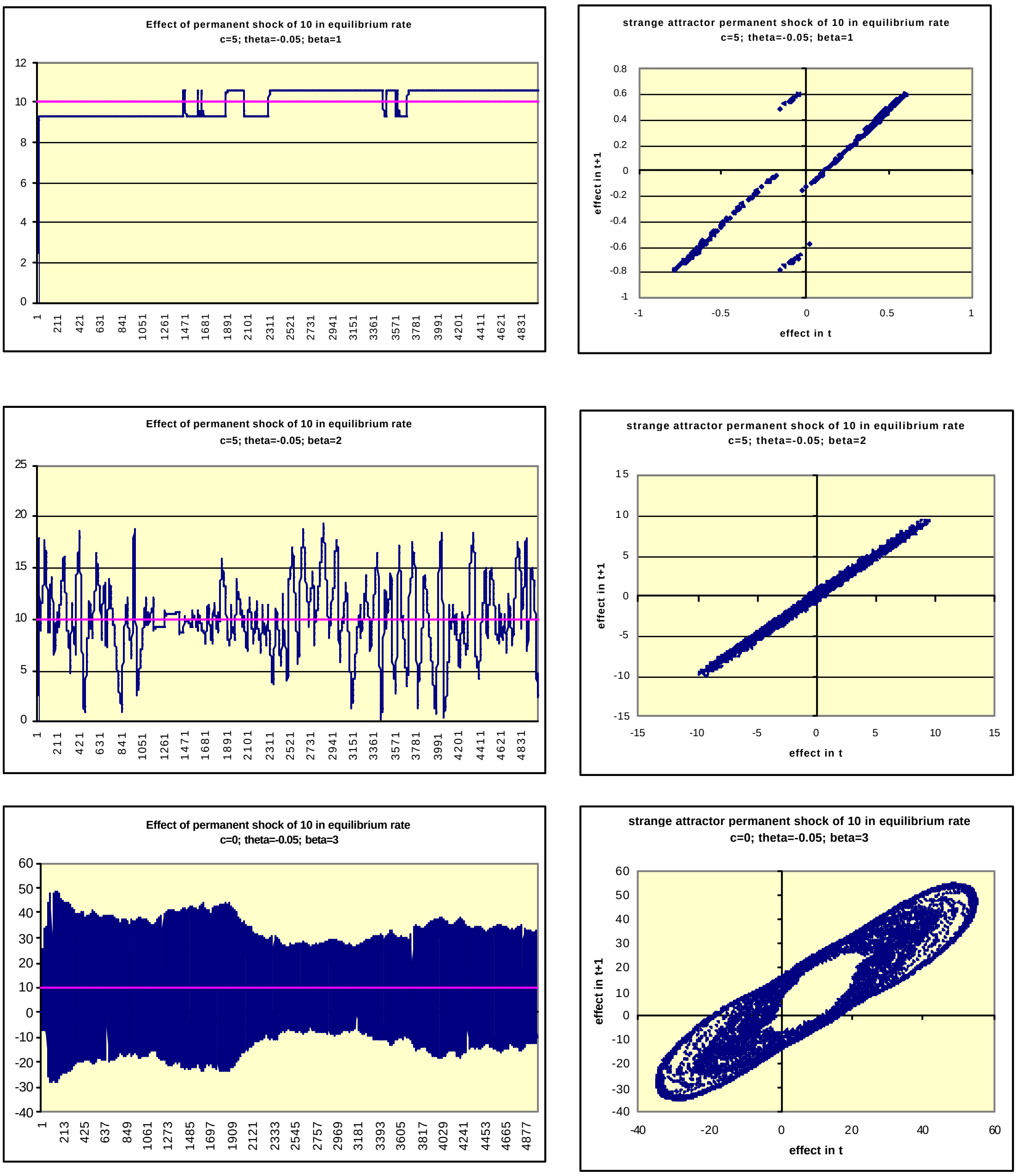

Effect of permanent shock of 10 in equilibrium rate
c=0; theta=-0.05; beta=0.5
strange attractor permanent shock of 10 in equilibrium rate
c=0; theta=-0.05; beta=0.5
effect in t+1
effect in t
Effect of permanent shock of 10 in equilibrium rate
c=0; theta=-0.05; beta=2
strange attractor permanent shock of 10 in equilibrium rate
c=0; theta=-0.05; beta=2
effect in t+1
effect in t
Effect of permanent shock of 10 in equilibrium rate
c=0; theta=-0.05; beta=2.5
strange attractor permanent shock of 10 in equilibrium rate
c=0; theta=-0.05; beta=2.5
effect in t+1
effect in t

