Bubbling and Crashing Exchange Rates

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Abstract

We develop a simple model of the foreign exchange market in which agents optimize their portfolio and use different forecasting rules. They check the profitability of these rules ex post and select the more profitable one.This model produces two kinds of equilibria, a fundamental and a bubble one. In a stochastic environment the model generates a complex dynamics in which bubbles and crashes occur at unpredictable moments. We also analyse the empirical relevance of the model

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1 Introduction

Since the publication of Kindleberger's "Manias, Panics, and Crashes" in 1978, a large literature has flourished on the theory and the empirics of bubbles and crashes in financial markets. Two schools of thought can be detected. One school has developed from the contributions of Blanchard in the late '70s on rational bubbles (see Blanchard(1979), Blanchard and Watson(1982)). In this view, bubbles can occur in rational expectations (RE) models when the time of the crash is not known with certainty. It is then in the interest of rational agents to "ride the bubble"[1]. A second school of thought has concentrated on the non rational sources of the emergence of bubbles (herding behaviour, bandwagon effects[2]).

Both approaches suffer from some inherent defects. The rational expectations approach has come under increasing criticism both from a theoretical and empirical point of view. The main criticism is that it assumes that individual agents are capable of storing and processing in their individual brains all the relevant information existing in the outside world; an information set which in its complexity by far surpasses the complexity of the individual brain. The response to this criticism has usually been that the rational expectations assumptions is nothing but an assumption about the model-consistency of expectations. The price for this requirement of logical consistency, however, is that in order to make it plausible that agents operating in such a model can be assumed to understand its structure, rational expectations models have to be kept exceedingly simple. As a result, the informational problems agents face in a complex environment is assumed away[3]. All this would not really be a problem if the rational expectations models would pass the only important scientific test which is the empirical verification. Unfortunately, the accumulated empirical evidence in the financial markets is not favourable for the rational expectations model. Too many anomalies have been detected in the financial markets contradicting the rational expectations paradigm. (See De Grauwe and Grimaldi(2003) for a discussion of these anomalies in the foreign exchange market). Apart from this general criticism of the rational expectations model, the Blanchard-Watson rational bubble model can be criticised for the fact that it predicts the occurrence of bubbles whose features are not found in empirical evidence (e.g. exponentially distributed bubbles, symmetry between bubble and crash phases, see Manderlbrot(1997) and Lux and Sornette (2002)).

[1] There is of course a strong "negationist" tradition in the ratioanl expectations literature which denies the existence of bubbles and crashes. In a rational expectations efficient market world bubbles and crashes are meaningless concepts, since the exchange rate always reflects the changing and often volatile information on underlying fundamental variables. A well-known proponent of this view is Garber(2000).

[2] For a recent insightful analysis see Shiller(2000). There is also a strong tradition in the behavioural finance literature analysing anomalies in the financial markets that can trigger bubbles and crashes. See Schleiffer(2000) and Thaler(1994).

[3] A recent booming literature tries to deal with this problem by introducing adaptive learning. This literature has led to important new insights. For interesting contributions introducing least squares learning see Evans and Honkapohja(2001).

The second approach, focusing on the irrationality of agents, provides a wealth of insights in the complexity of human behaviour. The problem it faces, however, is that it has to make as many special assumptions about human behaviour as the number of phenomena it wishes to explain. As a result, it has not yet developed into a scientific alternative for explaining bubbles and crashes in the financial markets.

The aim of this paper is to suggest a possible third alternative for understanding why bubbles and crashes occur in the foreign exchange markets. We develop a simple model of the exchange rate in which we relax the RE hyphotesis. We assume that individual agents are not capable of using all available information in the model, and that they select different simple forecasting rules. Although agents have potentially access to the same kind of information they differ in the interpretation and in the use of such information[4].

We assume two kinds of agents, fundamentalists who use fundamental information and technical traders (chartists) who extrapolate past exchange rate changes. Empirical evidence that supports the profitability of technical trading rules is provided, among others, by Taylor and Allen(1992) and LeBaron(1999). Both the fundamentalists and the chartists evaluate the fitness of their respective forecasting rules based on the risk adjusted profits and they decide to switch to the most profitable one. Thus, agents are not irrational in our model. They know that they cannot comprehend the full complexity of the underlying model. As a result, the rational response is to try different forecasting rules and to select the one that performs best.

We show that bubbles and crashes occur as a result of the interaction between agents using different forecasting rules. We analyse under which conditions bubbles occur and we study the nature of these bubbles. Next we check if our model reproduces the statistical properties of the exchange rate movements. We argue that our model is capable of explaining the 'anomalies' that have been detected by the empirical evidence which could not be explained by the traditional models. In particular most of the empirical findings document that the exchange rate returns have excess kurtosis and fat tails (see de Vries(2001), Lux T. (1997, 1998), Lux and Marchesi (1999, 2000). This evidence is difficult to rationalise in rational expectations efficient market exchange rate models, since there is little evidence of fat tails in the fundamental variables that drive the exchange rate in these models. Other empirical anomalies have been uncovered over the years. One is the "excess volatility" puzzleof the exchange rate, i.e. the volatility of the exchange rate by far exceeds the volatility of the underlying economic variables (Baxter and Stockman (1989) and Flood and Rose (1995)). We show that this puzzle can easily be rationalized in our model. Finally we compute the profits and losses that chartists and fundamentalists make depending on the accuracy of their respective forecasting rules.

[4] The fact that agents interpret in a different way the same information has been confirmed empirically by Kandel and Pearson(1995).

2 The model

In this section we develop a simple model of the exchange rate . The model consists of three building blocks. First, agents select their optimal portfolio using a mean-variance utility framework. Second they make forecasts about the future exchange rate based on simple but different rules. Third, they evaluate these rules ex-post by comparing their risk-adjusted profitability.

2.1 The optimal portfolio

We assume agents of different types i depending on their beliefs about the future exchange rate. Each agent can invest in two assets, a domestic and a foreign one. The agents' utility function can be represented by the following equation:

$$U(W_{t+1}^{i}) = E_t(W_{t+1}^{i}) - \frac{1}{2}\mu V^{i}(W_{t+1}^{i}) \tag{1}$$

where W_{t+1}^{i} is the wealth of agent of type i at time t+1, E_t.is the expectation operator, μ is the coefficient of risk aversion and $V^{i}(W_{t+1}^{i})$ represents the conditional variance of wealth of agent i. The wealth is specified as follows:

$$W_{t+1}^{i} = (1+r^{*})\, s_{t+1} d_t^{i} + 1 + r\left(W_t^{i} - s_t d_t^{i}\right) \tag{2}$$

where r and r^{*} are respectively the domestic and the foreign interest rates, s_{t+1} is the exchange rate at time t+1, $d_{i,t}$ represents the holdings of the foreign assets by agent of type i at time t. Thus, the first term on the right-hand side of 2 represents the value of the foreign portfolio in domestic currency at time t+1 while the second term represents the value of the domestic portfolio at time t+1.

Substituiting equation 2 in 1 and maximising the utility with respect to $d_{i,t}$ allows us to derive the standard optimal holding of foreign assets by agents of type i :

$$d_{i,t} = \frac{(1+r^{*})\, E_t^{i}\left(s_{t+1}\right) - (1+r)\, s_t}{\mu \sigma_{i,t}^{2}} \tag{3}$$

The optimal holding of the foreign asset depends on the expected excess return corrected for risk. The market demand for foreign assets at time t is the sum of the individual demands, i.e.:

$$\sum_{i=1}^{N} n_{i,t} d_{i,t} = D_t \tag{4}$$

where $n_{i,t}$ is the number of agents of type i.

Market equilibrium implies that the market demand is equal to the market supply $X_t$which we assume to be exogenous[5]. Thus,

[5] The market supply is determined by the net current account and by the sales or purchases of foreign exchange of the central bank. We assume both to be exogenous. In an extension of this paper we intend to endogenise the market supply.

$$X_t = D_t \tag{5}$$

Substituting the optimal holdings into the market demand and then into the market equilibrium equation and solving for the exchange rate s_t yields the equilibrium exchange rate:

$$s_t = \left(\frac{1+r^*}{1+r}\right) \frac{1}{\sum_{i=1}^{N} \frac{n_{i,t}}{\sigma_{i,t}^2}} \left[\sum_{i=1}^{N} n_{i,t} \frac{E_t^i\left(s_{t+1}\right)}{\sigma_{i,t}^2} - \mu \frac{X_t}{1+r}\right] \tag{6}$$

In order to model the expectations formation we assume that there are two types of agents: chartists and fundamentalists. As a result equation 6 specialises to :

$$s_t = \left(\frac{1+r^*}{1+r}\right) \frac{1}{\left(\frac{n_{f,t}}{\sigma_{f,t}^2} + \frac{n_{c,t}}{\sigma_{c,t}^2}\right)} \left[n_{f,t} \frac{E_t^f\left(s_{t+1}\right)}{\sigma_{f,t}^2} + n_{c,t} \frac{E_t^c\left(s_{t+1}\right)}{\sigma_{c,t}^2} - \mu \frac{X_t}{1+r}\right] \tag{7}$$

Thus the exchange rate is determined by the expectations of fundamentalists, E_t^f , and chartists E_t^c about the future exchange rate. These forecasts are weighted by their respective variances $\sigma_{f,t}^2$ and $\sigma_{c,t}^2$. Thus, when for example the chartists' forecasts have a high variance the weight of the chartists in the determination of the market exchange rate is reduced.

2.2 The forecasting rules

We now specify how fundamentalists and chartists.form their expectations of the future exchange rate. In a second step we will specify how they take into account the risk as measured by the variances.

The fundamentalists base their forecast on a comparison between the market and the fundamental exchange rate, i.e. they forecast the market rate to return to the fundamental rate in the future. In this sense they use a negative feedback rule that introduces a mean reverting dynamics in the exchange rate . The speed with which the market exchange rate returns to the fundamental is assumed to be determined by the speed of adjustment in the goods market which is assumed to be in the information set of the fundamentalists (together with the fundamental exchange rate itself). Thus, the forecasting rule for the fundamentalists is :

$$E_t^f\left(\Delta s_{t+1}\right) = -\psi\left(s_t - s_t^*\right) \tag{8}$$

where s_t^* is the fundamental exchange rate at time t , which is assumed to follow a random walk and $0 < \psi < \infty$.

