

# An Empirical Equilibrium Model of a Decentralized Asset Market\*

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## Abstract

I estimate a search-and-bargaining model of a decentralized market to quantify the effects of trading frictions on asset allocations and asset prices, and to quantify the effects of intermediaries that facilitate trade. Using business-aircraft data, I find that, relative to the Walrasian benchmark, 12 percent of the assets are misallocated, and prices are approximately 24.5-percent lower. Dealers play an important role in reducing frictions: In a market with no dealers, 15.2 percent of the assets would be misallocated. Perhaps surprisingly, in a market with no dealers, prices would *increase* by 1.7 percent, because sellers' outside options improve relative to buyers', thus counteracting the effects of higher search costs and slower trade on asset prices.

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# 1 Introduction

How large are trading frictions? How do they affect the allocations of assets? Their prices? What is the role of intermediaries in reducing trading frictions? This paper estimates a structural model of a decentralized market to provide quantitative answers to these questions.

Many assets trade in decentralized markets. Classic examples are financial assets such as bonds and derivatives, consumer durable goods such as cars and houses, and firms' capital assets such as plants and equipment. The fundamental characteristics of decentralized markets are that agents must search for trading partners and that, once a buyer and a seller meet, they must bargain to determine a price. Moreover, in response to trading frictions, almost all decentralized markets have intermediaries. Indeed, starting with Demsetz (1968), trading frictions have been used to explain the existence and behavior of intermediaries. The key role of intermediaries in such markets is to reduce frictions, thereby facilitating trade.

In this paper, I lay out a model of trading in decentralized markets with two-sided search and bilateral bargaining to formalize the effects of trading frictions on asset allocations and prices, and the effects of intermediaries in alleviating these frictions. I then quantify the role of frictions and of dealers, estimating this model using data on business jet aircraft.

The theoretical framework combines elements from Rubinstein and Wolinsky (1987) and Duffie, Gârleanu and Pedersen (2005), adapting them to capture key features of real asset markets. A flow of agents enters the market in every period, seeking to acquire an aircraft. They contact sellers at a rate that depends on the mass of aircraft for sale and on traders' search ability, and they contact dealers at another rate that reflects dealers' inventories and search ability. Dealers hold inventories because they do not meet buyers and sellers simultaneously (Grossman and Miller, 1988), and they extract surplus by shortening the time that buyers and sellers have to wait in order to trade (Rubinstein and Wolinsky, 1987). When agents meet or meet dealers, they bargain over the terms of trade. Gains from trade arise from heterogeneous valuations of holding the aircraft. For example, an aircraft owner, usually a corporation, wishes to sell because its valuation for the aircraft has dropped, perhaps because the firm's profitability has declined. The model analytically derives the equilibrium allocation of aircraft to agents and to dealers, as well as the price negotiated

between agents and the price negotiated between agents and dealers. Equilibrium allocations and prices depend in an intuitive way on agents' and dealers' search abilities, bargaining powers and valuations.

I estimate the model by using data on the secondary market of business jet aircraft—a typical decentralized market. The data are well-suited to studying the effects of search frictions and the role of intermediaries. In particular, they report the number of aircraft for sale and number of aircraft transactions in each month, and their ratio is informative on the magnitude of trading delays. Similarly, the data report dealers' inventories and dealers' transactions, and their ratio is informative on the role of dealers in reducing delays. In addition, the data report two series of prices: retail prices between final users of the aircraft; and wholesale prices between aircraft owners (as sellers) and dealers (as buyers). Their differences are useful in understanding how much dealers are able to command by supplying immediacy of trade (and, thus, sellers are willing to forego).

The estimation reveals that trading delays are non-trivial: On average, aircraft stay on the market approximately five months before a seller is able to finalize a sale. The quantitative importance of these delays depends on how frequently agents seek to trade, determined by a drop in their valuations; this happens, on average, every four and a half years. Moreover, the estimation implies that the “short” sides of the market—i.e., sellers and dealers, in particular—enjoy strong bargaining powers, capturing almost the entire surplus of transactions.

I use the estimated parameters to simulate two counterfactual scenarios. In the first one, I quantify the role of trading frictions on asset allocations and prices by computing a Walrasian market equilibrium. The estimates imply that trading frictions generate moderate inefficiencies: Compared to the Walrasian benchmark, 12 percent of all business aircraft are misallocated, and aircraft prices are 24.5-percent lower. Two forces affect allocations and prices in the estimated model relative to this Walrasian benchmark: search costs and trading delays. The parameter estimates imply that trading delays quantitatively account for almost all differences, and the effect of search costs on allocations and prices is negligible. In the second counterfactual, I quantify the effect of dealers on the decentralized market equilibrium computing a decentralized market with no dealers. The estimates imply that dealers play an important role in reducing frictions: In a market without them, 15.2 percent

of the assets would be misallocated. Perhaps surprisingly, in a market with no dealers prices would *increase* by 1.7 percent. The reason is that sellers' outside options improve relative to buyers', thus counteracting the effects of higher search costs and slower trade on asset prices. Therefore, an interesting conclusion of this counterfactual analysis is that it may be difficult to infer exclusively from asset prices the effects of changes in trading frictions (or, more generally, changes in trading mechanisms and institutions) on market inefficiencies.

This paper makes two main contributions. First, the paper provides a framework suited to empirically analyzing decentralized asset markets. Search models have proved to be useful in understanding key features of labor markets, and, more recently, researchers have started to apply search models to financial markets. To my knowledge, this paper is the first to estimate a bilateral search model that investigates the microstructure of the market of a capital asset/durable good, quantifying the effects of trading delays and of intermediaries that facilitate exchange. Second, the empirical findings suggest that, even within a well-defined asset class such as business aircraft, trading frictions are a non-trivial impediment to the efficient allocation of assets and have significant effects on asset prices. Thus, the paper innovates on recent works that study the process of asset reallocations (Ramey and Shapiro, 1998, 2001; Maksimovic and Phillips, 2001; Schlingemann et al., 2002; Eisfeldt and Rampini, 2006; Gavazza, 2011a and 2011b. For an empirical model of business transfers, see, also, Holmes and Schmitz, 1995) by quantifying its inefficiencies. This is a necessary step to understanding how asset markets work and, therefore, to the design of any policy that affects them.

The paper proceeds as follows. Section 2 reviews the literature. Section 3 presents some institutional details on the business-aircraft market. Section 4 introduces the data. Section 5 presents the theoretical model and Section 6 estimates it. Section 7 performs counterfactual analysis. Section 8 concludes. The appendices include analytic derivations and additional estimates of the model on different sample periods.

## 2 Related Literature

This paper contributes to the important literature that analyzes decentralized markets. The theoretical literature is vast. The most closely related papers examine bilateral search

markets, in which both buyers and sellers search for a trading counterpart and prices are determined through bilateral bargaining (Rubinstein and Wolinsky, 1985 and 1987; Gale, 1987; Mortensen and Wright, 2002; Duffie, Gârleanu and Pedersen, 2005 and 2007; Miao, 2006). The main focus of these theoretical papers is to investigate whether the equilibrium converges to the competitive outcome as frictions vanish. To my knowledge, this paper is the first to estimate a bilateral search model of a real asset market to quantify the effects of trading frictions—i.e., the focus of the theoretical literature.

The paper further contributes to the literature on intermediaries. Several papers investigate the role of brokers/dealers in financial markets and their inventory-management policies; for a survey, see O’Hara (1995). Spulber (1999) presents a thorough analysis of intermediaries between customers and suppliers. Weill (2007) presents a search-and-bargaining model to understand how intermediaries provide liquidity by accumulating inventories when selling pressure is large, and then dispose of those inventories after that selling pressure has subsided. Recent empirical analysis of non-financial intermediaries include Hall and Rust (2000), who focus on the inventory investment of a single steel wholesaler, and Gavazza (2011a), who focuses on the role of lessors in reallocating commercial aircraft. To my knowledge, this paper presents the first structural empirical analysis of the role of intermediaries in a search-and-bargaining framework.

This paper is also related to the empirical literature on the structural estimation of search models. Most applications focus on labor markets. Eckstein and Wolpin (1990) pioneered this literature by estimating the model of Albrecht and Axell (1984). Van den Berg and Ridder (1998) and Bontemps, Robin, Van der Berg (1999, 2000) estimate the Burdett and Mortensen (1998) model. Other recent applications include Postel-Vinay and Robin (2002) and Cahuc, Postel-Vinay and Robin (2006). Eckstein and van der Berg (2006) provide a thorough survey of this literature. One key difference between the current paper and previous research is that this paper seeks to understand how search frictions affect the *level* of asset prices and of asset allocations, while most papers that structurally estimate search models of the labor market focus on how search frictions affect wage *dispersion* across workers (notable exceptions are Gautier and Teulings, 2006 and 2010). Search models have also been applied to housing markets, and Carrillo (forthcoming) is the closest empirical paper.

Finally, this paper is related to a few papers that investigate aircraft transactions. Using

data on commercial aircraft transactions, Pulvino (1998, 1999) finds that airlines under financial pressure sell aircraft at a 14-percent discount, and that distressed airlines experience higher rates of asset sales than non-distressed airlines. Using data on business jets similar to the data that I use in this paper, Gilligan (2004) finds empirical evidence consistent with the idea that leasing ameliorates the consequences of information asymmetries about the quality of used aircraft. Gavazza (2011b) empirically investigates whether trading frictions vary with the size of the asset market in commercial aircraft markets. However, none of these papers quantifies the importance of trading frictions estimating a structural model.

### 3 Business-Aircraft Markets

For several reasons, the business-aircraft market is an interesting context in which to investigate the effects of search frictions and the role of intermediaries.

First, used business aircraft trade in decentralized markets, organized around privately-negotiated transactions. Almost all buyers (and sellers) are wealthy individuals or corporations, that use the aircraft to fly their executives. To initiate a transaction, a prospective seller must contact multiple potential buyers or can sell its aircraft to a dealer. For buyers, comparing two similar aircraft for sale is costly since aircraft sales involve the material inspection of the aircraft, which could be in two different locations. Thus, aircraft markets share many features with other over-the-counter markets for financial assets (mortgage-backed securities, corporate bonds, bank loans, derivatives, etc.) and for real assets (real estate), in which trading involves material and opportunity costs (Duffie, Gârleanu and Pedersen, 2005 and 2007). Moreover, compared to financial markets and other equipment markets, business-aircraft markets are “thin”: Slightly more than 17,000 business jet aircraft were operated worldwide in December 2008. In thin markets, the search costs to find high-value buyers are usually large (Ramey and Shapiro, 2001; Gavazza, 2011b).

Second, intermediaries play an important role in mediating transactions. Most intermediaries operate as brokers, matching buyers and sellers. Some larger intermediaries operate also as dealers, acquiring aircraft for inventories.

Finally, business aircraft are registered goods with all major “life” events (date of first flight, maintenance, scrappage, etc.) recorded, so detailed data are available. In the next

section, I describe them.

## 4 Data

Patterns in the data suggest that trading delays are an important feature of aircraft markets. Moreover, the available data dictate some of the modeling choices of this paper. Hence, I describe the data before presenting the model. This description also introduces some of the identification issues that I discuss in more detail in Section 6.

### 4.1 Data Sources

I combine two distinct datasets. The first is an extensive database that tracks the history of business-aircraft transactions. The second reports the average values of several aircraft models, similar to “Blue Book” prices. I now describe each dataset in more detail.

*Aircraft Transactions*—This database is compiled by AMSTAT, a producer of aviation-market information systems.<sup>1</sup> It provides summary reports that track business-aircraft market transactions. For each month from January 1990 to December 2008 and for each model (e.g., Cessna Citation V or VI), the dataset reports information on the active fleet (e.g., the number of active aircraft; number of new deliveries); information on aircraft for sale (e.g., the number of aircraft for sale, by owners and by dealers; the average vintage); and information on completed transactions (e.g., the total number of transactions; the number of retail-to-retail, retail-to-dealer, dealer-to-dealer and dealer-to-retail transactions). The dataset also reports detailed aircraft characteristics of each model (e.g., average number of seats, maximum range, fuel consumption). I restrict the analysis to business jets, thus excluding turbo-propellers.