Figure 6



Effect of permanent shock of 10 in equilibrium rate
c=5; theta=-0.05; beta=1
strange attractor permanent shock of 10 in equilibrium rate
c=5; theta=-0.05; beta=1
effect in t+1
effect in t
Effect of permanent shock of 10 in equilibrium rate
c=5; theta=-0.05; beta=2
strange attractor permanent shock of 10 in equilibrium rate
c=5; theta=-0.05; beta=2
effect in t+1
effect in t
Effect of permanent shock of 10 in equilibrium rate
c=0; theta=-0.05; beta=3
strange attractor permanent shock of 10 in equilibrium rate
c=0; theta=-0.05; beta=3
effect in t+1
effect in t

2.4 Towards more complexity

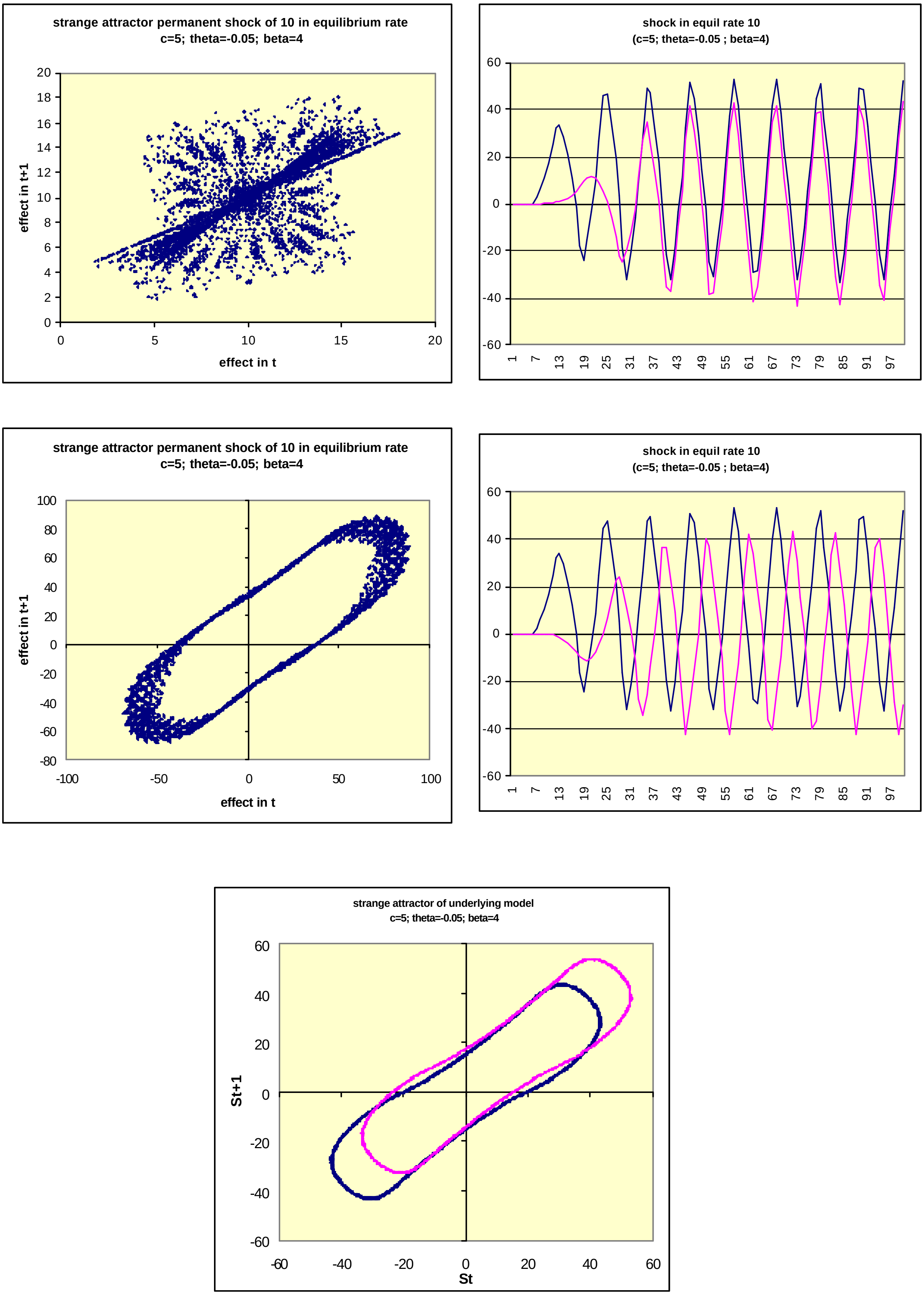
In the previous section we have shown that the complexity of the strange attractor of the effect of the shock in the equilibrium rate depends on the parameters values of the underlying model. Can we have different degrees of complexity even though we keep unchanged the parameter of the model? The surprising answer is yes. We show that the degree of complexity can change when we keep the parameters of the model unchanged but when the shock is applied with different initial conditions. However, this result is obtained only when the extrapolation parameter is sufficiently high (beta >2.8). We illustrate this result in figure 7 where we present two strange attractors of the effect of the same shock using exactly the same model but with a different initial condition. The latter is sufficient to produce dramatically different strange attractors.