The chartists forecast the future exchange rate by extrapolating past exchange rate movements. Their forecasting rule can be specified as :

$$E_t^c\left(\Delta s_{t+1}\right) = \beta \sum_{i=0}^{T} \alpha_i \Delta s_{t-i} \tag{9}$$

Thus, the chartists compute a moving average of the past exchange rate changes and they extrapolate this into the future exchange rate change. The degree of extrapolation is given by the parameter β. Note that in contrast to the fundamentalists, chartists do not take into account information concerning the fundamental exchange rate. In this sense they can be considered to be pure noise traders .In a way this chartist rule can also be seen as reflecting herding behaviour, i.e. chartists do not take fundamental information into account because they feel too uncertain about their meaning, but they closely watch the movements of the exchange rate as a way to detect "market sentiments". If the latter are positive, they buy; if they are negative, they sell.

Our choice to give a prominent role to chartists' rules of forecasting is based on empirical evidence. The evidence that chartism is used widely to make forecasts is overwhelming (see Cheung and Chinn(1989), Taylor and Allen(1992)). It remains important, however, to check if the model is internally consistent. In particular, the chartists' forecasting rule must be shown to be profitable within the confines of the model. If these rules turn out to be unprofitable, they will not continue to be used. We return to this issue when we let the number of chartists be determined by the profitability of the chartists' forecasting rule.

We now analyse how fundamentalists and chartists evaluate the risk in volved in forecasting. The latter is measured by the variance terms in equation 7, which we define as the weighted average of the squared (one period ahead) forecasting errors made by chartists and fundamentalists, respectively. Thus,

$$\sigma_{i,t} = \sum_{k=1}^{\infty} \gamma_k \left[E_{t-k}^i \left(s_{t-k+1} \right) - s_{t-k+1} \right]^2 \tag{10}$$

where γ_k are geometrically declining weights.

2.3 Fitness of the rules

The next step in our analysis is to specify how agents evaluate the fitness of these two forecasting rules. The general idea that we will follow is that agents use one of the two rules, compare their (risk adjusted) profitability *ex post* and then decide whether to keep the rule or switch to the other one. Thus, our model is in the logic of evolutionary dynamics, in which simple decision rules are followed. These rules will continue to be followed if they pass some "fitness" test (profitability test). Another way to interpret this is as follows. When great uncertainty exists about how the complex world functions, agents use a trial and error strategy. They try a particular forecasting rule until they find out that other rules work better. Such a trial and error strategy can be considered to be a rational strategy when agents cannot understand the full complexity of the underlying model. It can even be argued that it is a more rational strategy than the strategy followed by agents in rational expectations models, where these agents have the ambition to understand the underlying model in all its complexity.

We start by specifying the dynamics that governs the number of chartists and fundamentalists, namely n_{ct} and n_{ft}. In order to do so, we describe how the number of chartists and fundamentalists changes from period t-1 to period t:

$$n_{c,t} = n_{c,t-1} + n_{f,t-1} p_t^{fc} - n_{c,t-1} p_t^{cf} \tag{11}$$

$$n_{f,t} = n_{f,t-1} + n_{c,t-1} p_t^{cf} - n_{f,t-1} p_t^{fc} \tag{12}$$

where $n_{c,t}$ and $n_{f,t}$ are the number of chartists and fundamentalists in period t[6]; p_t^{cf} represents the fraction of the chartists who decide to become fundamentalists in period t, and p_t^{fc} is the fraction of the fundamentalists who decide to become chartists in period t.

These fractions are assumed to be a function of the profitability of the forecasting rules and the risk associated with their use. The fractions are specified as follows[7]:

$$p_t^{fc} = \frac{exp\left[\gamma \pi'_{c,t-1}\right]}{exp\left[\gamma \pi'_{c,t-1}\right] + exp\left[\gamma \pi'_{f,t-1}\right]} \tag{13}$$

$$p_t^{cf} = \frac{exp\left[\gamma \pi'_{f,t-1}\right]}{exp\left[\gamma \pi'_{c,t-1}\right] + exp\left[\gamma \pi'_{f,t-1}\right]} \tag{14}$$

where $\pi'_{c,t-1}$and $\pi'_{f,t-1}$ are the risk adjusted net profits made by chartists' and fundamentalists' forecasting the exchange rate in period t-1, i.e. $\pi'_{c,t-1} = \pi_{c,t-1} - \mu\sigma^2_{c,t-1}$ and $\pi'_{f,t-1} = \pi_{f,t-1} - C - \mu\sigma^2_{f,t-1}$. We assume that the fundamentalists make a fixed cost C cor the collection and processing of fundamental information, while the collection of inormation by chartists is assumed to be costless[8].

Equations 13and 14 can be interpreted as follows. When the risk adjusted profits of the chartists' rule increases relative to the risk adjusted net profits of the fundamentalists rule, then the fraction of the fundamentalists who become chartists in period t increases, and vice versa. The parameter which regulates such switches is γ.This parameter can be interpreted as the rate with which the chartists and fundamentalists revise their forecasting rules. With an increasing γ agents revise their forecasts very frequently. In the limit when γ goes to infinity

[6]It should be noted that our modelling approach shares important features with the multi-agent based models in which market partecipants evolve over time. However, in contrast with such models the characteristics of our model are independent of the total number of agents in the market.

[7]This specification of the decision rule is often used in discrete choice models. For an application in the market for differentiated products see Anderson, S., de Palma, A., Thisse, J.-F., 1992. The idea has also been applied in financial markets. See Brock and Hommes (1997) and Lux(1998).

[8]This asymmetry in the treatment of the cost of information for fundamentalists and chartists is not crucial for our results.

agents revise the forecasting rules instantaneously. When γ is low, chartists and fundamentalists revise their forecasts relatively slowly. When γ is equal to zero they do not revise their rules. In the latter case the fraction of chartists and fundamentalists is constant and equal to 0.5. Thus γ is a measure of inertia in the decision to switch to the more profitable rule .As will be seen this parameter is of great importance in generating bubbles.

Chartists and fundamentalists make a profit when they correctly forecast the direction of the exchange rate movement. They make a loss if they wrongly predict the direction of its movements. The profit (the loss) they make equals the one-period return of investing $1.

3 Solution of the model

In this section we investigate the properties of the solution of the model. We first study its deterministic solution. This will allow us to analyse the characteristics of the solution that are not clouded by exogenous noise. We use simulation techniques since the non-linearities do not allow for a simple analytical solution. We select " reasonable" values of the parameters, i.e. those that come close to empirically observed values. As we will show later these are also parameter values for which the model replicates the observed statistical properties of exchange rate movements. We will also subject the analysis to an extensive sensitivity analysis.

We first concentrate on the fixed point solutions of the model. In figure 1 we show the solutions of the exchange rate for different initial conditions. We plot the fixed point solutions (attractors) as a function of the different initial conditions[9]. On the horizontal axis we set out the different initial conditions. These are initial shocks to the deterministic system. The vertical axis shows the solutions corresponding to these different initial conditions. The fundamental exchange rate was normalized at 0. We find two types of fixed point solutions. First, for small disturbances in the initial conditions the fixed point solutions coincide with the fundamental exchange rate. We will call these solutions the *fundamental* solutions. Second, for large disturbances in the initial conditions, the fixed point solutions diverge from the fundamental. We will call these attractors, *bubble* attractors. It will become clear why we label these attractors in this way. The larger is the initial shock (the noise) the farther the fixed points are removed from the fundamental exchange rate. The border between these two types of fixed points is characterised by discontinuities. This has the implication that in the neigborhood of the border a small change in the initial condition (the noise) can have a large effect on the solution.

The different nature of these two types of fixed point attractors can also be seen from an analysis of the chartists' weights that correspond to these different fixed point attractors. We show these chartists weight as a function of the initial concicions in figure 2.

[9] These fixed point solutions of the exchange rate where obtained by running simulations of 100,000 periods. Each time the exchange rate converged to a fixed point.

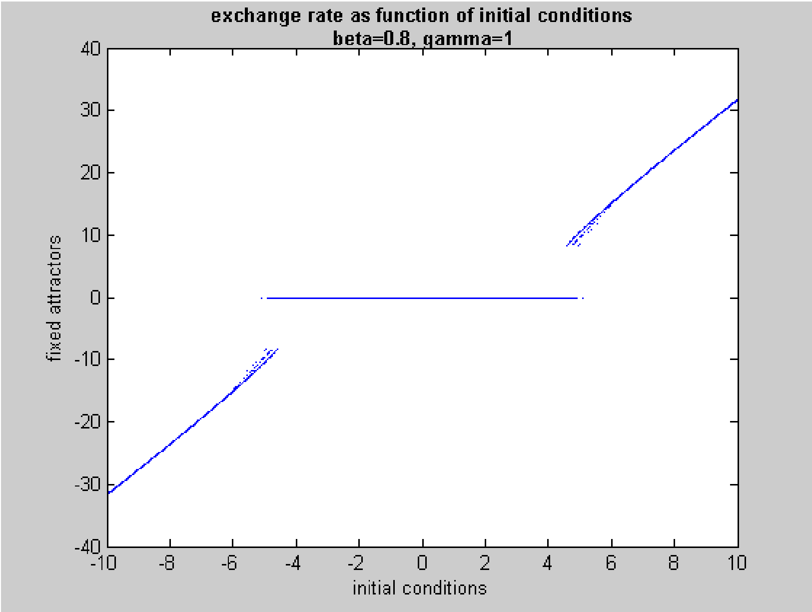

Figure 1:

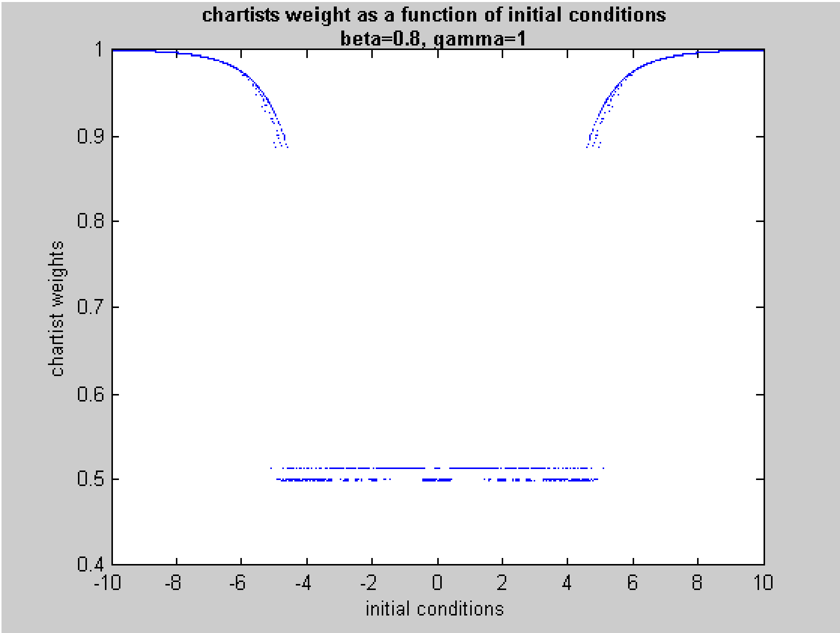

Figure 2:

We find, first, that for small initial disturbances the chartists' weight converges to a number slightly above 50% of the market. Thus when the exchange rate converges to the fundamental rate, the weight of the chartists and the fundamentalists are approximately equal to 50%. For large initial disturbances, however, the chartists' weight converges to 1. Thus, when the chartists take over a sufficiently large part of the market, the exchange rate converges to a bubble attractor. The meaning of a bubble attractor can now be understood better. It is an exchange rate equilibrium that is reached when the number of fundamentalists has become sufficiently small (the number of chartists has become sufficiently large) so as to eliminate the mean reversion dynamics. It will be made clearer in the next section why fundamentalists drop out of the market. Here it suffices to understand that such equilibria exist. It is important to see that these bubble attractors are fixed point solutions. Once we reach them, the exchange rate is constant. Such a constant exchange rate occurs then as a result of two situations. One is that the chartists have taken over the whole market. In this case, chartists who extrapolate the past movements, will forecast no change. At the same time, since the fundamentalists have left the market, there is no force acting to bring back the exchange rate to its fundamental value.The other case is when the fundamentalists still have a small market share which exerts some mean reverting pressure. This pressure is, however, offset by the extrapolating pressure exerted by the chartists. Thus two types of equilibria exist: a fundamental equilibrium where chartists and fundamentalists co-exist, and a bubble equilibrium where the chartists have completely or almost completely crowded out the fundamentalists.

These two types of equilibria differ in another respect. The *fundamental* equilibrium can be reached from many different initial conditions. It is locally stable, i.e after small disturbances the system returns to the same (fundamental) attractor. In contrast there is one and only one initial condition that will lead to a particular *bubble* equilibrium. This implies that a small disturbance leads to a displacement of the bubble solution. Note again that the border between these two types of equilibria is characterized by discontinuities and complexity, i.e. small disturbances can lead to either a fundamental or a bubble equilibrium.

It is useful to compute the attractors for different values of the fundamental exchange rate keeping initial conditions constant. We show such an exercise in figure 3. We now present different fundamental values of the exchange rate on the horzontal axis while keeping the initial condition unchanged. We have set the initial condition for the exchange rate equal to 4. We obtain the following results. First when the fundamental shock and the initial condition are opposite in sign, the exchange rate converges to its fundamental value. This can be seen by the fact that for negative values of the fundamental shocks, the attractors are on a 45■ line so that the equilibrium exchange rate equals its fundamental value. In the range of fundamental shocks between 0 and 4 we obtain bubble equilibria. This is the range in which the initial shock (noise) has the same sign as the fundamental shock. When the positive fundamental shock becomes large relative to the positive initial shock the system returns to a fundamental equilibrium. Thus, bubble equilibria arise when the fundamental shock and the

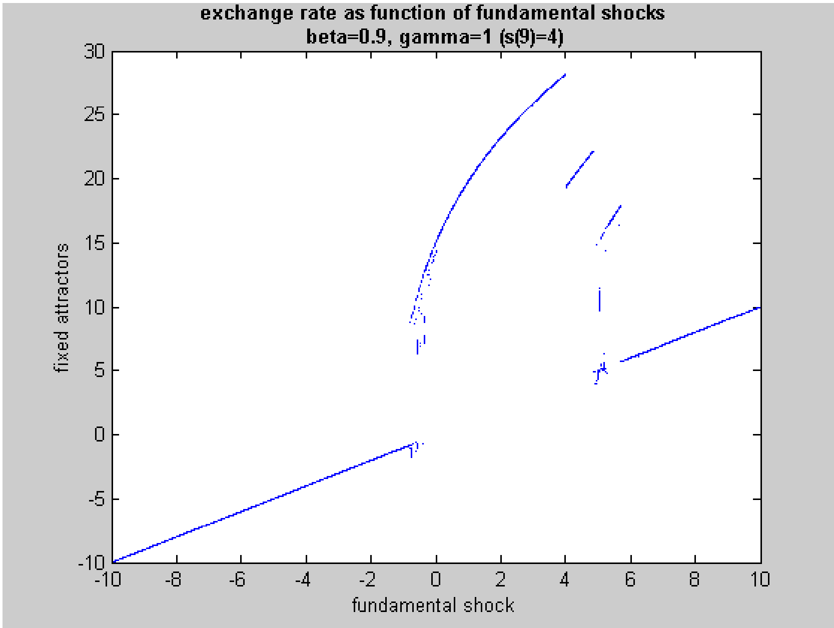

Figure 3:

noise have the same sign, and when the noise is relatively large with respect to the fundamental shock. With sufficiently large fundamental shocks (relative to the noise) the equilibrium exchange rate is forced back to its fundamental value. In appendix 1 we show some additional simulations for smaller and larger initial conditions. These simulations confirm that as the noise increases relative to the fundamental shocks the range of bubble equilibria increases and *vice versa*.

The previous results allow us to understand not only why bubbles can arise. They also shed light on why bubbles tend to crash. The noise that triggers a bubble is temporary. Fundamental shocks, however, typically have a large permanent component[10]. Thus in a stochastic environment small fundamental shocks accumulate to large cumulative fundamental changes. These cumulative changes in the fundamental exchange rate at some point become overwhelming leading to a crash. We will return to this result when we present the stochastic simulations of the model.

4 The anatomy of bubbles and crashes

In the previous section we identified the existence of two different types of fixed point solutions, i.e. a fundamental solution characterised by the fact that the

[10] In the simulations reported here a fundamental shock is permenent.

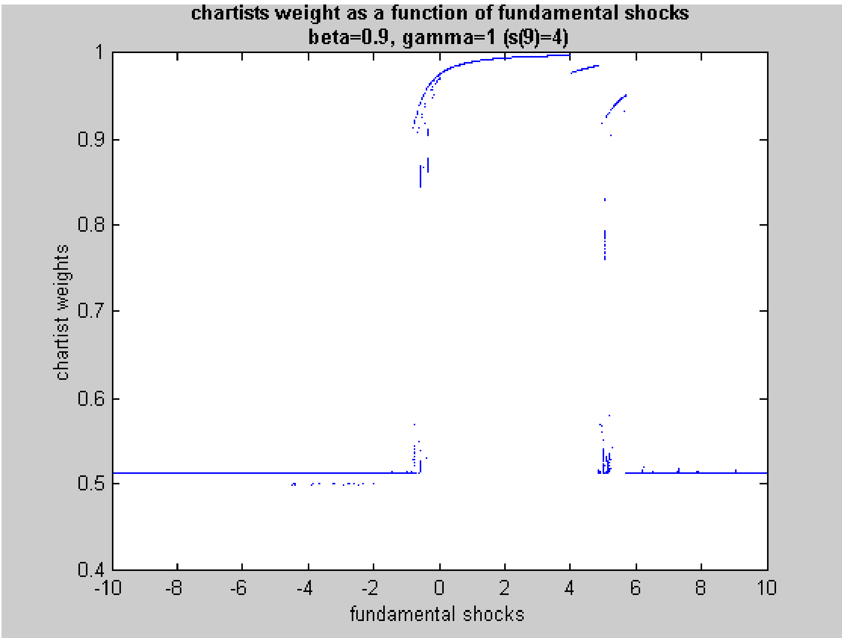

Figure 4:

exchange rate converges to its fundamental value while chartists and fundamentalists "co-habitate", and a bubble solution in which the exchange rate deviates from its fundamental value and in which chartists dominate the market. In this section we show that in combination with stochastic shocks in the fundamental exchange rate these features of the model lead to the emergence of bubbles and crashes.

The way we proceed is to calibrate the model in such a way that it replicates the statistical properties of observed exchange rate movements. We describe this procedure in section 6. Here we present the results of simulations in the time domain using this calibrated model. We start by presenting a case study of a typical bubble and crash scenario as produced by the stochastic version of the model. In the next section we will analyse more systematically the factors that determine the frequency with which such bubbles and crashes occur. Figure **??** shows the exchange rate and its fundamental value in the time domain; figure **??** shows the weight of the chartists in the same time domain. These two figures allow us to analyze a number of common features of a typical endogenously generated bubble and crashes in a stochastic environment.