*Aircraft Prices*—I obtained business-aircraft prices from the *Aircraft Bluebook Historical Value Reference*.<sup>2</sup> This dataset is an unbalanced panel, reporting quarterly historic values

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<sup>1</sup>[http://www.amstatcorp.com/pages/pr\\_stat.html](http://www.amstatcorp.com/pages/pr_stat.html). The website mentions that: “AMSTAT’s customers are aircraft professionals, whose primary business is selling, buying, leasing and/or financing business aircraft, as well as providers of related services and equipment.”

<sup>2</sup>The dataset is available at <http://www.aircraftbluebook.com>. The website describes the dataset as: “The Aircraft Bluebook Historical Value Reference is specifically designed for lease companies, bankers, aircraft dealers, or anyone who needs to know the pricing history of an individual aircraft.”

of different vintages for the most popular business-aircraft models during the period 1990-2008. Two series are reported: average retail prices and average wholesale prices. Average retail prices report prices between final users of the aircraft, and average wholesale prices report average transaction prices between an aircraft owner (as a seller) and a dealer (as a buyer). All prices are based on the company's experience in consulting, appraisal and fleet evaluation. All values are in U.S. dollars, and I have deflated them using the GDP Implicit Price Deflator, with 2005 as the base year.

It is important to note that the construction of the *Aircraft Bluebook Historical Value Reference* implies that the retail and wholesale price series are free of several biases. First, wholesale prices refer to sales *to* dealers, thus *before* dealers could make any improvement to the aircraft. Second, the database reports historical retail and wholesale prices even for those model-vintage pairs for which only one unit (i.e., one serial number) exists, indicating that the price series are not affected by sellers' selection based on aircraft quality (observable or unobservable to both trading parties).

## 4.2 Data Description

Table 1 provides summary statistics of the main variables used in the empirical analysis. Panel A refers to the Aircraft Transactions Dataset, a sample containing 161 models (the definition of a model is quite fine in this dataset), comprising a total of 26,237 aircraft model-month observations. For each model, the stock of ACTIVE AIRCRAFT equals approximately 90 units, of which approximately ten are on sale in a given month. The number of transactions is small relative to the number of aircraft on sale: On average, there are 0.58 RETAIL-TO-RETAIL TRANSACTIONS and 0.83 DEALER-TO-RETAIL TRANSACTIONS per model-month pair (I include Lease transactions in DEALER-TO-RETAIL TRANSACTIONS). There are also a few DEALER-TO-DEALER TRANSACTIONS, 0.28 per month, on average, suggesting that dealers smooth their inventories by trading with other intermediaries. The TOTAL NUMBER OF TRANSACTIONS, defined as the sum of RETAIL-TO-RETAIL TRANSACTIONS and DEALER-TO-RETAIL TRANSACTIONS, indicates that, on aggregate, approximately 1.5 aircraft per model trades in a given month and that a dealer is the seller to the final retail user in approximately 60 percent of these transactions.

The average age of aircraft for sale is 18.43 years, with a large dispersion across observa-

TABLE 1: Summary statistics

PANEL A: AIRCRAFT TRANSACTIONS	OBS.	MEAN	ST. DEV.	MEDIAN
MODELS OF AIRCRAFT	26,237	161		
ACTIVE AIRCRAFT	26,237	89.79	107.28	49
AIRCRAFT FOR SALE	26,237	10.91	15.11	6
–BY OWNERS	26,237	7.75	11.94	4
–BY DEALERS	26,237	3.16	4.78	2
AVERAGE AGE, AIRCRAFT FOR SALE	23,225	18.43	11.09	19
RETAIL-TO-RETAIL TRANSACTIONS	26,237	0.58	1.12	0
RETAIL-TO-DEALER TRANSACTIONS	26,237	0.83	1.48	0
DEALER-TO-DEALER TRANSACTIONS	26,237	0.28	0.78	0
DEALER-TO-RETAIL TRANSACTIONS	26,237	0.83	1.50	0
TOTAL NUMBER OF TRANSACTIONS	26,237	1.42	2.29	0
PANEL B: AIRCRAFT PRICES				
MODELS OF AIRCRAFT	31,524	72		
RETAIL PRICE (IN \$1,000)	31,524	7,607	8,534	4,343
WHOLESALE PRICE (IN \$1,000)	31,524	6,731	7,555	3,849
AGE (IN YEARS)	31,524	14.43	9.69	13.25

Notes—This table provides summary statistics of the variables used in the empirical analysis. Panel A presents summary statistics for the Aircraft Transactions dataset. Each observation represents a model-month pair. Panel B presents summary statistics for the Blue Book prices dataset. Each observation represents a model-vintage-quarter tuple. Aircraft prices are in thousands of U.S. dollars and have been deflated using the GDP Implicit Price Deflator, with 2005 as the base year.

tions (model-month pairs).<sup>3</sup> This dispersion indicates that aircraft of many vintages are on the market simultaneously. For example, Figure 1 plots the histogram of the average ages across models of aircraft on the market in January 2001, showing that the youngest and the oldest vintages are frequently on the market for sale. This pattern is in stark contrast with the evidence from other asset markets in which replacement purchases are the main motives for trade, such as, for example, the car market. In particular, in the car market, the fraction of cars sold increases and then decreases with the age of the car and, thus, is lowest for the youngest and the oldest vintages (Stolyarov, 2002; Gavazza and Lizzeri, 2010).<sup>4</sup> Hence,

<sup>3</sup>The number of observations of AVERAGE AGE, AIRCRAFT FOR SALE is lower than all other variables because there are no aircraft for sale in some model-month pairs.

<sup>4</sup>The intuition for this result in car markets is that, because of transaction costs, only few households sell their cars after one or two periods, so the resale rates are increasing when cars are relatively new. In addition, consumers are unlikely to purchase old cars that they would be scrapping after just a few periods upon purchase, so the resale rates are decreasing when cars are old.

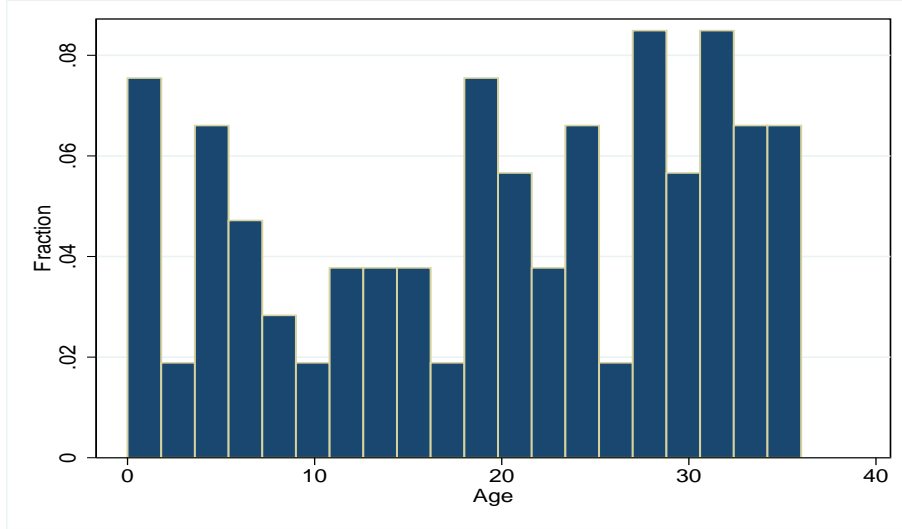


FIG. 1: Histogram of the average ages across models of aircraft on the market in January 2001.

Figure 1 suggests that replacement is not the first-order motive for trade.

Panel B provides summary statistics for the Aircraft Prices dataset. This sample contains 72 models (indicating that the definition of a model is coarser than in the Aircraft Transactions Dataset), comprising a total of 31,524 aircraft model-vintage-quarter observations. The average RETAIL PRICE of an aircraft in the sample is 7.6 million (year 2005) dollars, and the average WHOLESALE PRICE is 6.7 million (year 2005) dollars. Moreover, there is substantial variation in both prices: The standard deviation of retail prices is 8.5 million dollars and of wholesale prices is 7.5 million dollars.

The two datasets provide a rich description of the business-aircraft market and are well-suited to investigating the importance of frictions and the role of dealers. Specifically, three key patterns suggest that trading delays may be non-trivial and that dealers reduce them. First, the difference between the number of AIRCRAFT FOR SALE (on average, 11 aircraft per model-month) and the TOTAL NUMBER OF TRANSACTIONS (on average, 1.5 per model-month) means that aircraft stay on the market for several months before selling, indicating that trading delays are substantial. Second, the ratio between RETAIL-TO-DEALER TRANSACTIONS and AIRCRAFT FOR SALE BY DEALERS is higher than the ratio between TOTAL NUMBER OF TRANSACTIONS and AIRCRAFT FOR SALE, suggesting that dealers are faster than owners at turning aircraft over.<sup>5</sup> Third, the difference between the RETAIL PRICE and

<sup>5</sup>These differential trading patterns are difficult to explain through a model of “thin” markets—i.e.,

the WHOLESALE PRICE is quite large (on average, 13 percent), corroborating that frictions are relevant in this market and that dealers are able to command a substantial markup by supplying immediacy of trade.

With all their advantages, however, the datasets pose some challenges. In my view, the main limitation is that both datasets provide aggregate statistics of the market for different models. This limitation implies that a model with rich heterogeneity of agents and dealers, while theoretically feasible, would not be identified with these aggregated data. For example, the aircraft-price dataset does not allow the identification of rich heterogeneity in asset valuations, which would be possible if transaction prices were available. Therefore, the model will allow a parsimonious binary distribution of valuations, high and low. In Section 8, I will discuss the implications of this limited heterogeneity for the interpretation of the empirical results. Similarly, the aircraft-transaction dataset reports only aggregate dealers' inventories, limiting the possibility of identifying heterogeneity across dealers. Finally, an additional limitation is that the aircraft-transaction dataset does not report if retail buyers or sellers hired a broker to search for trading counterparts, although these intermediaries are popular in business-aircraft markets.

## 5 Model

In this section, I lay out a model of a decentralized market with two-sided search to theoretically investigate the effects of search frictions on asset allocations and prices. The model combines elements from Rubinstein and Wolinsky (1987) and Duffie, Gârleanu and Pedersen (2005), adapting them to capture key features of real asset markets, in a similar way to that of a growing literature that applies Diamond's (1982) seminal paper to financial markets.

I model frictions of reallocating assets explicitly. In particular, each agent contacts another agent randomly, and this is costly for two reasons: 1) there is an explicit cost  $c_s$  of searching, and 2) there is a time cost in that all agents discount future values by the discount rate  $\rho > 0$ .

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markets with a very small number of buyers and sellers wishing to trade at non-perfectly synchronized times. The differential patterns are also comparable between more- and less-popular aircraft.

## 5.1 Assumptions

Time is continuous and the horizon infinite. A mass  $\mu$  of risk-neutral agents enters the economy at every instant. All entrants have an exogenous parameter  $z = z_h$  that measures their valuation for an aircraft. This valuation parameter is a Markov chain, switching from  $z_h$  to  $z_l < z_h$  with intensity  $\lambda$ ;  $z_l$  is an absorbing state. The valuation processes of any two agents are independent. Hence, the mass of high-valuations agents is equal to  $\frac{\mu}{\lambda}$ .

There is a constant flow  $x$  of new aircraft entering the economy in every instant. Aircraft depreciate at the instantaneous rate  $\delta_2$  and are scrapped when they reach age  $T$ . Thus, the total mass  $A$  of aircraft is equal to  $xT$ . An aircraft of age  $a$  generates an instantaneous flow of utility equal to  $z(\delta_0 + \delta_1 e^{-\delta_2 a})$  to its owner with valuation  $z$ . I assume that  $A < \frac{\mu}{\lambda}$ , which implies that the “marginal” owner in a Walrasian market is a high-valuation agent.