How can this surprising phenomenon be explained? To answer this question we show the strange attractors of the underlying model with and without the shock at the bottom of figure 7. The effect of the positive shock in the equilibrium rate is to shift the strange attractor of the underlying model upwards. Note that that these “strange attractors” have a lower level of complexity compared to the strange attractor of the effect of the shock. In fact, they resemble closed loops indicating that the dynamics is quasi-periodic.

The different nature of the strange attractors of the effect of the shock can be explained by the different initial conditions that lead the initial exchange rates after the shock to be on different positions on the new loop. As a result, in the first simulation the post and pre-shock exchange rates move very much in-step as can be seen in the right-hand side of figure 7. In the second simulation the post and pre-shock exchange rates move out-of-step (see figure 7). In the latter case the variance of the effect of the shock in equilibrium rate is much larger than in the former case but the complexity of these movements, as shown by the strange attractor, is smaller.

Finally note that the previous results hold when the size of the shock is sufficiently large. For small shocks the pre and post-shock exchange rate always move in-step, so that the strange attractor does not change in nature.

Figure 7


strange attractor permanent shock of 10 in equilibrium rate
c=5; theta=-0.05; beta=4
effect in t+1
effect in t
shock in equil rate 10
(c=5; theta=-0.05 ; beta=4)
strange attractor permanent shock of 10 in equilibrium rate
c=5; theta=-0.05; beta=4
effect in t+1
effect in t
shock in equil rate 10
(c=5; theta=-0.05 ; beta=4)
strange attractor of underlying model
c=5; theta=-0.05; beta=4
St+1
St

3. Gains and losses of speculation

The results we have shown in the previous sections are based on the implicit assumption that the losses (and gains) by chartists and fundamentalists are bounded. If this is not the case, e.g. if the fundamentalists would make unlimited losses they would disappear from the market, thereby affecting the dynamics of the model[3]. In order to check whether losses (and gains) are bounded we compute the cumulative forecast errors that chartists and fundamentalists make. Assuming that a forecast error of 1 represents a loss of $1, the cumulative forecast errors represent the cumulative losses. We show the results in figures 8 and 9. Figure 8 represents the cumulative gains and losses of chartists (vertical axis) and fundamentalists (horizontal axis) in the same period. We observe that there is a negative relation between the cumulative gains and losses of the chartists and the fundamentalists.

Figure 9 shows the sum of the cumulative gains and losses made by chartists and fundamentalists in period t (horizontal axis) and t+1 (vertical axis). Thus, figure 9 represents the strange attractor of the cumulative gains and losses incurred by speculators. We find that these gains and losses behave in a complex manner and can therefore be represented by strange attractors. These strange attractors are bounded, implying that the cumulative gains (and losses) are bounded.

[3] A way to make this assumption explicit is to introduce a budget constraint in the model. However we do not do this here and leave that for further research.