First, once a bubble emerges, it sets in motion bandwagon effects. As the exchange rate moves steadily in one direction, the use of extrapolative forecasting rules becomes more profitable, thereby attracting more chartists in the market. This is clearly visible from a comparison of figure **??** with figure **??**. We observe

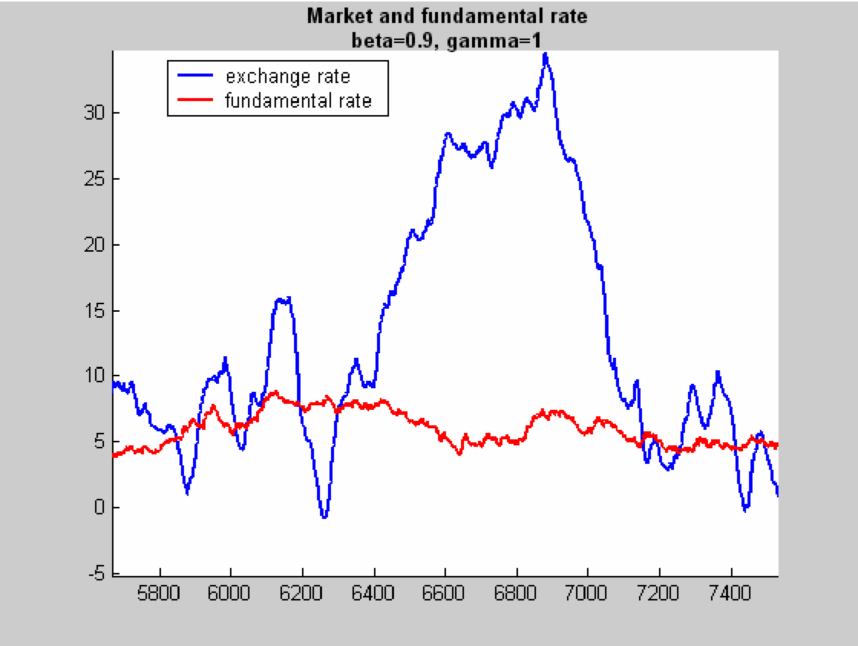
Market and fundamental rate
beta=0.9, gamma=1
exchange rate
fundamental rate
30
25
20
15
10
5
0
-5
5800
6000
6200
6400
6600
6800
7000
7200
7400

Figure 5:

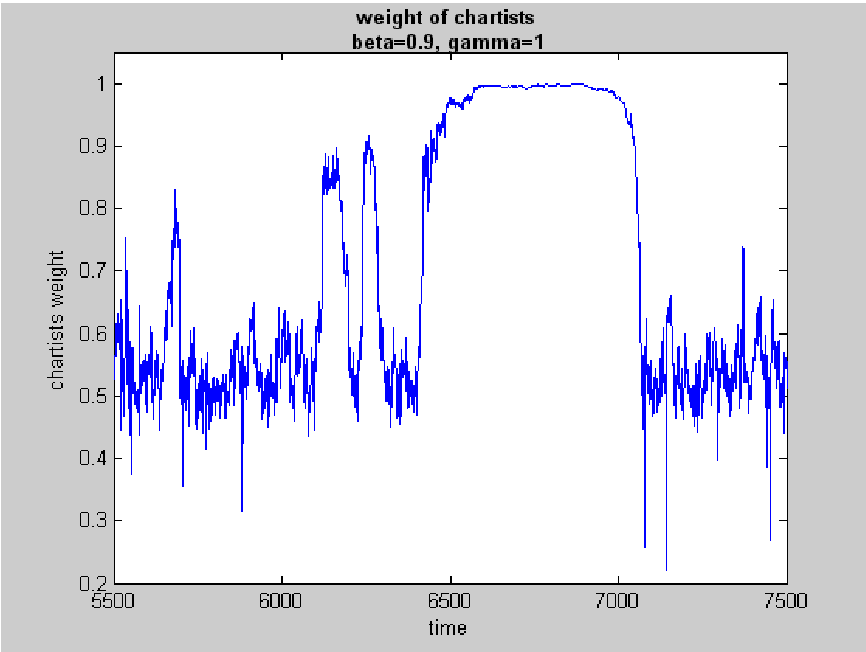
weight of chartists
beta=0.9, gamma=1
chartists weight
time
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
5500
6000
6500
7000
7500

Figure 6:

that the upward movement in the exchange rate coincides with an increase in the weight of chartists in the market. We have checked this feature in many bubbles produced by the model. In appendix 2 we show another example of a bubble, and we present the results of a causality test which shows that the exchange rate leads the weight of chartists during a bubble and the subsequent crash. Thus, typically a bubble starts after the exchange rate has moved in one direction, thereby attracting extrapolating chartists which in turn reinforces the exchange rate movement.

Second, a sustained upward (downward) movement of the exchange rate will not develop into a full scale bubble if at some point the market does not get sufficiently dominated by the chartists. As can be seen from figure **??** at the height of the bubble the chartists have almost 100% of the market. Put differently, an essential characteristic of a bubble is that at some point almost nobody is willing to take a contrarian fundamentalist view. The market is then dominated by agents who extrapolate the bubble into the future. This raises the question of why fundamentalists do not take an opposite position thereby preventing the bubble from developing. After all, the larger the deviation of the exchange rate from the fundamental the more the fundamentalists expect to make profit from selling the foreign currency. Yet they do not, and massively leave the marketplace to the chartists. The reason why they do so, is that during the bubble phase the profitability of chartism increases dramatically precisely because so many chartists enter the market thereby pushing the exchange rate up and making chartism more profitable. There is therefore a self-fulfiling dynamics in the profitability of chartism.

The limit of this dynamics is reached when almost everybody has become a chartist. We arrive at our next characteristics of the bubble-crash dynamics. When almost everybody is a chartist the self-reinforcing upward movement in the exchange rate and in profitability slows down, increasing the expected relative profitability of fundamentalists. This is so because while the bubble developed, the expected profits from fundamentalism also increased. However, these were overwhelmed by the self-fulfilling profitabilitty of chartism. When the latter tends to slow down, fundamentalism becomes attractive again. A small movement of the exchange rate can then trigger a fast decline in the share of chartism, back to its normal level of a tranquil market.A crash is set in in motion.

We have described the dynamics of bubbles in qualitative terms, although the underlying model that produces it is quantitative in nature. However, its complexity is such that sometimes a qualitative and fuzzy description comes closer to understanding its nature. We also did this because our qualitative description of the bubble and crashes is reminiscent of the classic description of bubbles and crashes by Kindleberger in his " Manias, Panics, and Crashes. A History of Financial Crises", which despite its qualitative nature has led to a deeper understanding of the nature of bubbles and crashes than has been possible by quantitative models. The remarkable feature of our mathematical model is that it produces a story that is very close to Kindleberger's story[11].

5 The frequency of bubbles

In the previous sections we showed that a very simple model is capable of generating bubbles and crashes that have the basic features of bubbles and crashes observed in financial markets. All we need is the existence of agents who maximize the utility of their portfolio, make forecasts based on the use of different forecasting rules and who switch to the more profitable of these rules. An important issue here concerns the frequency with which bubbles occur in our model. We analyse this issue by simulating the stochastic version of the model and by counting the number of periods the exchange rate is involved in a bubble. We define a bubble here to be a deviation of the exchange rate from its fundamental value by more than three times the standard deviation of the fundamental variable. We show the result of such an exercise in figure **??** for different values of the extrapolation parameter β. It shows the percentage of time the exchange rate is involved in a bubble dynamics. We observe that when β is smaller than 1 the frequency of the occurrence of bubbles is reasonable. For values of β larger than 1 this frequency increases exponentially. Thus the extrapolation by chartists is an important parameter affecting the frequency with which bubbles occur.

The results obtained in figure **??** are determined by the existence of bubble equilibria in the deterministic version of the model. Therefore, it is useful to connect figure **??** with a figure that plots the exchange rate solutions obtained in the deterministic version of the model. We show this in figure 8 where we set out the equilibrium exchange rate on the vertical axis as a function of β (horizontal axis). We see that for values of $\beta < 0.85$, the exchange rate converges to its fundamental value (normalized to 0). When $\beta > 0.85$ we obtain bubble equilibria that increasingly deviate from the fundamental value. Note that when $0.88 < \beta < 0.9$ we have a complex structure. The equilibrium jumps back and forth between the fundamental and a bubble. Thus, in a way figure 8 predicts what should happen in a stochastic environment. When $\beta < 0.85$ the equilibrium exchange rate converges to its fundamenal value. Around this fundamental value a basin of attraction exists which pulls the exchange rate. Only when the noise is sufficiently high will the exchange rate be attracted to a bubble equilibrium (see figure 1 where we showed that with $\beta = 0.8$ a sufficiently high initial shock will pull the exchange rate towards a bubble equilibium). Thus when $\beta < 0.85$ bubbles will be relatively infrequent events. When β increases above 0.85, however, bubble equilibria appear, increasing the probability of bubbles in a stochastic environment. Note however that even when β is larger enough (e.g. 0.9) to produce only bubble equilibria in the deterministic version

[11] A bubble is:"... a sharp rise in price of an asset or a range of assets in a continous process, with the initial rise generating expectations of further rises and attracting new buyers-generally speculators, interested in profits from trading in the asset rather than its use as earning capacity". Kindleberger(1987).

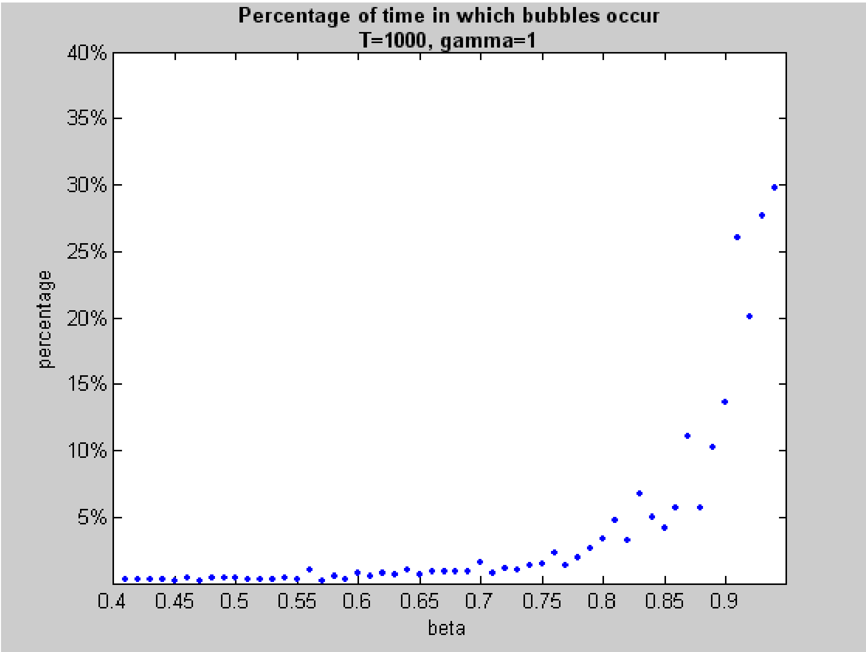
Percentage of time in which bubbles occur
T=1000, gamma=1
percentage
40%
35%
30%
25%
20%
15%
10%
5%
0.4
0.45
0.5
0.55
0.6
0.65
0.7
0.75
0.8
0.85
0.9
beta