Each agent can own either zero or one aircraft.<sup>6</sup> Agents can trade aircraft: A given agent wishing to trade (either a buyer or a seller) makes contacts with other agents pairwise independently at Poisson arrival times with intensity  $\gamma > 0$ . Thus, given that matches are determined at random, the arrival rate  $\gamma_s$  for a seller (the rate at which he meets buyers) is  $\gamma_s = \gamma\mu_b$ , and the arrival rate  $\gamma_b$  for a buyer is  $\gamma_b = \gamma\mu_s$ , where  $\mu_b$  and  $\mu_s$  are the endogenous equilibrium masses of buyers and sellers, respectively. In addition, there is an endogenous mass  $\mu_d$  of independent used-aircraft dealers that meets traders also through a search process. Each dealer has a flow cost equal to  $k$  and has, at most, one unit of inventory. An agent wishing to trade meets dealers pairwise independently at Poisson arrival times with intensity  $\gamma' > 0$ . Thus, a buyer meets a dealer with an aircraft to sell at rate  $\gamma_{bd} = \gamma'\mu_{do}$ , and a dealer with an aircraft to sell meets a buyer at rate  $\alpha_{ds} = \frac{\gamma_{bd}\mu_b}{\mu_{do}} = \gamma'\mu_b$ , where  $\mu_{do}$  is the endogenous mass of dealers with an aircraft to sell. The rate  $\gamma_{bd}$  and  $\alpha_{ds}$  represent the sum of the intensity of buyers’ search for dealers and dealers’ search for buyers. A seller meets a dealer willing to buy an aircraft at rate  $\gamma_{sd} = \gamma'\mu_{dn}$ , and a dealer willing to buy an aircraft meets a seller at rate  $\alpha_{db} = \frac{\gamma_{sd}\mu_s}{\mu_{dn}} = \gamma'\mu_s$ , where  $\mu_{dn}$  is the endogenous mass of dealers with an aircraft to sell. There is free entry into the dealers’ market.

Once a buyer and a seller meet, or they meet a dealer, parties negotiate a price to trade. I assume that a buyer and a seller negotiate a price according to generalized Nash bargaining,

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<sup>6</sup>Hence, I do not consider quantity decisions, as do Duffie, Gârleanu and Pedersen (2005 and 2007), Miao (2006), Vayanos and Wang (2007), Vayanos and Weill (2008), and Weill (2007 and 2008). See Lagos and Rocheteau (2009) for a model that considers quantity decisions.

where  $\theta_s \in [0, 1]$  denotes the bargaining power of the seller. Similarly, when an agent meets a dealer, they negotiate a price, and  $\theta_d \in [0, 1]$  denotes the bargaining power of the dealer.<sup>7</sup>

## 5.2 Solution

There are four types of agents in the model economy and two types of dealers: high- and low- valuation owners and non-owners, and dealers with and without an aircraft for sale. I denote their types  $ho, lo, hn, ln, do, dn$ , respectively.

The owner of an aircraft with valuation  $z$  can put it up for sale or keep operating it. In the former case, he meets potential trading partners at rate  $\gamma_s$  and a dealer at rate  $\gamma_d$ . In the latter case, he enjoys the flow profit  $z$ . Similarly, an agent with no aircraft can meet active sellers at rate  $\gamma_b$  and a dealer at rate  $\gamma_d$ , or can exit the market. Intuitively, agents' optimal choices depend on their valuation: Non-owners prefer to search when their valuation is high, and owners prefer to sell their aircraft when their valuations are low. Hence, gains from trade arise only when low-valuation owners meet high-valuation non-owners. When these agents meet, they trade.

I now formally derive the value functions for all agents of the economy and the transaction price at which trade occurs.<sup>8</sup> These value functions allow us to pin down the equilibrium conditions and derive the endogenous distribution of owners' valuations. This distribution describes in an intuitive way how frictions generate allocative inefficiencies and affect aircraft prices.

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<sup>7</sup>The model assumes symmetry of information about the quality of the asset. Several institutional features of aircraft markets support this assumption. First, the aviation authorities often regulate aircraft maintenance. Second, maintenance records are frequently available, and all parties can observe the entire history of owners of each aircraft. Finally, all transactions involve a thorough material inspection of the aircraft.

<sup>8</sup>The analysis focuses on decentralized markets, thus on aircraft of age  $a < T$ , neglecting the issue of replacement of aircraft (at age  $T$  or before). Similarly, I neglect how the flow  $x$  of vintage-0 goods is allocated. These issues can be reconciled with the model by assuming that agents with age- $T$  aircraft acquire age-0 aircraft (at a premium that makes them indifferent between buying the new aircraft or searching for a used one). Moreover, in the estimation of Section 6, I will check that agents do not want to replace aircraft that is less than 40 years old.

### 5.2.1 Agents' Value Functions

Let  $V_{ho}(a)$  be the value function of an agent with valuation  $z_h$  that owns an aircraft of age  $a < T$  and is not seeking it to sell it.  $V_{ho}(a)$  satisfies:

$$\rho V_{ho}(a) = z_h (\delta_0 + \delta_1 e^{-\delta_2 a}) + \lambda (\max \{V_{lo}(a), W_{lo}(a)\} - V_{ho}(a)) + V'_{ho}(a). \quad (1)$$

Equation (1) has the usual interpretation of an asset-pricing equation. An agent with valuation  $z_h$  enjoys the flow utility  $z_h (\delta_0 + \delta_1 e^{-\delta_2 a})$  from an aircraft of age  $a$ . At any date, at most, one possible event might happen to him: At rate  $\lambda$ , his valuation drops to  $z_l$ , in which case he chooses between continuing to operate the aircraft (enjoying the value  $V_{lo}(a)$ ) and actively seeking to sell it (enjoying value  $W_{lo}(a)$ ). Thus, the agent obtains a capital loss equal to  $\max \{V_{lo}(a), W_{lo}(a)\} - V_{ho}(a)$ . Moreover, the aircraft depreciates continuously, so that the agent has a capital loss equal to  $V'_{ho}(a)$ .

Similarly, the value function  $W_{ho}(a)$  of an agent with valuation  $z_h$  that owns an aircraft of age  $a$  and is actively seeking to sell it satisfies:

$$\begin{aligned} \rho W_{ho}(a) = & z_h (\delta_0 + \delta_1 e^{-\delta_2 a}) - c_s + \lambda (\max \{V_{lo}(a), W_{lo}(a)\} - W_{ho}(a)) + \\ & \gamma_s \max \{p(a) + \max \{V_{hn}, W_{hn}\} - W_{ho}(a), 0\} + \\ & \gamma_{sd} \max \{p_B(a) + \max \{V_{hn}, W_{hn}\} - W_{ho}(a), 0\} + W'_{ho}(a). \end{aligned} \quad (2)$$

An agent with valuation  $z_h$  enjoys the flow utility  $z_h (\delta_0 + \delta_1 e^{-\delta_2 a})$  from an aircraft of age  $a$ , but he pays the flow cost  $c_s$  while he is actively seeking to sell it. At any date one of three possible events, at most, might happen to him: 1) At rate  $\lambda$ , his valuation drops to  $z_l$ . In this case, the agent chooses between keeping the aircraft (enjoying the value  $V_{lo}(a)$ ) and actively seeking to sell it (enjoying value  $W_{lo}(a)$ ). Hence, the agent obtains a capital loss equal to  $\max \{V_{lo}(a), W_{lo}(a)\} - V_{ho}(a)$ . 2) At rate  $\gamma_{sd}$ , the agent meets a potential buyer and chooses between trading the aircraft or keeping it. If he trades it at price  $p(a)$ , he then chooses between actively searching for another aircraft or not, thus obtaining a capital gain equal to  $p(a) + \max \{V_{hn}, W_{hn}\} - V_{ho}(a)$ . If he keeps it, he obtains a capital gain of zero. 3) At rate  $\gamma_{sd}$ , he meets a dealer and chooses between trading the aircraft or keeping it. If he trades it at price  $p_B(a)$ , he then chooses between actively searching for another aircraft

or not, thus obtaining a capital gain equal to  $p_B(a) + \max\{V_{hn}, W_{hn}\} - V_{ho}(a)$ . If he keeps it, he obtains a capital gain of zero. Moreover, the aircraft depreciates continuously, so that the agent has a capital loss equal to  $W'_{ho}(a)$ .

The value functions  $V_{lo}(a)$  and  $W_{lo}(a)$  of an agent with valuation  $z_l$  that owns an aircraft of age  $a$  satisfy the following Bellman equations, respectively:

$$\rho V_{lo}(a) = z_l (\delta_0 + \delta_1 e^{-\delta_2 a}) + V'_{lo}(a). \quad (3)$$

$$\begin{aligned} \rho W_{lo}(a) = & z_l (\delta_0 + \delta_1 e^{-\delta_2 a}) - c_s + \gamma_s \max\{p(a) + \max\{V_{ln}, W_{ln}\} - W_{lo}(a), 0\} + \\ & \gamma_{sd} \max\{p_B(a) + \max\{V_{ln}, W_{ln}\} - W_{lo}(a), 0\} + W'_{lo}(a). \end{aligned} \quad (4)$$

The interpretation of equations (3) and (4) is now straightforward. An agent with valuation  $z_l$  enjoys the flow utility  $z_l (\delta_0 + \delta_1 e^{-\delta_2 a})$  from an aircraft of age  $a$ . If he does not seek to sell the aircraft, the only event that affects his utility is the depreciation of the aircraft, generating a capital loss equal to  $V'_{lo}(a)$ . If he seeks to sell the aircraft, his flow utility is reduced by the search cost  $c_s$ . Then, at any date, he meets a potential buyer at rate  $\gamma_s$ , in which case he chooses between selling the aircraft at price  $p(a)$ —thus obtaining a capital gain equal to  $p(a) + \max\{V_{ln}, W_{ln}\} - V_{lo}(a)$ —or keeping it—thus obtaining a capital gain of zero. Similarly, he meets a dealer at rate  $\gamma_{sd}$ , in which case he chooses between trading the aircraft at price  $p_B(a)$ —thus, obtaining a capital gain equal to  $p_B(a) + \max\{V_{ln}, W_{ln}\} - V_{lo}(a)$ —or keeping it—thus obtaining a capital gain of zero. Moreover, the aircraft depreciates continuously, so that the agent has a capital loss equal to  $W'_{lo}(a)$ .

The value functions of high- and low- valuation agents with no aircraft satisfy:

$$\begin{aligned} \rho W_{hn} = & -c_s + \lambda (\max\{V_{ln}, W_{ln}\} - W_{hn}) + \\ & \gamma_b \int \max\{\max\{V_{ho}(a), W_{ho}(a)\} - p(a) - W_{hn}, 0\} dF(a) + \\ & \gamma_{bd} \int \max\{\max\{V_{ho}(a), W_{ho}(a)\} - p_A(a) - W_{hn}, 0\} dF(a), \end{aligned} \quad (5)$$

$$\rho V_{hn} = 0, \quad (6)$$

$$\rho W_{ln} = -c_s + \gamma_b \int \max\{\max\{V_{lo}(a), W_{lo}(a)\} - p(a) - W_{ln}, 0\} dF(a) + \quad (7)$$

$$\gamma_{bd} \int \max\{\max\{V_{lo}(a), W_{lo}(a)\} - p_A(a) - W_{ln}, 0\} dF(a),$$

$$\rho V_{ln} = 0. \quad (8)$$

Equation (5) says that a high-valuation agent with no aircraft who is paying the search cost  $c_s$  has a capital loss equal to  $\max\{V_{ln}, W_{ln}\} - W_{hn}$  when, at rate  $\lambda$ , his valuation drops from high to low; has a capital gain equal to  $\max\{\max\{V_{ho}(a), W_{ho}(a)\} - p(a) - W_{hn}, 0\}$  when, at rate  $\gamma_b$ , he meets a seller of an aircraft of age  $a$ ; and he has a capital gain equal to  $\max\{\max\{V_{ho}(a), W_{ho}(a)\} - p_A(a) - W_{hn}, 0\}$  when, at rate  $\gamma_{bd}$ , he meets a dealer selling an aircraft of age  $a$ . Since the  $hn$ -agent does not know the age of the aircraft that the counterparty will have, he takes expectation over the possible capital gains that arise from the different vintages. Similarly, equation (7) says that a low-valuation agent with no aircraft who is paying the search cost  $c_s$  to search for a counterparty has an expected capital gain equal to  $\max\{\max\{V_{lo}(a), W_{lo}(a)\} - p(a) - W_{ln}, 0\}$  when he meets a potential seller of an aircraft of age  $a$ , and an expected capital gain equal to  $\max\{\max\{V_{lo}(a), W_{lo}(a)\} - p_A(a) - W_{ln}, 0\}$  when he meets a dealer. Equations (6) and (8) say that agents without aircraft who are not searching have a zero value.