Figure 8

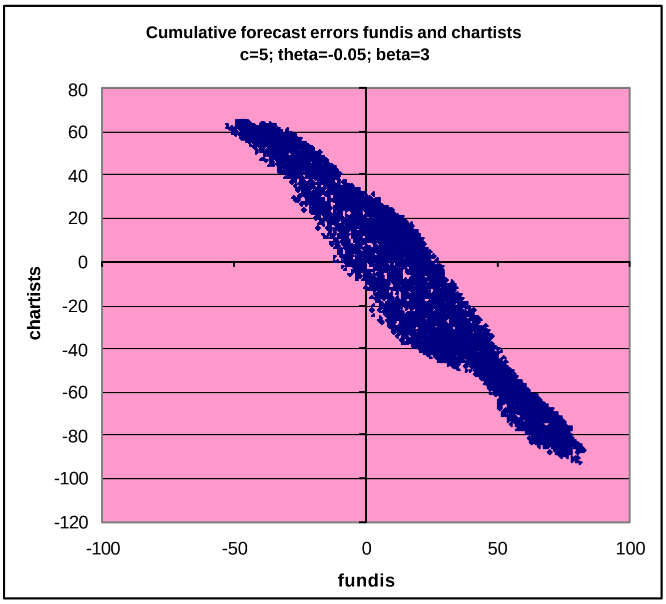
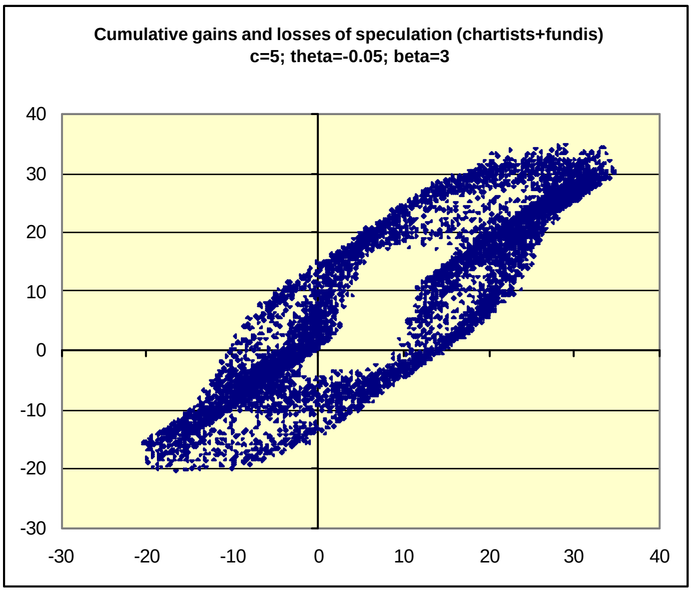


Figure 9



4. The size of the shocks matters

In this section we analyse how the size of shocks affect the dynamics of the exchange rate. We consider large and small shocks by changing the variance of the exogenous noise ε_t, which is independent of the noise in the underlying fundamentals. We show the dynamics of the exchange rate in figures 10, 11 and 12 for increasing values of the variance of the noise. In figure 10 we show the dynamics of the market rate when the exogenous noise is very low (variance equal to 1). We find that with low exogenous noise the nature of the strange attractor of the effect of the increase in the equilibrium rate is complex and its shape changes depending on the initial conditions. When the noise in the market is very high (variance equal to 100) (see figure 12), then the effect of an increase in the equilibrium rate quickly converges to a fix point, i.e. the increase of the equilibrium rate is quickly translated in an equal increase in the market rate. As a result, the effect of the shock is very predictable. We also show the case of an intermediate value of the variance of the exogenous noise (variance equal to 10) in figure 11. Not surprisingly, in this case the complexity of the strange attractor is lower than in the case of a very small noise.