Figure 7:

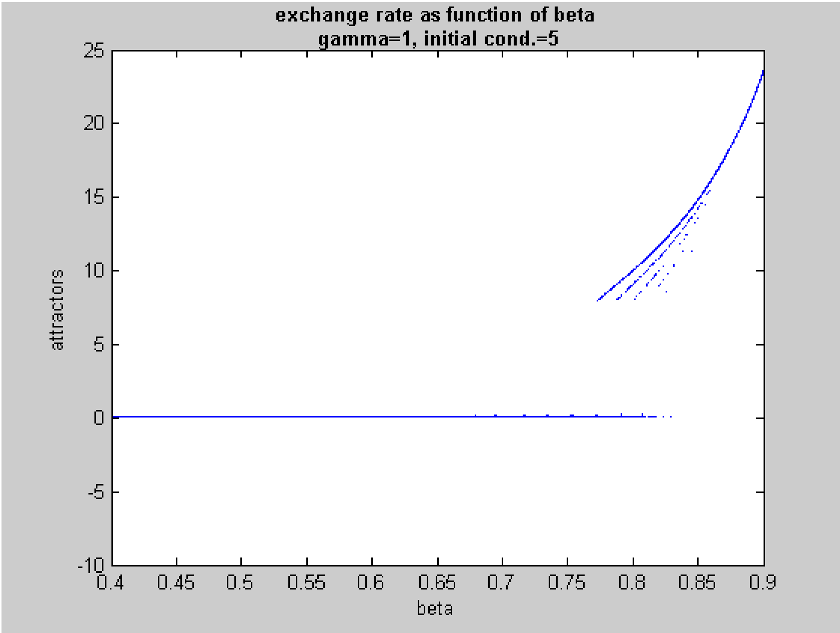

Figure 8:

of the model, the probability of a bubble is not 1 in the stochastic version. The reason is that the noise can lead the exchange rate within the basin of attraction around the fundamental or, more importantly, that the shocks in the fundamentals displace the basin of attraction leading to a crash in the bubble.

From the analysis of the different types of equilibria made in section 3, we predicted that bubbles are more likely to occur when the size of the noise is large relative to the size of the shocks in the fundamental. We now test this prediction in the stochastic environment. We proceed as follows. We first simulate the model assuming that the variance of the noise is 50% higher than the variance of the shocks in the fundamentals. We then perform a simulation with the reverse assumption, i.e. the variance of the shocks in the fundamentals is 50% higher than the variance of the noise. We show the results in figures XXX and **??**. We find that the percentage of time in which bubbles occur is about twice as high when the variance of the noise is high relative to the variance of the shocks in the fundamental. This result is interesting for the following reason. It is often said that if the authorities reduce the variability of the fundamental variables (inflation, money growth, etc.) the exchange rate will also become less volatile. Our results indicate that this may not be so. If the noise is high relative to volatility of fundamentals, bubbles may even become more frequent[12].

[12] It is also worth pointing out that despite the dramatic decline of the variability of important fundamental variables like inflation during the last twenty years, there is no evidence

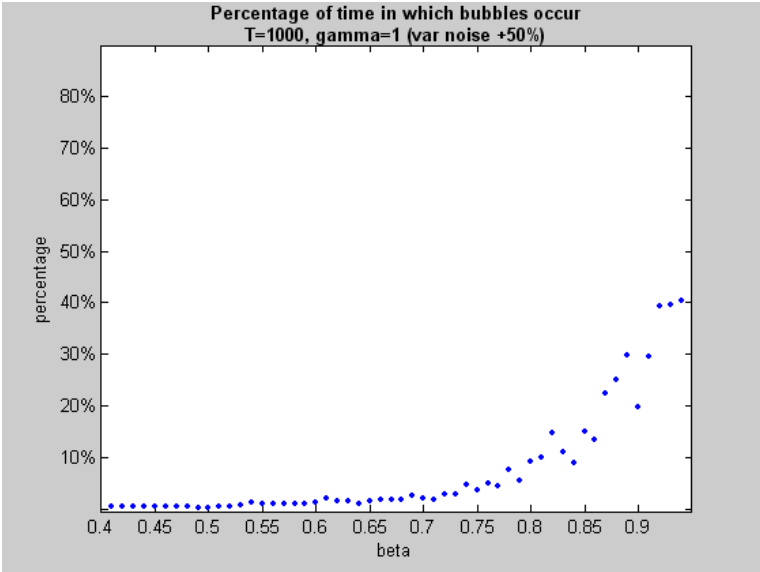

Figure 9:

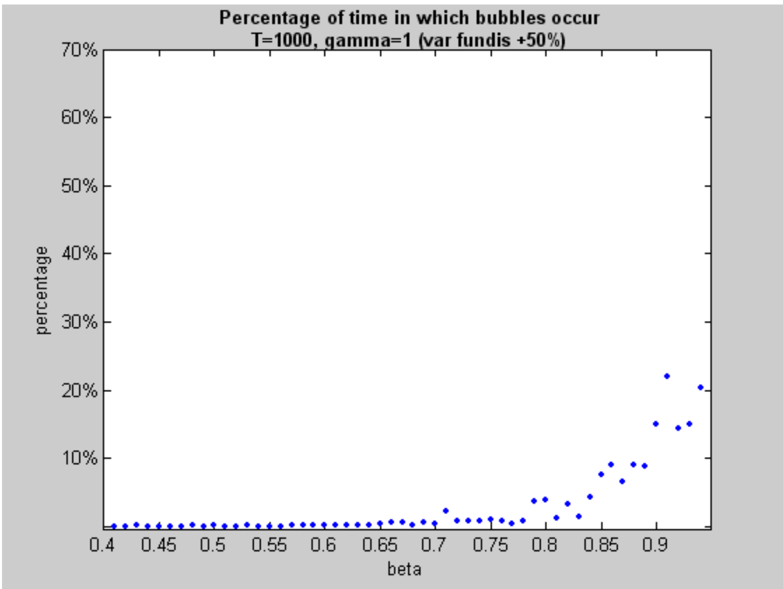

Figure 10:

The frequency of the occurrence of bubbles also depends on the parameter γ which measures the rate with which chartists and fundamentalists revise their forecasting rules. We have called this parameter *rate of revision.* In a way, γ also measures the speed with which agents learn about the profitability of the other rule and revise their forecasts. The lower is this parameter the less frequently agents will revise their forecasting rules. In the limit when $\gamma = 0$ the agents never revise their forecasts which could be interpreted as a world which agents perceive to be stationary.

In order to illustrate the importance of this parameter, we first show the results of the deterministic simulations in figures **??**. We observe that for values of γ lower than (approximately) 1.2 the exchange rate converges to its fundamental value. For higher values we obtain bubble equilibria[13]. Note also a zone of complexity where the location of the bubble equilibria is very sensitive to small changes in the parameter γ. In figure 12 we show the results of the stochastic simulation under the same parameter configuration. We observe that for low values of γ the occurence of bubbles is very infrequent. As γ increases the frequency of bubbles increases significantly.

The previous results allow us to shed some additional light on the nature of bubbles and crashes. As we have seen before, bubbles arise because agents are attracted by the profitability of the extrapolating (chartist) rule, and this attraction in turn makes this forecasting rule more profitable, leading to a self-fulfilling increase in profitability. For this dynamics to work, agents' decision to switch must be sufficiently sensitive to the relative profitabilities of the rules. If it is not, no bubble equilibria can arise, as is the case when γ does not exceed 1. The larger is γ the more likely it is that these self-fulfilling bubble equilibria arise. The interesting aspect of this result is that in a world where agents quickly react to changing profit opportunities, bubbles become more likely than in a world where agents do not react quickly to these new profit opportunities. One way to interpret this result could be the following. When γ is low, agents do not quickly adjust their forecasting rules to changing relative profitabilities. This must be a world that they perceive to be stationary in which there is no need to switch in and out of different forecasting rules. In such a world, bubbles are unlikely to occur. Conversely, in a world where agents react to every whimp in relative profitabilities bubbles will be a frequent occurrence.

The policy implication of this result is that by increasing the inertia in the system so that agents react less quickly to changes in relative profitabilities of forecasting rules, the authorities could reduce the probability of the occurrence of bubbles. How this can be done and whether some form of taxation of exchange transactions can do this, is a question we want to analyse in future research.

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that the exchange rates of the major currencies have become less volatile.

[13] In appendix 3 we show a similar figure where we have set $\beta = 0.9$. In that case the critical value of γ which produces bubble equilibria is lowered.