We can simplify the value functions of all types of agents  $\{ho, lo, hn, ln\}$  recognizing that gains from trade arise only when high-valuation non-owners meet low-valuation owners. Thus, the relevant value functions are:

$$\rho V_{ho}(a) = z_h (\delta_0 + \delta_1 e^{-\delta_2 a}) + \lambda (W_{lo}(a) - V_{ho}(a)) + V'_{ho}(a), \quad (9)$$

$$\rho W_{lo}(a) = z_l (\delta_0 + \delta_1 e^{-\delta_2 a}) - c_s + \gamma_s (p(a) + V_{ln} - W_{lo}(a)) + \gamma_d (p_B(a) + V_{ln} - W_{lo}(a)) + W'_{lo}(a), \quad (10)$$

$$\rho W_{hn} = -c_s + \lambda (V_{ln} - W_{hn}) + \gamma_b \int (V_{ho}(a) - p(a) - W_{hn}) dF(a) + \gamma_d \int (V_{ho}(a) - p_A(a) - W_{hn}) dF(a),$$

$$\rho V_{ln} = 0.$$

### 5.2.2 Dealers' Value Functions

As in Rubinstein and Wolinsky (1987), dealers can extract surplus by shortening the time that buyers and sellers have to wait in order to trade. They are capacity-constrained and cannot store more than one aircraft. Thus, the value functions  $J_{do}(a)$  and  $J_{dn}$  of dealers

with and without an aircraft for sale, respectively, satisfy:

$$\rho J_{do}(a) = -k + \alpha_{ds}(p_A(a) + J_{dn} - J_{do}(a)) + J'_{do}(a), \quad (11)$$

$$\rho J_{dn} = -k + \alpha_{db} \int (J_{do}(a) - p_B(a) - J_{dn}) dF(a). \quad (12)$$

Equation (11) says that a dealer who owns an aircraft of age  $a$  pays the flow cost  $k$  while he is actively seeking to sell it. At any date, at rate  $\alpha_{ds}$ , the dealer meets a potential buyer. In this case, the dealer trades the aircraft at a negotiated price  $p(a)$ , thus obtaining a capital gain equal to  $p_A(a) + J_{dn} - J_{do}(a)$ . Moreover, the aircraft depreciates continuously, so the dealer has a capital loss equal to  $J'_{do}(a)$ . Similarly, equation (12) says that a dealer without aircraft pays the flow cost  $k$  while he is actively seeking to purchase one. At rate  $\alpha_{db}$ , this dealer meets a potential seller of an aircraft of age  $a$ , thus enjoying a capital gain equal to  $J_{do}(a) - p_B(a) - J_{dn}$ . Since the  $dn$ -dealer does not know the age of the aircraft that the counterparty will have, he takes expectation over the possible capital gains that arise from the different vintages.

Dealers' free-entry requires that  $J_{dn} = 0$ —i.e., dealers' expected capital gain is exactly equal to its fixed operating cost.<sup>9</sup>

### 5.2.3 Distribution of Agents

Let  $\mu_i$  be the masses of agents whose state is  $i \in \{ho, lo, hn, ln, do, dn\}$ . Consider a small interval of time of length  $\epsilon$ . Up to terms in  $o(\epsilon)$ , in equilibrium, these masses evolve from time  $t$  to time  $t + \epsilon$  according to:

$$\begin{aligned} \mu_{lo}(t + \epsilon) &= \lambda \epsilon \mu_{ho}(t) + (1 - \gamma_s \epsilon - \gamma_{sd} \epsilon) \mu_{lo}(t), \\ \mu_{ho}(t + \epsilon) &= \gamma_b \epsilon \mu_{hn}(t) + \gamma_{bd} \epsilon \mu_{hn}(t) + (1 - \lambda \epsilon) \mu_{ho}(t), \\ \mu_{hn}(t + \epsilon) &= \epsilon \mu + (1 - \lambda \epsilon - \gamma_b \epsilon - \gamma_{bd} \epsilon) \mu_{hn}(t), \\ \mu_{ln}(t + \epsilon) &= \lambda \epsilon \mu_{hn}(t) + \gamma_s \epsilon \mu_{lo}(t) + \gamma_{sd} \epsilon \mu_{lo}(t), \\ \mu_{do}(t + \epsilon) &= \alpha_{db} \epsilon \mu_{dn}(t) + (1 - \alpha_{ds} \epsilon) \mu_{do}(t), \\ \mu_{dn}(t + \epsilon) &= \alpha_{ds} \epsilon \mu_{do}(t) + (1 - \alpha_{db} \epsilon) \mu_{dn}(t). \end{aligned}$$

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<sup>9</sup>Since the matching function exhibits increasing returns to scale, the free entry of dealers maximizes (constrained) efficiency. Under constant returns to scale, the mass of dealers could be smaller or larger than the socially optimal mass depending on the bargaining parameter (Hosios, 1990).

The intuition for these equations is simple. For example, the first equation states that the mass of low-valuation agents with an asset results from the flows of three sets of agents: 1) the inflow of high-valuation owners whose valuation just dropped—the term  $\lambda\epsilon\mu_{ho}(t)$ ; 2) the outflow of low-valuation owners that found a dealer—the term  $\gamma_d\epsilon\mu_m(t)$ ; and 3) the mass of low-valuation owners that have not found a buyer—the term  $(1 - \gamma_s\epsilon)\mu_{lo}(t)$ . The intuition for the other equations is similar.

Steady state implies that the total mass of agents with high valuation  $\mu_{hn} + \mu_{ho}$  is equal to  $\frac{\mu}{\lambda}$ , that the total mass of owners  $\mu_{ho} + \mu_{lo} + \mu_{do}$  is equal to the mass of assets  $A$ , and that the total mass of dealers  $\mu_d$  is equal to  $\mu_{do} + \mu_{dn}$ . Hence,  $\mu_{hn} + A - \mu_{lo} - \mu_{do} = \frac{\mu}{\lambda}$ . In turn, since  $A < \frac{\mu}{\lambda}$ , this implies that  $\mu_{hn} > \mu_{lo} + \mu_{do}$ —i.e., sellers are the “short” side of the market. Moreover, the masses of assets sold and purchased by dealers must be equal to the mass of assets purchased and sold to dealers:  $\alpha_{ds}\mu_{do} = \gamma_{bd}\mu_{hn}$  and  $\alpha_{db}\mu_{dn} = \gamma_{sd}\mu_{lo}$ . Furthermore, steady state requires  $\alpha_{ds}\mu_{do} = \gamma_{bd}\mu_{hn} = \alpha_{db}\mu_{dn} = \gamma_{sd}\mu_{lo}$  since dealers’ aggregate inventories do not change over time.

Thus, rearranging and taking the limit for  $\epsilon \rightarrow 0$ , the masses of agents are:

$$\begin{aligned}\mu_{lo} &= \frac{\lambda}{\gamma_s + \gamma_{sd}}\mu_{ho}, \\ \mu_{ho} &= \frac{\gamma_b\mu_{hn} + \gamma_{bd}\mu_{hn}}{\lambda}, \\ \mu_{hn} &= \frac{\mu}{\lambda + \gamma_b + \gamma_{bd}}, \\ \mu_{ln} &= \lambda\mu_{hn} + \gamma_s\mu_{lo} + \gamma_{sd}\mu_{lo} = \mu, \\ \mu_{do} &= \frac{\alpha_{db}}{\alpha_{ds}}\mu_{dn} = \frac{\gamma_{sd}}{\alpha_{ds}}\mu_{lo}.\end{aligned}$$

All these steady-state equalities and the equilibrium condition  $\mu_{ho} + \mu_{lo} + \mu_{do} = A$  allows us to solve for the endogenous masses  $\mu_i$   $i \in \{ho, lo, hn, ln, do, dn\}$  as a function of the exogenous parameters  $A, \mu$  and  $\lambda$ , and the exogenous parameters of the matching functions  $\gamma$  and  $\gamma'$ .

Letting  $\gamma$  increase,  $\lim_{\gamma \rightarrow +\infty} \mu_{lo}$  and converges to 0: When frictions vanish, no low-valuation agent operates an aircraft. Similarly,  $\lim_{\gamma' \rightarrow +\infty} \mu_{do}$  and converges to 0. Thus, the masses  $\mu_{lo}$  and  $\mu_{do}$  and the ratios  $\frac{\mu_{lo}}{A}$  and  $\frac{\mu_{do}}{A}$ —i.e., the masses and the fractions of low-valuation agents and dealers actively seeking to sell aircraft—are measures of assets inefficiently allocated in the economy. Based on the parameters estimated from the data, in Section 7.1, I compute these measures to quantify the effects of trading frictions on asset

allocations.

### 5.2.4 Prices

When a buyer and a seller meet, the negotiated price

$$p(a) = (1 - \theta_s)(W_{lo}(a) - V_{ln}) + \theta_s(V_{ho}(a) - W_{hn}) \quad (13)$$

is the solution to the following symmetric-information bargaining problem:

$$\begin{aligned} & \max_{p(a)} [V_{ho}(a) - p(a) - W_{hn}]^{1-\theta_s} [p(a) + V_{ln} - W_{lo}(a)]^{\theta_s} \\ \text{subject to: } & V_{ho}(a) - p(a) - W_{hn} \geq 0 \text{ and } p(a) + V_{ln} - W_{lo}(a) \geq 0. \end{aligned}$$

Similarly, the ask and bid prices  $p_A(a)$  and  $p_B(a)$  satisfy:

$$p_A(a) = (1 - \theta_d)(J_{do}(a) - J_{dn}) + \theta_d(V_{ho}(a) - V_{hn}), \quad (14)$$

$$p_B(a) = (1 - \theta_d)(J_{do}(a) - J_{dn}) + \theta_d(W_{lo}(a) - V_{ln}). \quad (15)$$

## 5.3 Equilibrium

We can solve the system of differential equations (9), (10) and (11) and obtain (Appendix A reports the calculations):

$$\begin{aligned} V_{ho}(a) &= \beta_0 + \beta_1 e^{-\delta_2 a}, \\ W_{lo}(a) &= \omega_0 + \omega_1 e^{-\delta_2 a}, \\ J_{do}(a) &= \eta_0 + \eta_1 e^{-\delta_2 a}, \end{aligned}$$

where  $\beta_i$ ,  $\omega_i$  and  $\eta_i$   $i = 0, 1$  depend on all other parameters of the model. The coefficients  $\omega_1$  and  $\beta_1$  are the discounted values of weighted averages of the valuations  $z_l$  and  $z_h$ , where the relevant discount factor is the user cost of capital  $\rho + \delta_2$ , and the weights depend on the trading delays. For example, the value function  $V_{ho}$  of high-value owners takes into account that, when their valuation switches to the low value, they will not be able to sell their aircraft immediately, thus weighting the high and the low valuation according to their

expected durations, whereas  $\beta_1$  converges to  $z_h$  as  $\gamma$  and  $\gamma'$  grow large,

Moreover, we can solve the system of linear equations (13)-(15) to obtain that prices are equal to:

$$p(a) = \phi_0 + \phi_1 e^{-\delta_2 a}, \quad (16)$$

$$p_A(a) = \tau_0 + \tau_1 e^{-\delta_2 a},$$

$$p_B(a) = \psi_0 + \psi_1 e^{-\delta_2 a}, \quad (17)$$

where the coefficients are linear combinations of the coefficients of the value functions. For example, the intercept  $\phi_0$  is equal to  $(1 - \theta_s)\omega_0 + \theta_s(\beta_0 - W_{hn})$  and the slope  $\phi_1 = (1 - \theta_s)\omega_1 + \theta_s\beta_1$  is, again, the discounted value of a weighted average of the valuations  $z_l$  and  $z_h$ .

In Section 6, I use equations (16) and (17) to estimate parameters of the model. Based on these parameters, I compute Walrasian prices and compare them to observed prices to quantify the effects of trading frictions on asset prices.

## 6 Estimation and Identification

I estimate the model using the data on business aircraft described in Section 4, assuming that they are generated from the model's steady state. I set the unit of time to be one quarter. I estimate two versions of the model. In the first one, all aircraft models are homogeneous, so that the only heterogeneous characteristic across aircraft is their age. In the second one, the parameters  $\lambda$ ,  $\gamma_s$ ,  $\gamma_{sd}$ ,  $\alpha_{ds}$ ,  $z_h$ ,  $z_l$ ,  $\delta_2$ ,  $\delta_0$  and  $c_s$  are functions of observable characteristics of the aircraft and of the market.

Unfortunately, the data lack some detailed information to identify all parameters. Therefore, I have to fix some values. Specifically, the discount rate  $\rho$  is traditionally difficult to identify, and I fix it to  $\rho = .015$ . Moreover, I set  $\delta_1 = 1$ , as this parameter is not separately identified from the baseline valuation  $z_l\delta_0$  (this is just a normalization).