Figure 10: variance of noise=1

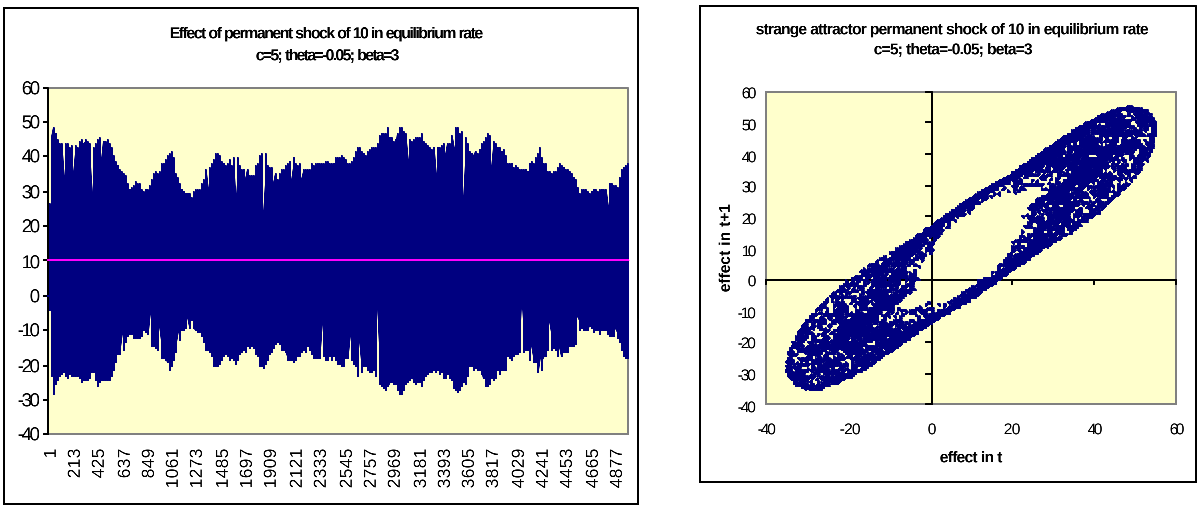


Figure 11: variance of noise = 10

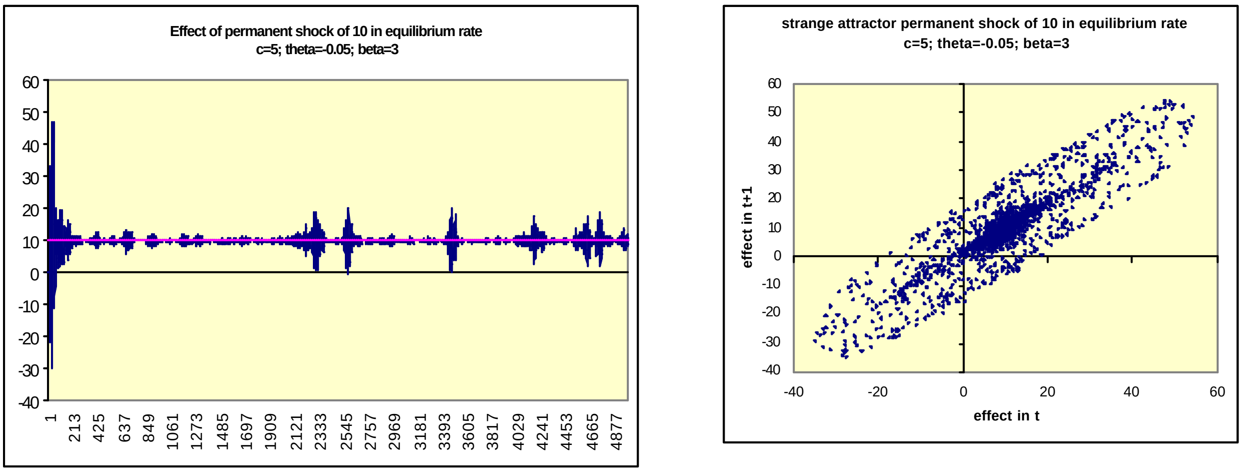
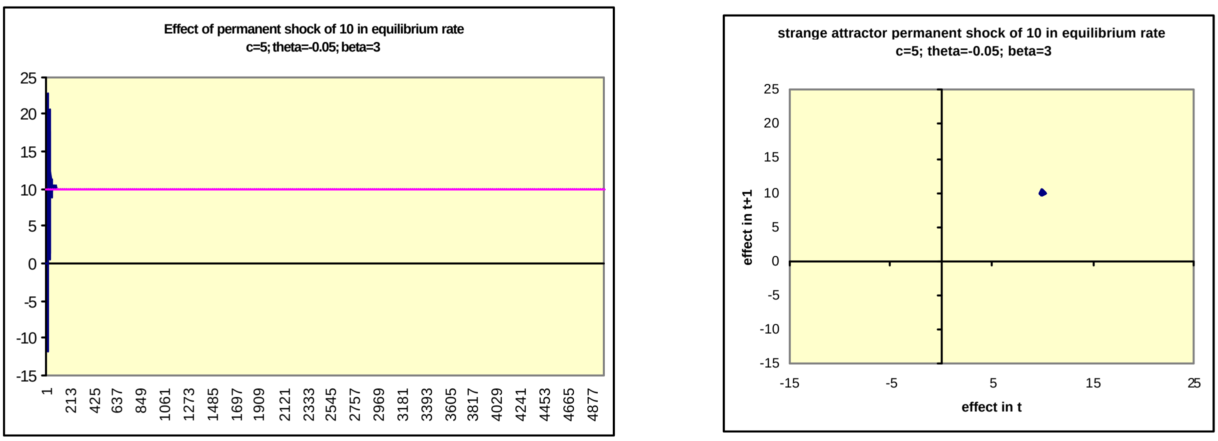