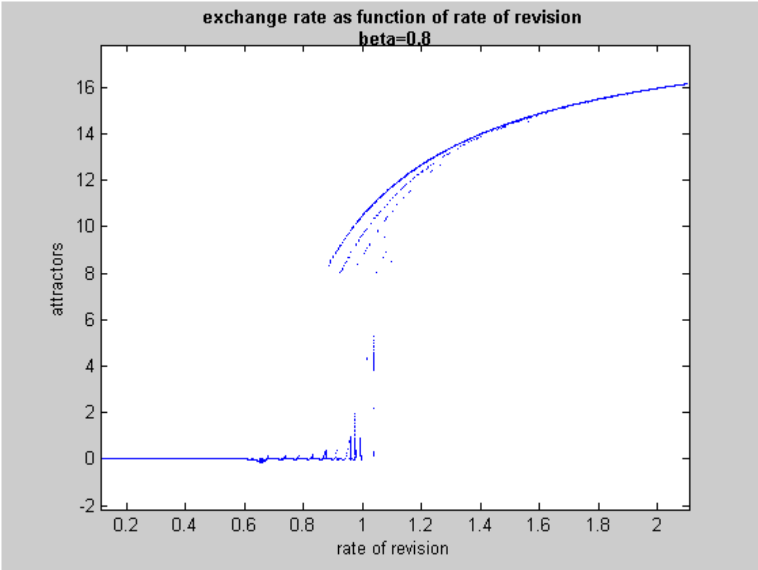

Figure 11:

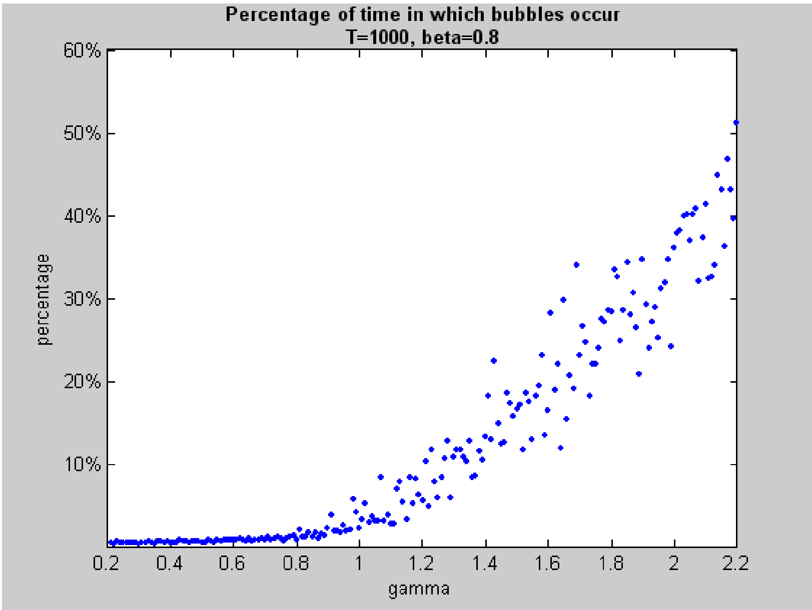

Figure 12:

6 Empirical relevance of the model

In this section we analyse how well our model mimics the empirical anomalies and puzzles that have been uncovered by the flourishing empirical literature. We calibrate the model such that it replicates the observed statistical properties of exchange rate movements. In order to do so we selected a parameter configuration that mimics these properties most closely. We discuss these different statistical properties in the following sections.

6.1 Fat tails and excess kurtosis

It is well known that the exchange rate changes do not follow a normal distribution. Instead it has been observed that the distribution of exchange rate changes has more density around the mean than the normal and exhibits fatter tails than the normal (see de Vries(2001)). This phenomenon was first discovered by Mandelbrot (1963), in commodity markets. Since then, fat tails and excess kurtosis have been discovered in many other asset markets including the exchange market. In particular, in the latter the returns have a kurtosis typically exceeding 3[14] and a measure of fat tails (Hill index) ranging between 2 and 5 (see Koedijk, Stork and de Vries (1992), Huisman, et al.(2002)). It implies that most of the time the exchange rate movements are relatively small but that occasionally periods of turbulence occur with relatively large exchange rate changes. However, it has also been detected that the kurtosis is reduced under time aggregation. This phenomenon has been observed for most exchange rates (Lux(1998), Calvet and Fisher(2002)). We checked whether this is also the case with the simulated exchange rate changes in our model.

The model was simulated using normally distributed random disturbances (with mean = 0 and standard deviation = 1). We computed the kurtosis and the Hill index of the simulated exchange rate returns. We computed the Hill index for 4 different samples of 2000 observations. In addition, we considered three different cut-off points of the tails (2.5%, 5%, 10%). We show the results of the kurtosis and of the Hill index in table 1. We find that for a broad range of parameter values the kurtosis exceeds 3 and the Hill index indicates the presence of fat tails. Finally we check if the kurtosis of our simulated exchange rate returns declines under time aggregation. In order to do so, we chose different time aggregation periods and we computed the kurtosis of the time-aggregated exchange rate returns. We found that the kurtosis declines under time aggregation. In table 2 we show the results for some sets of parameter values[15]. This suggests that the speculative dynamics of the model transforms normally distributed noise in the exchange rate into exchange rate movements with tails that are significantly fatter than the normal distribution and with more density around the mean. Thus, our model mimics an important empirical regularity, i.e. that exchange rate movements are characterised by tranquil periods (oc-

[14] The normal distibution has a kurtosis index equal to 3.

[15] Another empirical regularity of the distribution of exchange returns is its symmetry. We computed the skewness, and we could not reject that the distribution is symmetric.

Table 1: Kurtosis and Hill index

	kurtosis	**median Hill index**		
Parameter values		**2.5% tail**	**5% tail**	**10% tail**
C=0, beta=0.9, gamma=0.001	6.9	6.1	5.4	4.4
C=0, beta=0.9, gamma=0.5	11.0	7.4	5.7	4.3
C=0, beta=0.9, gamma=1	48.1	7.1	5.5	4.3
C=0, beta=0.9, gamma=5	25.0	7.1	5.9	4.2
C=0, beta=0.8, gamma=0.001	6.9	6.6	5.2	4.4
C=0, beta=0.8, gamma=0.5	3.8	5.9	5.7	4.3
C=0, beta=0.8, gamma=1	9.3	7.3	5.7	4.6
C=0, beta=0.8, gamma=5	21.4	7.4	5.9	4.6

Table 2: Kurtosis and time aggregation

Parameter values	5 period returns	10 period returns	25 period returns	50 period returns
C=0, beta=0.9, gamma=0.001	177.7	30.9	2.9	3.2
C=0, beta=0.9, gamma=0.5	39.2	17.0	3.7	2.7
C=0, beta=0.9, gamma=1	43.3	33.4	6.7	2.9
C=0, beta=0.9, gamma=5	46.1	40.4	9.6	2.9
C=0, beta=0.8, gamma=0.001	185.7	14.3	8.5	2.5
C=0, beta=0.8, gamma=0.5	175.6	4.5	9.39	12.2
C=0, beta=0.8, gamma=1	74.1	16.2	3.0	2.2
C=0, beta=0.8, gamma=5	90.5	30.4	3.3	2.9

curring most of the time) and turbulent periods (occurring infrequently).This phenomenon has been also called *intermittency phenomenon* (see Lux(1998)).

6.2 The " excess volatility" puzzle

In this section we analyse another important empirical regularity, which has been called the "excess volatility" puzzle, i.e. the volatility of the exchange rate by far exceeds the volatility of the underlying economic variables. Baxter and Stockman (1989) and Flood and Rose (1995) found that while the movements from fixed to flexible exchange rates led to a dramatic increase in the volatility of the exchange rate no such increase could be detected in the volatility of the underlying economic variables. This contradicted the 'news' models that predicted that the volatility of the exchange rate can only increase when the variability of the underlying fundamental variables increases (see Obstfeld and Rogoff (1996) for a recent formulation of this model)[16].

[16]In addition, Goodhart (1989) and Goodhart and Figlioli (1991) found that most of the changes in the exchange rates occur when there is no observable news in the fundamental

In order to deal with this puzzle we compute the noise to signal ratio in the simulated exchange rate. We derive this noise to signal ratio as follows:

$$var(s) = var(f) + var(n) \tag{15}$$

where var(s) is the variance of the simulated exchange rate, var(f) is the variance of the fundamental and var(n) is the residual variance (noise) produced by the non-linear speculative dynamics which is uncorrelated with var(f). Rewriting (15) we obtain

$$\frac{var(n)}{var(f)} = \frac{var(s)}{var(f)} - 1 \tag{16}$$

The ratio $var(n)/var(f)$ can be interpreted as the noise to signal ratio. It gives a measure of how large the noise produced by the speculative dynamics is with respect to the exogenous volatility of the fundamental exchange rate. We simulate this noise to signal ratio for different values of the extrapolation parameter beta (see figure **??**). In addition, since this ratio is sensitive to the time interval over which it is computed we checked how it changes depending on the length of the time interval. In particular, we expect that the noise-to-signal ratio is larger when it is computed on a short than on a long time horizon. We show the results in figure **??** which assumes the same parameter configuration as **??**.

First, we find that with increasing β the noise to signal ratio increases. This implies that when the chartists increase the degree with which they extrapolate the past exchange rate movements, the noise in the exchange rate, which is unrelated to fundamentals, increases. Thus, the signal about the fundamentals that we can extract from the exchange rate becomes more clouded when the chartists extrapolate more. Second, we find that when the time horizon increases the noise-to-signal ratio declines. This is so because over long time horizons most of the volatility of the exchange rate is due to the fundamentals' volatility and very little to the endogenous noise. In contrast, over short time horizons the endogenous volatility is predominant and the signal that comes from the fundamentals is weak. This is consistent with the empirical findings following Meese and Rogoff(1983) celebrated studies. This literature tells us that when the forecasting horizon increases the performance of forecasting based on fundamentals tends to improve relative to random walk forecasting (see Mark(1995), Faust, et al. (2002)). .

economic variables. This finding contradicted the theoretical models (based on the efficient market hypothesis), which imply that the exchange rate can only move when there is news in the fundamentals.