## 6.1 Homogeneous Aircraft

I estimate all other parameters using the generalized method of moments (GMM), matching key moments of the data with the corresponding moments of the model. More precisely, the first moment is the fraction of aircraft for sale, which, in the model, is equal to:

$$\frac{\mu_{lo} + \mu_{do}}{A} = \frac{\lambda(\alpha_{ds} + \gamma_{sd})}{\lambda\alpha_{ds} + \lambda\gamma_{sd} + \gamma_s\alpha_{ds} + \alpha_{ds}\gamma_{sd}}. \quad (18)$$

The second moment is the fraction of aircraft for sale by dealers (or dealers' inventories), which is equal to:

$$\frac{\mu_{do}}{A} = \frac{\lambda\gamma_{sd}}{\lambda\alpha_{ds} + \lambda\gamma_{sd} + \gamma_s\alpha_{ds} + \alpha_{ds}\gamma_{sd}}. \quad (19)$$

The third moment is the fraction of retail-to-retail transactions to total aircraft, which is equal to:

$$\frac{\gamma_s\mu_{lo}}{A} = \frac{\lambda\gamma_s\alpha_{ds}}{\lambda\alpha_{ds} + \lambda\gamma_{sd} + \gamma_s\alpha_{ds} + \alpha_{ds}\gamma_{sd}}. \quad (20)$$

The fourth moment is the fraction of dealer-to-retail transactions to total aircraft, which is equal to:

$$\frac{\alpha_{ds}\mu_{do}}{A} = \frac{\lambda\gamma_{sd}\alpha_{ds}}{\lambda\alpha_{ds} + \lambda\gamma_{sd} + \gamma_s\alpha_{ds} + \alpha_{ds}\gamma_{sd}}. \quad (21)$$

Equations (18)-(21) identify the parameters  $\lambda$ ,  $\gamma_s$ ,  $\gamma_{sd}$  and  $\alpha_{ds}$ . In particular, Table 1 reported that the fraction of aircraft for sale is non-trivial, thus indicating that trading delays are relevant. Hence, the model can fit this key feature of the data very well. Similarly, Table 1 dealers appear faster at trading aircraft than sellers, and the model captures this difference through the trading parameters  $\gamma_s$ ,  $\gamma_{sd}$  and  $\alpha_{ds}$ .

The parameters  $\lambda$ ,  $\gamma_s$ ,  $\gamma_{sd}$  and  $\alpha_{ds}$  allow me to compute the ratios  $\frac{\mu_{lo}}{A}$  and  $\frac{\mu_{do}}{A}$ . Furthermore, I match the mass of total assets  $A$  to the average total number of aircraft, and the mass of dealers  $\mu_d$  to the average total number of dealers. Having estimates of  $\lambda$ ,  $\gamma_s$ ,  $\gamma_{sd}$ ,

$\alpha_{ds}$ ,  $\frac{\mu_{lo}}{A}$ ,  $\frac{\mu_{do}}{A}$ ,  $A$  and  $\mu_d$ , I can recover  $\gamma_b$ ,  $\mu$ ,  $\gamma$  and  $\gamma'$  by solving the system:

$$\begin{aligned}\frac{\mu}{\lambda} &= \mu_{hn} + \mu_{ho}, \\ \mu_{hn} &= \frac{\mu - \gamma_d \mu_{lo}}{\lambda + \gamma_b}, \\ \mu &= \lambda \mu_{hn} + \gamma_s \mu_{lo} + \gamma_d \mu_{lo}, \\ \gamma &= \frac{\gamma_s}{\mu_{hn}}, \\ \gamma' &= \frac{\gamma_{bd}}{\mu_{do}}.\end{aligned}$$

The price equations (16) and (17) identify other parameters of the model. More precisely, I construct the following moments to estimate  $\phi_0$ ,  $\phi_1$ ,  $\psi_0$ ,  $\psi_1$  and  $\delta_2$  :

$$\begin{aligned}\frac{1}{N_p} \sum \left[ (\log p(a) - \log(\phi_0 + \phi_1 e^{-\delta_2 a})) \frac{1}{\phi_0 + \phi_1 e^{-\delta_2 a}} \right] &= 0, \\ \frac{1}{N_p} \sum \left[ (\log p_B(a) - \log(\psi_0 + \psi_1 e^{-\delta_2 a})) \frac{1}{\psi_0 + \psi_1 e^{-\delta_2 a}} \right] &= 0, \\ \frac{1}{N_p} \sum \left[ (\log p(a) - \log(\phi_0 + \phi_1 e^{-\delta_2 a})) \frac{e^{-\delta_2 a}}{\phi_0 + \phi_1 e^{-\delta_2 a}} \right] &= 0, \\ \frac{1}{N_p} \sum \left[ (\log p_B(a) - \log(\psi_0 + \psi_1 e^{-\delta_2 a})) \frac{e^{-\delta_2 a}}{\psi_0 + \psi_1 e^{-\delta_2 a}} \right] &= 0, \\ \frac{1}{N_p} \sum \left[ (\log p(a) - \log(\phi_0 + \phi_1 e^{-\delta_2 a})) \frac{a \phi_1 e^{-a \delta_2}}{\phi_0 + \phi_1 e^{-\delta_2 a}} \right] &= 0, \\ \frac{1}{N_p} \sum \left[ (\log p_B(a) - \log(\psi_0 + \psi_1 e^{-\delta_2 a})) \frac{a \psi_1 e^{-a \delta_2}}{\psi_0 + \psi_1 e^{-\delta_2 a}} \right] &= 0,\end{aligned}$$

where  $N_p$  is the number of price observations. These moments correspond to the first-order conditions of a non-linear least squares estimation of the logarithm of the price functions  $p(a)$  and  $p_B(a)$ .

For any value of the bargaining parameters  $\theta_s$  and  $\theta_d$ , from the estimates of  $\phi_1$  and  $\psi_1$ , I can recover  $z_h$  and  $z_l$  by solving the system of linear equations (see Appendix A for

derivations):

$$\begin{aligned}
\phi_1 &= \theta_s \beta_1 + (1 - \theta_s) \omega_1, \\
\psi_1 &= (1 - \theta_d) \eta_1 + \theta_d \omega_1, \\
\omega_1 &= \frac{\left( \gamma_s \theta_s + \gamma_{sd} (1 - \theta_d) \frac{\alpha_{ds} \theta_d}{\rho + \delta_2 + \alpha_{ds} \theta_d} \right) \delta_1 z_h + (\delta_2 + \rho + \lambda) \delta_1 z_l}{(\delta_2 + \rho + \lambda) (\rho + \gamma_s \theta_s + \gamma_{sd} (1 - \theta_d) + \delta_2) - \left( \gamma_s \theta_s + \gamma_{sd} (1 - \theta_d) \frac{\alpha_{ds} \theta_d}{\rho + \delta_2 + \alpha_{ds} \theta_d} \right) \lambda}, \\
\beta_1 &= \frac{\rho \omega_1 - \delta_1 z_l + \gamma_s \theta_s \omega_1 + \gamma_{sd} (1 - \theta_d) \omega_1 + \omega_1 \delta_2}{\gamma_s \theta_s + \gamma_{sd} (1 - \theta_d) \frac{\alpha_{ds} \theta_d}{\rho + \delta_2 + \alpha_{ds} \theta_d}}, \\
\eta_1 &= \frac{\alpha_{ds} \theta_d \beta_1}{\rho + \delta_2 + \alpha_{ds} \theta_d}.
\end{aligned}$$

Similarly, from the estimates of  $\phi_0$  and  $\psi_0$ , I can recover  $c_s$ ,  $\delta_0$  and  $k$  by solving (again, see Appendix A for derivations):

$$\begin{aligned}
\phi_0 &= (1 - \theta_s) \omega_0 + \theta_s (\beta_0 - W_{hn}), \\
\psi_0 &= (1 - \theta_d) (\eta_0 - J_{dn}) + \theta_d \omega_0, \\
\omega_0 &= \frac{\frac{c_s - \delta_0 z_h}{\lambda + \rho} + \frac{c_s - \delta_0 z_l + \theta_s \gamma_s W_{hn} + \gamma_{sd} (1 - \theta_d) \frac{k + \rho J_{dn} + \theta_d \alpha_{ds} W_{hn}}{\rho + \theta_d \alpha_{ds}}}{\gamma_s \theta_s + \gamma_{sd} (1 - \theta_d) \frac{\alpha_{ds} \theta_d}{\rho + \alpha_{ds} \theta_d}}}{\frac{\lambda}{\lambda + \rho} - \frac{\gamma_s \theta_s + \gamma_{sd} (1 - \theta_d)}{\gamma_s \theta_s + \gamma_{sd} (1 - \theta_d) \frac{\alpha_{ds} \theta_d}{\rho + \alpha_{ds} \theta_d}}}, \\
\beta_0 &= \frac{\delta_0 z_h - c_s + \lambda \omega_0}{\rho + \lambda}, \\
\eta_0 &= \frac{-k + \alpha_{ds} \theta_d (\beta_0 + J_{dn} - W_{hn})}{\rho + \alpha_{ds} \theta_d}, \\
W_{hn} &= \frac{-c_s + \gamma_b (1 - \theta_s) \int (\beta_0 - \omega_0 + (\beta_1 - \omega_1) e^{-\delta_2 a}) dF(a)}{\rho + \lambda + \gamma_b (1 - \theta_s) + \gamma_{bd} (1 - \theta_d)} + \\
&\quad \frac{\gamma_{bd} (1 - \theta_d) \int (\beta_0 - \eta_0 + J_{dn} + (\beta_1 - \eta_1) e^{-\delta_2 a}) dF(a)}{\rho + \lambda + \gamma_b (1 - \theta_s) + \gamma_{bd} (1 - \theta_d)}, \\
J_{dn} &= \frac{-k + \alpha_{db} \theta_d \int (\eta_0 - \omega_0 + (\eta_1 - \omega_1) e^{-\delta_2 a}) dF(a)}{\rho + \alpha_{db} \theta_d} = 0,
\end{aligned}$$

where the last equation is dealers' free-entry/zero-profit condition and I use the uniform distribution on  $[0, 160]$  for  $F(a)$ .<sup>10</sup>

The bargaining parameters  $\theta_s$  and  $\theta_d$  are not point-identified. Rather, I estimate an

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<sup>10</sup>The uniform distribution is the exact distribution of vintages available for sale according to the model.

interval for their values by imposing restrictions on some parameters. Specifically, I require the following inequalities to hold at the estimated parameters:

$$\begin{aligned}
z_h &\geq z_l, \\
z_l &\geq 0, \\
c_s &\geq 0, \\
k &\geq 0, \\
W_{hn} &\geq 0, \\
W_{lo}(a) &\geq V_{lo}(a) \quad \text{for } a \leq 130.
\end{aligned}$$

Thus, I impose that: buyers’ valuation is higher than sellers’; sellers have non-negative valuations for the assets; the cost of search is non-negative; dealers’ fixed cost is non-negative; high-valuation agents with no aircraft have a positive value of searching; and low-valuation aircraft owners prefer to sell any aircraft younger than 130 quarters of age rather than operating it.<sup>11</sup> Since the bargaining parameters are set-identified only, the parameters  $z_h$ ,  $z_l$ ,  $\delta_0$ ,  $c_s$  and  $k$  are set-identified only, as well.

### 6.1.1 Estimates

Figure 2 displays the set of values of the bargaining parameters consistent with the constraints imposed by the model. The set is small and indicates that the “short” side of the market—i.e., sellers and dealers, in particular—captures the largest share of surplus. More precisely, almost all elements of the set imply that dealers capture more than 95 percent of any transaction’s surplus. Table 2 reports estimates of all other parameters. Since some parameters are combinations of other parameters, I compute standard errors by bootstrapping the data using 1,000 replications. In Appendix B, I perform the estimation on two separate sample periods—i.e., 1990-1999 and 2000-2008—confirming the robustness of the estimates reported in Figure 2 and Table 2 to different sample periods.

The magnitude of the parameter  $\lambda$  indicates that, on average, valuations switch from

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<sup>11</sup>Increasing (decreasing) the threshold of 130 quarters obviously increases (decreases) the number of constraints, thus decreasing (increasing) the values of the bargaining parameters consistent with the model, but does not dramatically change them.