Figure 12: variance of noise = 100



These surprising results can be explained as follows. When the exogenous noise is very large the fundamentalists observe that the market exchange rate deviates from the equilibrium rate by large amounts. Therefore, they are very active in the market and tend to dominate the chartists. This implies that the mean reverting process (negative feedback dynamics) dominates. As a result, the deviations of the market rate from the equilibrium rate tend to be adjusted quickly. This also implies that a shock in the equilibrium rate is transmitted into the market rate on one-to-one basis.

Things are very different when the noise is very low. In this case the chartists tend to dominate (at least around the equilibrium rate) because the relatively small deviations of the exchange market rate from its equilibrium value do not trigger much fundamentalists' action. As a result, the non-linear dynamics of the model, which is responsible for the complexity in the exchange rate movements, works fully. Therefore, the effect of an increase in the equilibrium rate cannot easily be predicted.

5. Conclusion

In this paper we have studied the transmission of a change in the fundamental (equilibrium) exchange rate on the market exchange rate. We made this analysis using a non-linear model of the exchange rate. This model is characterised by two non-linearities. The first one comes from the goods market. We used the empirical evidence, which shows that the speed of adjustment towards PPP is positively related to the size of the deviation from PPP, and we assumed that 'fundamentalists' use this information when they forecast the future exchange rate. This contrasts with the chartists who extrapolate past movements in the exchange rate. The second non-linearity introduced comes from transactions costs. There is a growing realisation that transaction costs matter and play a role in understanding some of the puzzles in international macroeconomics.

The extremely simple model developed in this paper is capable of generating very complex exchange rate dynamics. We showed that as a result of this complexity, a change in the equilibrium exchange rate has a very unpredictable effect on the exchange rate. In fact we showed that this effect has a chaotic structure even when the parameters are such that the underlying model is non-chaotic.

We also showed that when the exogenous noise in the model is low the complexity in the transmission of changes in the exchange rate is high. Conversely, when the exogenous noise increases the complexity in the transmission is reduced. We interpreted this in the following way. When the exogenous noise is large, the exchange rate is often driven very far from its equilibrium rate. As a result, fundamentalist action dominates in the foreign exchange market. This tightens the link between the fundamental and the exchange rate. When the exogenous noise is low, however, chartists are more important driving the exchange rate away from the equilibrium until fundamentalists take over. Put differently, in a regime of low exogenous noise, the non-linear nature of the model works fully producing complexity and endogenous noise. The latter obscures the transmission of changes in the equilibrium exchange rate, making it essentially unpredictable. Conversely, in a regime of high exogenous noise, the non-linear features of the model are inoperative most of the time, reducing complexity and making the effect of a change in the exchange rate more predictable.

The exchange rate regime of the major industrialised countries can be said to be characterised by relatively low noise, at least compared to the exchange rate regimes of high inflation countries. As a result, the non-linearities analysed in this paper are likely to be important. All this leads to the conclusion that the transmission process of shocks in the fundamental variables (e.g. inflation rates, output shocks) is clouded by the endogenous noise produced by these non-linearities, making it very difficult to predict how the exchange rate will react to changes in fundamentals.

The analysis performed in this paper was done in a model without explicit optimising behaviour by agents. This has become very unfashionable. We invoke two lines of defence. The first one, which has a long tradition in economics, is that agents do not optimise most of the time and follow rules of thumb that have shown to be performing in the past. The second line of defence is that we show how a very simple model is capable of generating a complex dynamics that comes much closer to understanding a number of puzzles in exchange rate economics than the models that assume that agents continuously solve a dynamic utility maximisation program.