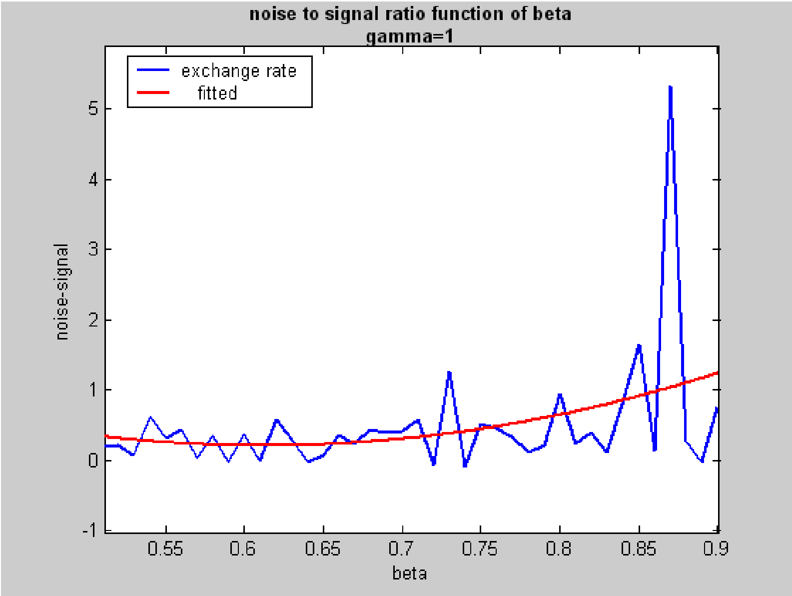

Figure 13:

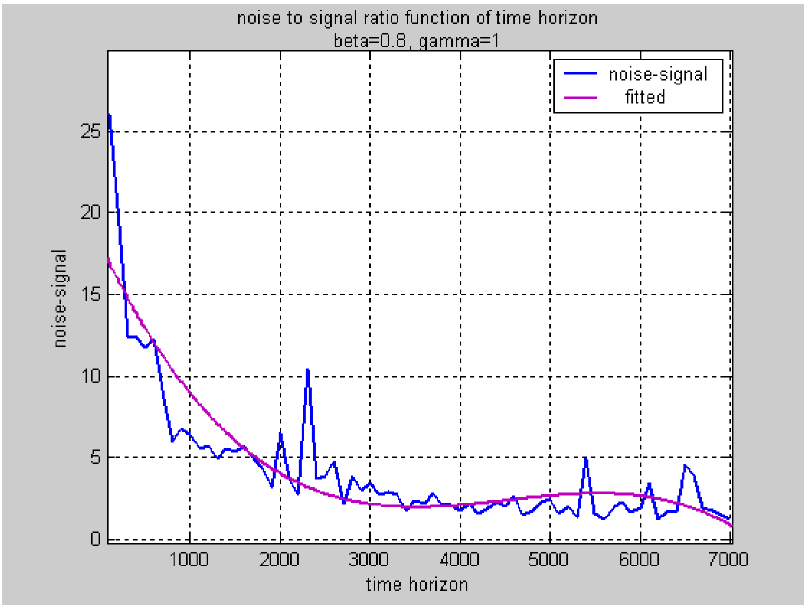

Figure 14:

7 Rationality and complexity

In the previous sections we discussed the statistical properties of our simulated exchange rates. We now analyse in more detail how those results are sensitive to the choice of γ, i.e. the rate of revision of the forecasts made by the agents. A first thing to note is that with a low γ our simulated exchange rates do not mimick the statistical properties of the exchange rate movements observed in reality (i.e. fat tails, excess kurtosis, excess volatility). Instead, with low values of γ, these simulated exchange rates exhibit statistical properties that are predicted by the efficient market rational expectations model. For example, with $\gamma = 0.001$ the variability of the simulated exchange rates is closely connected to the variability of the fundamentals and there is very little excess kurtosis. Thus in a way for low γ's it is as if the system is in a rational expectations equilibrium. Conversely, with increasing values of γ the variability of the simulated exchange rates is disconnected from the variability of the fundamentals, the distribution of exchange rate changes becomes leptokurtic and exhibits fat tails. We move into an equilibrium characterized by complexity. In this complex equilibrium the system departs from the predictions of the rational expectations model, and bubbles and crashes become possible. The interesting aspect of this result is that it is precisely when the system is in such an equilibrium that we come closest to the observed statistical properties of the exchange rate movements. This suggests that agents are sensitive to the relative profitability of forecasting rules, and that this behaviour is responsible for the observed statistical properties of the exchange rate returns.

We next compute how the accuracy and the fitness of the forecasting rules evolve for different values of γ i.e., for different rates of revision of the rules. We define the accuracy of a forecasting rule as the moving average of the forecasting errors (the difference between the forecast exchange rate and the actual exchange rate):

$$A_{i,t} = \sum_{k=1}^{\infty} \gamma_k \left[E_{t-k}^{i} \left(s_{t-k+1} \right) - s_{t-k+1} \right]^2 \tag{17}$$

It should be noted that the accuracy measure is the estimated variance in the forecasting rules (see equation 10) .

We also define a fitness measure. The fitness of a rule indicates how well the rule has been performing. It is similar to the accuracy except that the moving average of the forecasting errors is transformed as follows:

$$F_{i,t} = \begin{cases} D - A_{i,t} \, , & \text{if} \quad D > A_{i,t} \\ 0 & , \text{if} \quad D \leq A_{i,t} \end{cases} \tag{18}$$

where D is a threshold. From equation 18 it can be seen that the fitness measure captures only the forecasting errors larger than a certain threshold, D.

Table 3: Accuracy and Fitness forecasting rules

	Accuracy		**Fitness**	
Gamma	fundamentalists	chartists	fundamentalists	chartists
0.01	0.11	0.09	0	0
0.1	0.11	0.09	0	0
1	2.21	0.08	0.95	0
5	14.07	0.08	2.88	0
10	15.0	0.08	3.57	0

Table 4: Wealth as a function of gamma

Gamma	**chartists profits**	**fund. profits**	**total profits**
0.01	0.03	-0.03	0.00
0.1	0.04	-0.03	0.01
1	0.07	-0.04	0.03
5	0.12	-0.06	0.06
10	0.12	-0.06	0.06

In table 3 we represent the accuracy and the fitness measures of the forecasting rules.These were computed as the average accuracy and fitness measures obtained from running 100 simulations of 10000 periods.

It can be seen that the accuracy of the fundamentalists' forecasting rule and its fitness decrease with increasing γ. Conversely, the accuracy and the fitness of the chartists rule is practically not affected by increasing γ. This implies that under high γ regime , i.e. when agents revise their forecasting rules very frequently the accuracy and the fitness of the fundamentalists forecasting rules deteriorates significantly. This result is related to the increasing disconnection of the volatility of the exchange rate from the fundamenal variability and the increasing frequency of bubbles and crashes when γ increases.

An interesting issue is how the proftis of these different types of agents evolve with increasing γ. In table 4 we represent the profits of chartists and fundamentalists for different values of γ[17]. These are obtained in the same way as in table 3, i.e. as averages obtained from 100 simulation runs of 10000 periods.

The remarkable result is that as γ increases the profits made by chartists increase significantly while the profits of the fundamentalists decline. The gains of the chartists largely outweight the losses of the fundamentalists, and this is increasingly the case as γ increases. Thus in a complex environment total market profits are high, which is the result of large average profits of the chartists which

[17] Note that the fundamentalists' profits are net of the fixed costs of collecting information C, which was set equal to 0.05 in the simulations reported here.

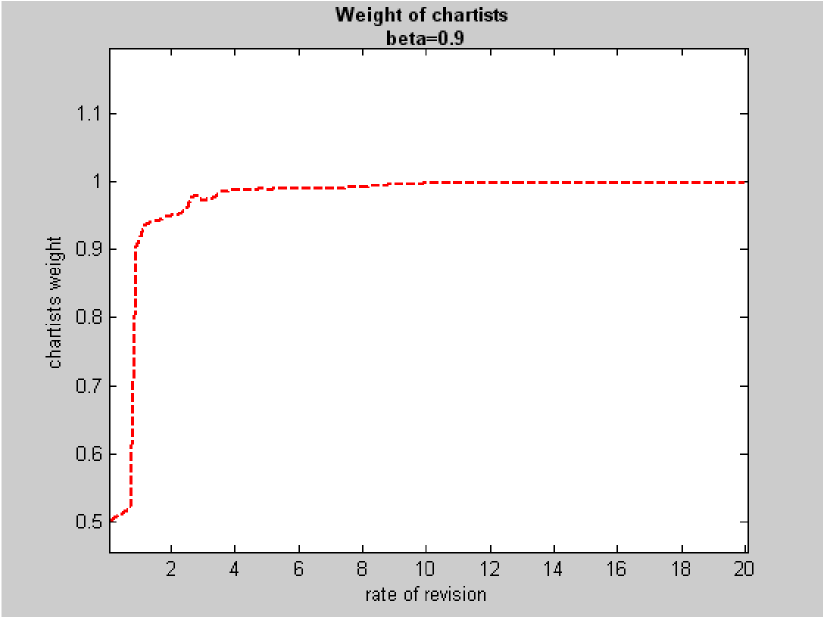

Figure 15:

more than compensates the losses of the fundamentalists. An interpretation of this result is that in a complex environment, the fundamentalists' forecasting rules loose much of their accuracy, making these rules unprofitable. Conversely, the chartists are better off in a noisy environment. This, in turn, leads to a higher profits. The result of all this is that as γ increases, the the chartists increasingly dominate the market. For sufficiently high values of γ the chartists' weight converges to 1 and the fundamentalists tend to disappear from the market. We show this feature in figure **??** where we plot the weight of the chartists as a function of γ (*rate of revision*).

Next, we analyse how frequently technical traders anf fundamentalists make profits and losses depending on the rate of revision, γ. In order to do so, we proceed as follows. First, we compute profits and losses. Second, we consider only profits and losses that are larger than a certain threshold that we set equal to 1.5 . Third, we compute how many times large profits and losses occur. The results are summarised in table5

For low values of γ both chartists and fundamentalists never face large losses and never make large profits. Thus, the size profits and losses and the frequency with which they are made is very much alike for both chartists and fundamentalists. However, for increasing values of γ agents differ greatly for their ability to make exceptionally large profits and losses. For values of $\gamma >1$, fundamen-

Table 5: Number of periods of large profits and large losses as a function of gamma (in percent)

Gamma	**Profits > 1.5**		**Losses < -1.5**	
	chartists	fundamentalists	chartists	fundamentalists
0.001	0.001	0	0	0
0.01	0	0	0	0
0.1	0.03	0	0	0
1	0.01	0.01	0	0
1.5	0.13	0	0	0.14
5	0.13	0	0	0.14
10	0.14	0	0	0.15

talists make large losses more frequently than chartists. For example, for $\gamma = 10$ they make losses, approximately, 0.15% of the time. Conversely chartists never face such large losses for any value of γ. Instead when γ is large, chartists make exceptionally large profits more frequently than fundamentalists. This asymmetry is related to the occurrence of bubbles. As we have seen, when γ is high, bubbles occur more frequently. This is when chartists can make exceptionally large profits, while fundamentalists then make very large losses. When the bubble crashes, chartists make losses, however, these remain limited as the chartists quickly ride on the downward trend in the market. The interesting feature of this result is that, in contrast to fundamentalist rules, chartism is a forecasting rule that has an built-in insurance against large losses. This is also one of the reasons why it tends to dominate in the market.