TABLE 2: Homogeneous Aircraft: GMM Estimates

	$\theta_s = .50$ $\theta_d = .96$	$\theta_s = .70$ $\theta_d = .98$	$\theta_s = .85$ $\theta_d = .99$	$\theta_s = .97$ $\theta_d = 1$
$\lambda$	0.0538 [0.0531; 0.0545]	0.0538 [0.0531; 0.0545]	0.0538 [0.0531; 0.0545]	0.0538 [0.0531; 0.0545]
$\gamma_s$	0.2072 [0.2022; 0.2119]	0.2072 [0.2022; 0.2119]	0.2072 [0.2022; 0.2119]	0.2072 [0.2022; 0.2119]
$\gamma_{sd}$	0.3702 [0.3614; 0.3786]	0.3702 [0.3614; 0.3786]	0.3702 [0.3614; 0.3786]	0.3702 [0.3614; 0.3786]
$\alpha_{ds}$	0.7943 [0.7793; 0.8101]	0.7943 [0.7793; 0.8101]	0.7943 [0.7793; 0.8101]	0.7943 [0.7793; 0.8101]
$\gamma$	0.00027 [0.00026; 0.00029]	0.00027 [0.00026; 0.00029]	0.00027 [0.00026; 0.00029]	0.00027 [0.00026; 0.00029]
$\gamma'$	0.0011 [0.0009; 0.0011]	0.0011 [0.0009; 0.0011]	0.0011 [0.0009; 0.0011]	0.0011 [0.0009; 0.0011]
$A$	10, 327 [10, 190; 10, 481]	10, 327 [10, 190; 10, 481]	10, 327 [10, 190; 10, 481]	10, 327 [10, 190; 10, 481]
$\mu$	529.0 [519.5; 538.9]	529.0 [519.5; 538.9]	529.0 [519.5; 538.9]	529.0 [519.5; 538.9]
$\mu_d$	742.6 [747.6; 742.6]	742.6 [747.6; 742.6]	742.6 [747.6; 742.6]	742.6 [747.6; 742.6]
$\delta_2$	0.0211 [0.0207; 0.0215]	0.0211 [0.0207; 0.0215]	0.0211 [0.0207; 0.0215]	0.0211 [0.0207; 0.0215]
$\delta_0$	0.1612 [0.1538; 0.1693]	0.0766 [0.0731; 0.0804]	0.0328 [0.0310; 0.0346]	0.0045 [0.0030; 0.0059]
$z_h$	805, 768 [788, 500; 823, 310]	704, 527 [689, 070; 720, 330]	663, 358 [648, 800; 678, 430]	640, 016 [625, 960; 654, 680]
$z_l$	55, 856 [44, 150; 68, 239]	104, 769 [93, 360; 116, 820]	118, 490 [107, 390; 130, 550]	126, 805 [115, 940; 138, 740]
$c_s$	1, 270 [-1, 528; 3, 891]	7, 382 [5, 543; 9, 116]	5, 156 [3, 623; 6, 595]	2, 985 [1, 616; 4, 287]
$k$	333, 930 [325, 350; 343, 240]	218, 799 [212, 880; 225, 190]	168, 892 [164, 040; 174, 070]	140, 709 [136, 490; 145, 280]

Notes—This table reports GMM estimates of the parameters. 95-percent confidence intervals in brackets are obtained bootstrapping the data using 1,000 replications.

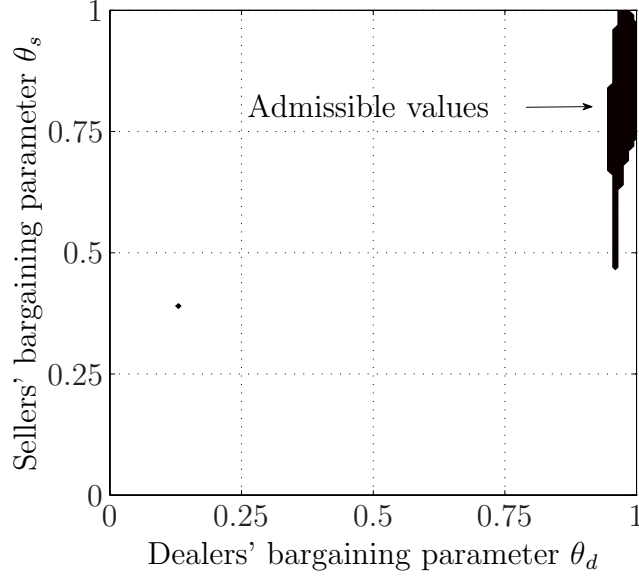


FIG. 2: Admissible values of the bargaining parameters.

high to low every four and a half years. The magnitude of the sum  $\gamma_s + \gamma_{sd}$  indicates that aircraft stay on the market approximately five months before a low-valuation owner is able to sell it. The magnitude of the parameter  $\alpha_{ds}$  indicates that, on average, it takes slightly less than four months for a dealer to find a buyer. These parameters  $\gamma_s$ ,  $\gamma_{sd}$  and  $\alpha_{ds}$  imply that trading delays are non-trivial in this market, and dealers play an important role in reducing them.

The parameter  $\delta_2$  indicates that aircraft depreciate by approximately eight percent every year, a decline comparable to that of larger commercial aircraft. Indeed, business aircraft older than thirty years are common: see Figure 1. The valuations  $z_h$  and  $z_l$  indicate that the difference between buyers' and sellers' valuations are large—i.e., gains from trade are large—and this difference is slightly larger when sellers and dealers have lower bargaining parameters. The parameter  $c_s$  indicates that search costs are small—they vary between \$5,000 and \$30,000 per year, depending on the bargaining parameter—suggesting that most inefficiencies arise because of trading delays. On average, each dealer trades one aircraft every  $\frac{1}{\alpha_{db}} + \frac{1}{\alpha_{ds}} \simeq 2.3$  quarters. Combined with the difference between retail and wholesale prices reported in Table 1 and dealers' free entry condition  $J_{dn} = 0$ , this implies a dealers' fixed flow cost  $k$  on the order of \$200,000 per quarter, indicating that the costs of “market-making” are not negligible. Overall, Table 2 indicates that varying the bargaining weights

does not greatly affect the other parameters that depend on these weights.

## 6.2 Heterogeneous Aircraft

I also estimate a specification in which the parameters  $\lambda$ ,  $\gamma_s$ ,  $\gamma_{sd}$ ,  $\alpha_{ds}$ ,  $z_h$ ,  $z_l$ , and  $\delta_2$  are functions of observable variables. More precisely, I fix the bargaining parameters to be equal to  $\theta_s = .85$  and  $\theta_d = .99$ —i.e., approximately their average estimated values; see Figure 2. Then, I specify the Poisson rates  $\lambda$ ,  $\gamma_s$ ,  $\gamma_{sd}$ ,  $\alpha_{ds}$  and the valuations  $z_h$  and  $z_l$  to be exponential functions (i.e.,  $z_h = \exp(\zeta_{z_h} X)$ ) and the depreciation rate  $\delta_2$  to be a logistic function (i.e.,  $\delta_2 = \frac{\exp(\zeta_{\delta_2} X)}{1 + \exp(\zeta_{\delta_2} X)}$ ) of observables  $X$ . These observables include aircraft characteristics—such as the flying range, the number of passengers, the maximum take-off weight, the fuel burn per hour, the maximum fuel, and the number of existing aircraft of the same model—and aggregate market characteristics—such as the level of the S&P 500 index and a quadratic yearly trend.

I estimate this version of the model using non-linear least squares: Based on a guess of the coefficients, I calculate the model’s value  $\hat{y}_i$  of observation  $i$  of outcome  $y_i$  observed in the data—i.e., the fraction of aircraft for sale, the fraction of aircraft for sale by dealers, the fraction of aircraft traded, the fraction of aircraft traded by dealers, and the log of retail and wholesale aircraft prices. Then, I construct the criterion

$$\frac{1}{N} \sum_{i=1}^N (y_i - \hat{y}_i)^2 \tag{22}$$

for each observed outcome. Finally, I sum over all criteria of the observed outcomes and choose the coefficients  $\zeta$  to minimize this sum.

### 6.2.1 Estimates

Table 3 reports the estimates of the coefficients. The effect of many variables is estimated rather imprecisely, or their effect is small—in particular, the effect of aircraft characteristics on the parameters  $\lambda$ ,  $\gamma_s$ ,  $\gamma_{sd}$ , and  $\alpha_{ds}$ . The reason is that the variation that identifies these coefficients—i.e., across-models variation in aircraft for sale, dealers’ inventories, aircraft transactions, and dealers’ transactions—does not depend on physical aircraft characteristics or is quite small. Instead, the variables affecting valuation parameters  $z_h$  and  $z_l$  exhibit

TABLE 3: Heterogeneous Aircraft: NLLS Estimates

	PARAMETER						
	$\lambda$	$\gamma_s$	$\gamma_{sd}$	$\alpha_{ds}$	$z_h$	$z_l$	$\delta_2$
CONSTANT	0.4405 [1.4280]	-5.0187 [1.5249]	2.3620 [1.3022]	1.6389 [1.5596]	11.3265 [0.0346]	10.5812 [0.1391]	-5.4715 [0.0404]
MEDIUM JET	0.0728 [0.7237]	-0.0867 [0.7322]	-0.0742 [0.7151]	0.0233 [0.7371]	0.3254 [0.0238]	0.2087 [0.1171]	-0.1314 [0.0221]
HEAVY JET	-0.1181 [7.6760]	0.0366 [7.6891]	0.0668 [7.6692]	0.0835 [7.6860]	-0.1373 [0.0467]	0.3863 [0.1098]	-1.6425 [0.0658]
MAX PASSENGERS	-0.0284 [0.2708]	-0.0050 [0.2745]	0.0245 [0.2639]	0.0295 [0.2771]	0.2520 [0.0077]	0.0903 [0.0090]	0.5205 [0.0105]
RANGE	0.0003 [0.0008]	-0.0013 [0.0008]	0.0008 [0.0008]	0.0010 [0.0008]	0.0005 [1.73e - 5]	0.0007 [1.0e - 4]	0.0008 [2.0e - 5]
MAXIMUM TAKE-OFF WEIGHT	-0.0152 [0.1685]	0.0752 [0.1693]	0.0678 [0.1675]	0.0500 [0.1703]	-0.0236 [0.0026]	0.1987 [0.0053]	-0.1196 [0.0038]
FUEL BURN PER HOUR	0.0345 [0.0072]	0.0158 [0.0073]	-0.0398 [0.0072]	-0.0349 [0.0073]	-0.0022 [0.0002]	-0.0127 [0.0004]	0.0001 [0.0002]
MAX FUEL	0.0021 [0.0034]	-0.0090 [0.0034]	0.0009 [0.0033]	0.0006 [0.0034]	-3.5e - 5 [3.26e - 5]	-0.0025 [0.0001]	-0.0001 [7.94e - 5]
LOG(ACTIVE)	-0.0764 [0.1701]	-0.0303 [0.1730]	-0.0097 [0.1684]	0.0701 [0.1742]	-0.0706 [0.0052]	0.0756 [0.0123]	-0.1620 [0.0054]
S&P 500	0.0017 [0.0005]	0.0014 [0.0006]	0.0018 [0.0001]	0.0024 [0.0006]	-5.8e - 5 [1.36e - 5]	0.0009 [4.0e - 5]	0.0001 [1.7e - 5]
YEAR-1980	0.0100 [0.2226]	-0.0743 [0.2234]	-0.0165 [0.2143]	-0.1077 [0.2223]	0.0090 [0.0024]	-0.1075 [0.0069]	-0.0503 [0.0032]
(YEAR-1980) SQUARED	-0.0056 [0.0121]	0.0033 [0.0122]	-0.0063 [0.0119]	-0.0016 [0.0122]	-0.0010 [0.0001]	0.0005 [0.0003]	0.0011 [0.0001]

Notes—This table reports NLLS estimates of the coefficients. Standard errors in brackets.

larger and more systematic variations, because price heterogeneity across models—the heterogeneity in the data that identifies these valuations—is quite large and is correlated to some aircraft characteristics, such as the number of passengers and fuel efficiency. Interestingly, market characteristics, captured by the level of the S&P 500 index, have a significant effect on the parameters. Specifically, a higher level of the U.S. stock market is correlated with lower trading frictions—i.e., higher trading rates—and higher valuations, consistent with a link between funding liquidity and market liquidity (Brunnermeier and Pedersen, 2009).

Overall, the estimates of Table 3 imply that, for most parameters, adding observable aircraft characteristics does not substantially improve the description of the market relative to the model with homogeneous assets, whose parameter estimates are reported in Table 2.

## 7 Counterfactual Analyses

I now perform two counterfactuals to answer the questions of how frictions and how dealers affect asset allocations and prices. In both cases, I use the parameter estimates from the model with homogeneous assets and bargaining parameters  $\theta_s = .85$  and  $\theta_d = .99$  reported in column (3) of Table 2 to compute the equilibrium in these counterfactual scenarios.

### 7.1 Quantifying the Effects of Frictions

The estimates of the parameters allow me to quantify the effect of trading frictions on the allocation and the prices of assets. Specifically, I compare asset allocations and asset prices with the Walrasian efficient benchmark, which is a special case of the model presented in Section 5 when trade is frictionless—i.e.,  $\gamma \rightarrow +\infty$  and  $c_s = 0$ .

In the Walrasian market, as soon as high-valuation agents enter the market, they immediately meet low-valuation sellers or dealers. Hence, in a frictionless market, no low-valuation agents and no dealers have aircraft—i.e.,  $\mu_{lo}^w = \mu_{do}^w = 0$ . Thus, the sum  $\mu_{lo} + \mu_{do}$  measures the mass of assets misallocated due to search frictions. The value of the fraction  $\frac{\mu_{lo} + \mu_{do}}{A} = \frac{\lambda(\alpha_{ds} + \gamma_{sd})}{\lambda\alpha_{ds} + \lambda\gamma_{sd} + \gamma_s\alpha_{ds} + \alpha_{ds}\gamma_{sd}}$  highlights that the allocative costs of trading frictions, captured by the parameters  $\gamma_s$ ,  $\gamma_{sd}$  and  $\alpha_{ds}$ , depend on how frequently agents seek to trade, captured by the parameter  $\lambda$ . Moreover, the ratio  $\frac{\mu_{lo} + \mu_{do}}{A}$  depends exclusively on param-

TABLE 4: Comparison with Walrasian Market

	(1)	(2)	(3)
	ESTIMATED MODEL	WALRASIAN MARKET	$c_s = 0$
$\mu_{lo} + \mu_{do}$	1,241 [1, 220; 1, 260]	0 [0; 0]	1,305 [1, 273; 1, 340]
$\frac{\mu_{lo} + \mu_{do}}{A}$	0.120 [0.1189; 0.1215]	0 [0; 0]	0.126 [0.1238; 0.1293]
$p(0)$	14,936,993 [14, 687, 000; 15, 184, 000]	19,804,958 [19, 499, 000; 20, 106, 000]	15,150,369 [14, 852, 000; 15, 450, 000]
$p(10)$	6,359,107 [6, 301, 400; 6, 415, 000]	9,329,647 [9, 228, 800; 9, 440, 100]	6,475,456 [6, 409, 200; 6, 537, 900]

Notes—This table reports counterfactual prices and allocations. 95-percent confidence intervals in brackets are obtained bootstrapping the data using 1,000 replications.

eters that I point-estimate using the aircraft transactions dataset, and not on any of the set-identified parameters.

Similarly, Walrasian prices solve the differential equation

$$\rho p^w(a) = z_h (\delta_0 + \delta_1 e^{-\delta_2 a}) + p^{w'}(a)$$

and, thus, are equal to  $p^w(a) = \frac{z_h \delta_0}{\rho} + \frac{z_h \delta_1 e^{-\delta_2 a}}{\rho + \delta_2}$ . Instead, in the estimated model with frictions, prices are equal to  $p(a) = \phi_0 + \phi_1 e^{-\delta_2 a}$ , as in equation (16).

Columns (1) and (2) in Table 4 report the calculations of these magnitudes for the model and the Walrasian benchmark, respectively, using the parameters estimated with  $\theta_s = .85$  and  $\theta_d = .99$ . Larger (smaller) values of the bargaining parameters have no effect on counterfactual allocations and decrease (increase) Walrasian prices, but the difference with the Walrasian prices reported in Table 4 is small. The estimates imply that trading frictions cause the misallocation of 1,241 aircraft, corresponding to  $\frac{\mu_{lo} + \mu_{do}}{A} = 12$  percent of all aircraft. The instantaneous welfare loss due to this misallocation is equal to the difference between the utility flows in the estimated model and in the Walrasian market, and this corresponds to  $(\mu_{lo}(z_h - z_l) + \mu_{do}z_h) \int (\delta_0 + \delta_1 e^{-\delta_2 a}) dF(a)$ . Using the uniform distribution on  $[0, 160]$  for  $F(a)$  and the other estimated parameters, the welfare loss equals \$179,008,120, or  $1 - \frac{\mu_{lo}z_l + \mu_{do}z_h}{Az_h} = 9.58$  percent of total potential welfare. Table 4 further shows that trading

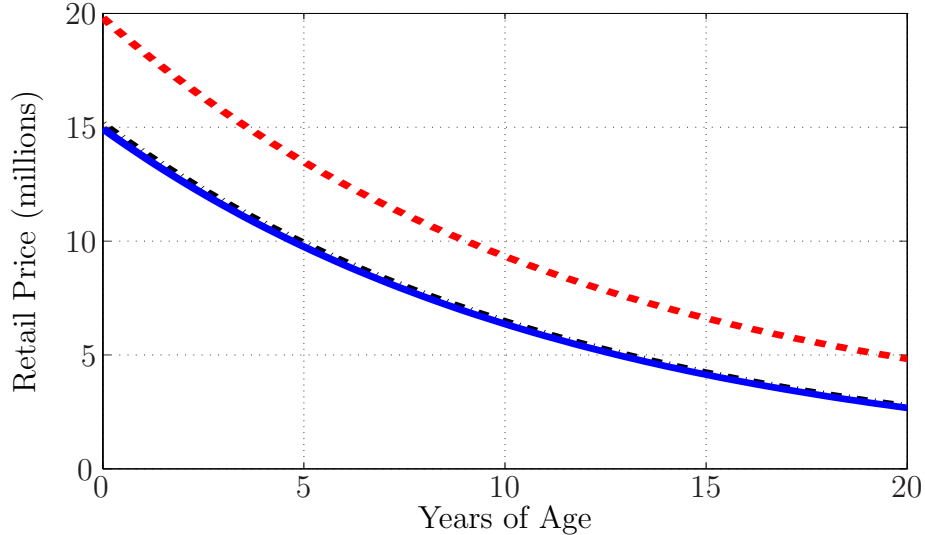


FIG. 3: Actual prices (solid line), Walrasian prices (dashed line) and prices without search costs (dash-dotted line) as a function of aircraft age.

frictions decrease the price of new aircraft by approximately  $\frac{p(0)-p^w(0)}{p^w(0)} = -24.5$  percent (five million dollars) relative to the Walrasian benchmark. The percent decrease of a ten-year-old aircraft is similar.

Two forces affect allocations and prices in the estimated model relative to the Walrasian market: search costs and trading delays. We can assess the relative contributions of these two forces by computing the equilibrium when  $c_s = 0$ , and column (3) in Table 4 reports these calculations. The direct effect of the elimination of search costs is that parties' outside options in bargaining increase, thereby affecting negotiated prices. In addition, this reduction in search costs has general-equilibrium effects by affecting dealers' margins. Thus, dealers' free-entry condition implies that their mass changes, as well. Since dealers' mass determines agents' buying and selling rates, the elimination of search costs affects equilibrium allocations and further affects equilibrium prices. More precisely, dealers' margins decrease, thereby decreasing their mass  $\mu_d$  and their aggregate inventories  $\mu_{do}$ . Hence, sellers' trading rates with dealers  $\gamma_{sd}$  decrease, thereby increasing the mass  $\mu_{lo}$  of sellers. Overall, column (3) says that the mass  $\mu_{lo} + \mu_{do}$  of misallocated aircraft increase when  $c_s = 0$ , indicating that inefficiencies may *increase* when search costs vanish—i.e., in contrast to a large search literature. Column (3) further shows that asset prices increases slightly when  $c_s = 0$ . Thus,

an interesting conclusion of the counterfactual analysis of column (3) is that it may be difficult to infer exclusively from asset prices the effects of changes in trading frictions (or, more generally, changes in trading mechanisms and institutions) on market inefficiencies.

Figure 3 plots the estimated and counterfactual prices, confirming that actual prices (the solid line) are always lower than Walrasian prices (the dashed line) and prices without search costs (the dash-dotted line). Moreover, the figure displays an additional interesting pattern: Walrasian prices decline at a faster rate than actual prices. The reason is that the willingness to pay for a marginally younger—i.e., better—aircraft is higher if there are no frictions.

Overall, these counterfactuals imply non-trivial effects on allocations and prices, illustrating clearly the importance of frictions in this decentralized market.

## 7.2 Quantifying the Effects of Dealers

The estimates of the parameters allow me to quantify the effects of dealers on the allocation and the prices of assets. Specifically, I compare asset allocations and asset prices with the counterfactual of no dealers—i.e.,  $\gamma' = 0$ .

If  $\gamma' = 0$ , then the distribution of agents becomes:

$$\begin{aligned}\mu_{lo} &= \frac{\gamma\mu_{lo}}{\gamma\mu_{hn}(\lambda + \gamma\mu_{lo})}\mu, \\ \mu_{ho} &= \frac{\gamma\mu_{lo}}{\lambda(\lambda + \gamma\mu_{lo})}\mu, \\ \mu_{hn} &= \frac{\mu}{\lambda + \gamma\mu_{lo}}.\end{aligned}$$

Using  $A = \mu_{ho} + \mu_{lo}$ , we obtain  $A = \frac{\gamma\mu_{lo}}{\lambda(\lambda + \gamma\mu_{lo})}\mu + \mu_{lo}$ , which we can solve for  $\mu_{lo}$ :

$$\mu_{lo} = \frac{(A\lambda\gamma - \mu\gamma - \lambda^2) + \sqrt{(A\lambda\gamma - \mu\gamma - \lambda^2)^2 + 4A\lambda^3\gamma}}{2\lambda\gamma}$$

(the other root is negative). This distribution of agents determines the counterfactual trading probabilities  $\gamma_s$  and  $\gamma_b$ , which are key components of the coefficients  $\phi_0$  and  $\phi_1$  of the equilibrium prices  $p^{nd}(a)$ .

Table 5 reports the calculations of these magnitudes for the model and the counterfactual market with no dealers, and Figure 4 plots the prices. The table and the figure show inter-

TABLE 5: Comparison with a Market with No Dealers

	ESTIMATED MODEL	NO DEALERS MARKET
$\mu_{lo} + \mu_{do}$	1, 241 [1, 220; 1, 260]	1, 573 [1, 549; 1, 596]
$\frac{\mu_{lo} + \mu_{do}}{A}$	0.120 [0.118; 0.121]	0.152 [0.150; 0.154]
$p(0)$	14, 936, 993 [14, 687, 000; 15, 184, 000]	15, 199, 078 [14, 952, 000; 15, 448, 000]
$p(10)$	6, 359, 107 [6, 301, 400; 6, 415, 000]	6, 203, 256 [6, 140, 800; 6, 258, 400]

Notes—This table reports counterfactual prices and allocations. 95-percent confidence intervals in brackets are obtained bootstrapping the data using 1,000 replications

esting features. The number of misallocated assets increases by 332 units, reaching a fraction of  $\frac{\mu_{lo}^{nd}}{A} = 15.2$  percent of all aircraft, a 3.2-percentage-points increase. The welfare gain due to dealers' reduction in misallocation is equal to  $(\mu_{lo}^{nd} - \mu_{lo} - \mu_{do})(z_h - z_l) \int (\delta_0 + \delta_1 e^{-\delta_2 a}) dF(a)$ . Using the uniform distribution on  $[0, 160]$  for  $F(a)$ , we compute this welfare gain to be equal to \$42, 223, 907, or  $1 - \frac{\mu_{lo}^{nd} z_l + \mu_{ho}^{nd} z_h}{\mu_{lo} z_l + \mu_{ho} z_h} = 1.2$  percent of actual welfare.