8 Conclusion

Bubbles and crashes in financial markets in general, and in the foreign exchange markets in particular, have occurred frequently, often with devastating effects. In this paper we provide a framework to analyse the emergence and the subsequent disappearance of bubbles in the foreign exchange market. We use a very simple model in which agents use an optimal portfolio in the mean-variance utility framework. The special feature of our model is that individual agents recognize that they are not capable of understanding and processing the complex information structure of the underlying model. As a result, they use simple rules to forecast the exchange rates. None of these rules is rational in the technical sense. Yet we claim that these agents act rationally within the context of the uncertainty they face. That is, agents check the 'fitness' (profitability) of the forecasting rule at each point in time and decide to reject the rule if it is less profitable (in a risk adjusted sense) than competing rules. Our model is in the tradition of evolutionary dynamics where agents use trial and error strategies. We assume that some of the forecasting rules are based on extrapolating past exchange rate movements (chartism) and others are based on mean reversion towards the fundamental rate.

The model generates two types of equilibria. The first one, which we called a fundamental equilibrium, is one in which the exchange rate converges to its fundamental value. The exchange rate, however, can also converge to a second type of equilibrium, which we called a bubble equilibrium, and which is reached in a self-fulfilling manner. An important feature of the bubble equilibrium is that chartism (extrapolative forecasting) takes over most of the market. We simulated the model in a stochastic environment and generated complex scenarios of bubbles and crashes. One interesting aspect of the model is that it explains both the emergence of the bubble and its subsequent crash.

We also analysed under what conditions bubbles and crashes occur. We find that when agents react quickly to changing relative profitabilities of the different forecasting rules, the frequency of bubbles increases. We also find that in this case the exchange rate dynamics is very complex and produces features such as fat tails, excess kurtosis and excess volatility; features that are also found in reality. In contrast, in order for our model to mimick the statistical properties of the exchange rate that are predicted by the rational expectations efficient market model, agents must be very slow in reacting to relative profitabilities of the forecasting rules.

Next we analysed the size of profits and losses and the frequency with which they are made. We found that there is an important asymmetry depending on how frequently agents revise their forecasting rules. When agents revise their rules frequently then fundamentalists make large losses more frequently than chartists. When agents revise their rules with a low frequence then both chartists and fundamentalists alike avoid large losses and large profits. This asymmetry is related to the occurrence of bubbles, which are sources of large profits for chartists using extrpolating forecasting rules and large losses for fundamentalists. The interesting feature of this result is that, in contrast to fundamentalist rules, chartism is a forecasting rule that has an built-in insurance against large losses. This is also one of the reasons why it tends to dominate in the market.

We also found that bubbles are more likely to emerge when the variance of the noise is high relative to the variance of the fundamental variable. Thus, policies that lead to less variability of the underlying fundamentals (e.g. inflation) do not necessarily reduce the probability of bubbles.

Finally we tested our model in the sense that we reproduced the statistical properties of exchange rate changes observed in reality, i.e. excess volatility, excess kurtosis, fat tails. Invariably we find that the parameter values that best mimick these properties are also those that produce significant probabilities of bubbles and crashes.

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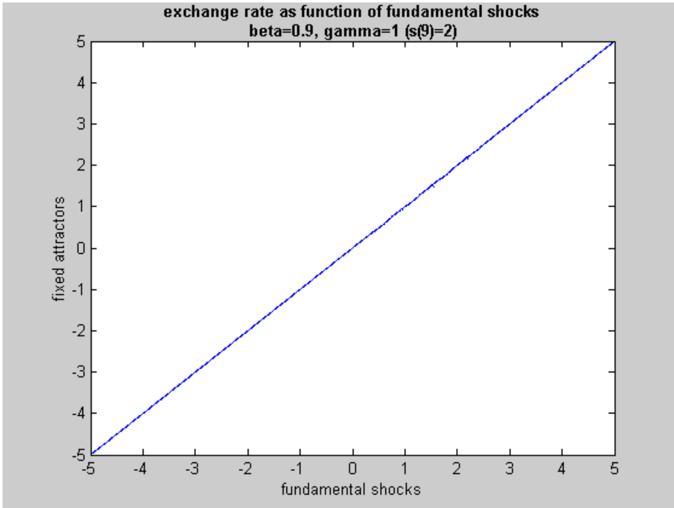


Figure 16:

10 Appendix 1: Fixed attractors and fundamental shocks: additional results

In this appendix we present additional simulations of the effect of shocks in the fundamental on the exchange rate. We assume different values of the initial conditions. The results are shown in figures A1, A2 and A3. When the initial condition (noise) is small (figure A1) no bubble equilibria exist and the exchange rate always coincides with its fundamental value. When the initial condition is gradually increased (figures A2 and A3) the range of bubble equilibria progressively increases.

11 Appendix 2: Causality tests between exchange rate and chartist weight

In this appendix we present the results of causality tests between the exchange rate and the weight of chartists during a bubble and crash episode. We simulated the model using the standard set of parameters, and we selected an episode during which a bubble and crash occurred. We show such an episode in figure A2. A visual inspection of the graph reveals that the exchange rate appears to lead the chartist weight. at least when the bubble starts and later when the bubble bursts. Note also that the crash occurs faster than the bubble phase, a feature we often find in our simulated bubbles and crashes. This has also been found in empirical data (see Sornette(2003))

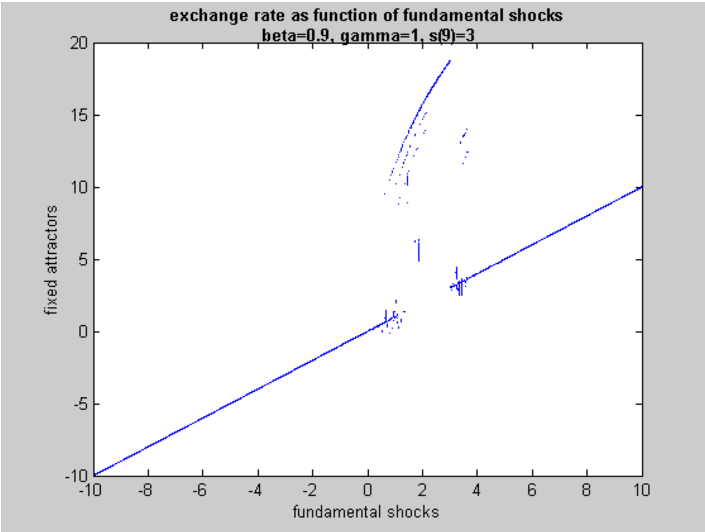


Figure 17:

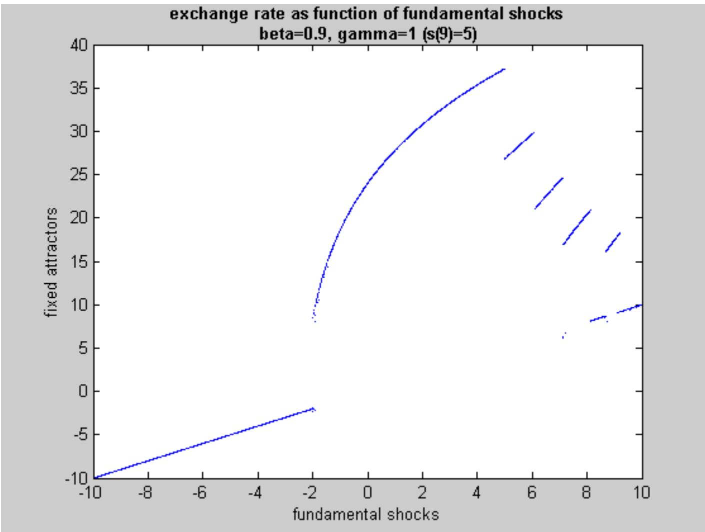


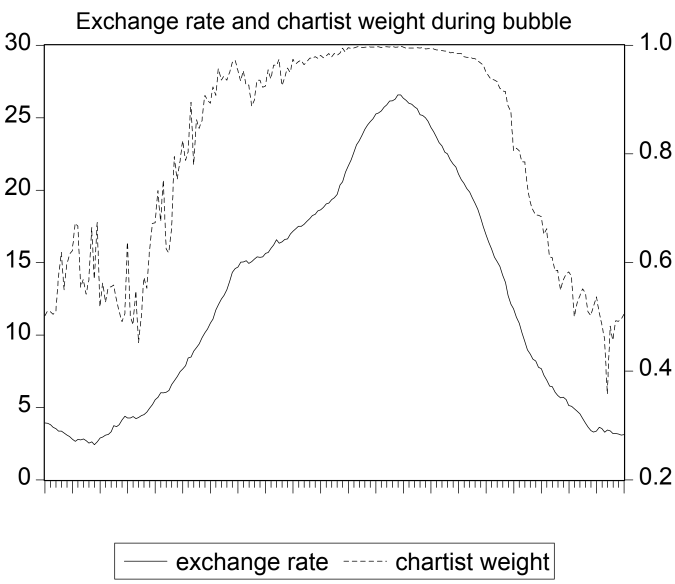
Figure 18:

Table 6: Granger causality tests

Null Hypothesis:	F-statistic	Probability
cw not Granger cause exchange rate	0.377	0.865
exchange rate not Granger cause cw	6.85	6.4E-06

Note: obs=211, lags=5.

Next we performed a Granger causality test on the exchange rate and the chartist weight during the bubble and crash episode represented in figure A2[18]. The result of this causality test is presented in table A1. We observe that we cannot reject the hypothesis that the exchange rate leads the chartists' weight during the bubble and crash episode, while we can reject the reverse. We find this feature in most bubble and crash episodes.



[18] We checked for stationarity and could not reject that the two series are stationary during the sample period.