Interestingly, Table 5 shows that prices of new aircraft increase by \$262, 084, a 1.7-percent increase. The intuition for why the effect on allocations is opposite to the effect on prices is that, without dealers, the volume of trade is lower. Therefore, at any point in time, the mass of high-valuation agents seeking to buy an aircraft is larger—i.e.,  $\mu_{hn}^{nd} > \mu_{hn}$ . Hence, it is easier for sellers to find a high-valuation buyer—i.e.,  $\gamma_s^{nd} > \gamma_s$ —thereby increasing their value function  $W_{lo}(a)$  and their outside option in bargaining. This effect dominates the opposite effects of higher search costs and slower trade on asset prices. Hence, this counterfactual analysis reiterates that it may be difficult to infer exclusively from asset prices the effects of changes in trading frictions. Moreover, Figure 4 displays an additional, related pattern: Prices in a market with no dealers decline at a faster rate than actual prices. The reason is that sellers' value function  $W_{lo}(a)$  increases relatively more for sellers of younger aircraft since they value the ability of selling the aircraft at a faster rate relatively more. In turn, the value function  $V_{ho}(a)$  of high-valuation owners increases relatively more for owners of younger aircraft since they value relatively more the ability of being able to sell the aircraft at a faster rate once their valuation changes. Since sellers capture almost the entire buyers'

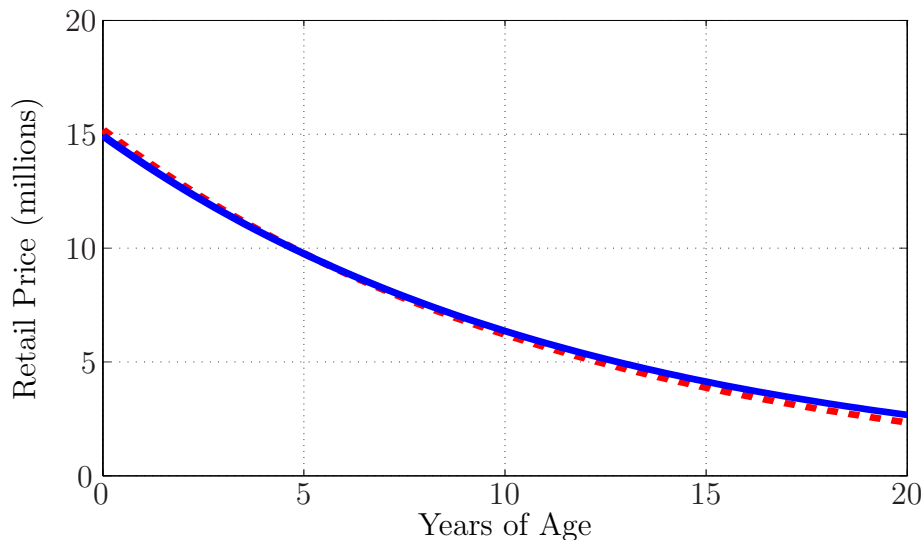


FIG. 4: Actual (solid line) and counterfactual (dashed line) aircraft prices if there were no dealers as a function of aircraft age.

surplus  $V_{ho}(a) - W_{hn}$ , prices of younger aircraft increase relatively more than prices of older ones.

## 8 Concluding Remarks

This paper provides a framework to empirically analyze decentralized asset markets. I apply this framework to investigate the effect of trading frictions on asset allocations and asset prices in the business-aircraft market. The estimated model implies that trading frictions generate moderate inefficiencies in such markets: Compared to the Walrasian benchmark, 12 percent of all business aircraft are misallocated, and aircraft prices are 24.5-percent lower. Moreover, dealers play an important role in reducing frictions: In a market with no dealers, 15.2 percent of the assets would be misallocated. Perhaps surprisingly, in a market with no dealers, prices would *increase* by 1.7 percent, because sellers' outside options improve relative to buyers', thus counteracting the effect of higher search costs and slower trade on asset prices.

The model proposed in the paper describes the key features of asset markets, and its estimation allows me to quantify the importance of frictions and of dealers. The simplicity of the estimation is appealing and suggests that the estimation of richer equilibrium models

of asset markets may be feasible. Nonetheless, the model also has some limitations. First, the supply of asset  $A$  is exogenous. This modeling assumption is common in many other models that focus on secondary markets and on trading frictions. A second limitation is that information on buyers is unavailable: The mass of new buyers  $\mu$  is exogenous, and their meeting rate  $\gamma_b$  and  $\gamma_{bd}$  are derived from the model. In particular, the identification of trading frictions relies exclusively on the sellers' side of the market. This limitation is common to many equilibrium search models, and part of the point of this paper is to show that, under the same assumptions, it is possible to quantify the effect of trading delays on allocations and prices. A third limitation is that there are no aggregate shocks. In reality, the business-aircraft market, like most other asset markets, exhibits non-trivial fluctuations in prices and trading volume, corresponding to fluctuations in supply and demand. The key advantage of a stationary framework is that the model can be solved analytically, thus making the estimation simple. Nonetheless, a potential concern is how sensitive the estimation results are to this stationarity assumption. Fourth, another limitation, quite common among search models, is that price is the only characteristic over which parties bargain, and no other characteristics, such as financing, are taken into consideration.

Finally, in my view, the main limitation is that the model allows for limited heterogeneity in agents' valuations and dealers' inventories. As explained in Section 4, this limitation stems from the aggregate nature of the data, which makes a model with richer heterogeneity difficult to identify. One important consequence of agents' limited heterogeneity is that the marginal buyer and the marginal seller in the estimated model and in the Walrasian market are the same. Instead, if agents' heterogeneity is richer, the marginal buyers and the marginal sellers differ in a model with or without frictions. More precisely, there exists a unique buyers' threshold in the valuation distribution, such that an agent that does not currently own an aircraft and whose valuation jumps above the threshold chooses to acquire one. Similarly, there exists a unique sellers' threshold valuation, such that an agent that currently owns an aircraft and whose valuation falls below the threshold chooses to sell it. When there are trading frictions, buyers' threshold is higher than sellers' threshold since frictions create a wedge that prevents sellers from selling and buyers from buying. Instead, in a frictionless Walrasian market, these thresholds converge, and the marginal buyer and the marginal seller coincide. This argument implies that the mass of misallocated assets may be

larger than the estimated one, since the latter does not take into account the misallocation of assets owned by agents between the thresholds that are not searching to sell or buy an aircraft, but would do so in a frictionless market. Similarly, it implies that Walrasian prices may be lower than the counterfactual values calculated in Section 7.1 since the marginal owner in a Walrasian market has a lower valuation than in the counterfactual market of Section 7.1.

## APPENDICES

### A Derivation of Solution

The system is composed by the following differential equations:

$$\rho V_{ho}(a) = z_h (\delta_0 + \delta_1 e^{-\delta_2 a}) - c_s + \lambda (W_{lo}(a) - V_{ho}(a)) + V'_{ho}(a), \quad (23)$$

$$\begin{aligned} \rho W_{lo}(a) &= z_l (\delta_0 + \delta_1 e^{-\delta_2 a}) - c_s + \gamma_s (p(a) + V_{ln} - W_{lo}(a)) + \\ &\quad \gamma_{sd} (p_B(a) + V_{ln} - W_{lo}(a)) + W'_{lo}(a), \end{aligned} \quad (24)$$

$$\begin{aligned} \rho W_{hn} &= -c_s + \lambda (V_{ln} - W_{hn}) + \gamma_b \int (V_{ho}(a) - p(a) - W_{hn}) dF(a) + \\ &\quad \gamma_{bd} \int (V_{ho}(a) - p_A(a) - W_{hn}) dF(a), \end{aligned} \quad (25)$$

$$\rho V_{ln} = 0,$$

$$\rho J_{do}(a) = -k + \alpha_{ds} (p_A(a) + J_{dn} - J_{do}(a)) + J'_{do}(a), \quad (26)$$

$$\rho J_{dn} = -k + \alpha_{db} \int (J_{do}(a) - p_B(a) - J_{dn}) dF(a) \quad (27)$$

$$p(a) = (1 - \theta_s) (W_{lo}(a) - V_{ln}) + \theta_s (V_{ho}(a) - W_{hn}),$$

$$p_A(a) = (1 - \theta_d) (J_{do}(a) - J_{dn}) + \theta_d (V_{ho}(a) - W_{hn}),$$

$$p_B(a) = (1 - \theta_d) (J_{do}(a) - J_{dn}) + \theta_d (W_{lo}(a) - V_{ln}).$$

I guess the following solutions:

$$V_{ho}(a) = \beta_0 + \beta_1 e^{-\delta_2 a}, \quad (28)$$

$$W_{lo}(a) = \omega_0 + \omega_1 e^{-\delta_2 a}, \quad (29)$$

$$J_{do}(a) = \eta_0 + \eta_1 e^{-\delta_2 a}. \quad (30)$$

These solutions imply that prices are:

$$\begin{aligned} p(a) &= \phi_0 + \phi_1 e^{-\delta_2 a}, \\ p_A(a) &= \tau_0 + \tau_1 e^{-\delta_2 a}, \\ p_B(a) &= \psi_0 + \psi_1 e^{-\delta_2 a}, \end{aligned}$$

where

$$\begin{aligned} \phi_0 &= (1 - \theta_s) \omega_0 + \theta_s (\beta_0 - W_{hn}), \\ \phi_1 &= (1 - \theta_s) \omega_1 + \theta_s \beta_1, \\ \tau_0 &= (1 - \theta_d) (\eta_0 - V_{dn}) + \theta_d (\beta_0 - W_{hn}), \\ \tau_1 &= (1 - \theta_d) \eta_1 + \theta_d \beta_1, \\ \psi_0 &= (1 - \theta_d) (\eta_0 - J_{dn}) + \theta_d \omega_0, \\ \psi_1 &= (1 - \theta_d) \eta_1 + \theta_d \omega_1. \end{aligned}$$

I now solve for the value of the coefficients. Substituting the guesses of  $V_{ho}(a)$  and  $W_{lo}(a)$  into equation (23), we obtain:

$$\rho (\beta_0 + \beta_1 e^{-\delta_2 a}) = z_h (\delta_0 + \delta_1 e^{-\delta_2 a}) - c_s + \lambda (\omega_0 + \omega_1 e^{-\delta_2 a} - \beta_0 - \beta_1 e^{-\delta_2 a}) - \delta_2 \beta_1 e^{-\delta_2 a},$$

which implies:

$$\beta_0 = \frac{z_h \delta_0 - c_s + \lambda \omega_0}{\rho + \lambda}, \quad (31)$$

$$\beta_1 = \frac{z_h \delta_1 + \lambda \omega_1}{\delta_2 + \rho + \lambda}. \quad (32)$$

Similarly, substituting the guesses of  $V_{ho}(a)$  and  $W_{lo}(a)$  into equation (24), we obtain:

$$\begin{aligned} \rho (\omega_0 + \omega_1 e^{-\delta_2 a}) &= z_l (\delta_0 + \delta_1 e^{-\delta_2 a}) - c_s + \gamma_s \theta_s (\beta_0 + \beta_1 e^{-\delta_2 a} - W_{hn} - \omega_0 - \omega_1 e^{-\delta_2 a}) + \\ &\quad \gamma_{sd} (1 - \theta_d) (\eta_0 + \eta_1 e^{-\delta_2 a} - J_{dn} - \omega_0 - \omega_1 e^{-\delta_2 a}) - \omega_1 \delta_2 e^{-\delta_2 a}, \end{aligned}$$

which implies:

$$\beta_0 = \frac{\rho\omega_0 - z_l\delta_0 + c_s + \gamma_s\theta_s(W_{hn} + \omega_0) - \gamma_{sd}(1 - \theta_d)(\eta_0 - J_{dn} - \omega_0)}{\gamma_s\theta_s},$$

$$\beta_1 = \frac{\rho\omega_1 - z_l\delta_1 + \gamma_s\theta_s\omega_1 - \gamma_{sd}(1 - \theta_d)(\eta_1 - \omega_1) + \omega_1\delta_2}{\gamma_s\theta_s}.$$

Substituting the guesses of  $V_{ho}(a)$ ,  $W_{lo}(a)$  and  $J_{do}(a)$  into (26), we obtain:

$$\rho(\eta_0 + \eta_1 e^{-\delta_2 a}) = -k + \alpha_{ds}\theta_d(\beta_0 + \beta_1 e^{-\delta_2 a} - \eta_0 - \eta_1 e^{-\delta_2 a} + J_{dn}) - \delta_2\eta_1 e^{-\delta_2 a},$$

which implies:

$$\eta_0 = \frac{-k + \alpha_{ds}\theta_d(\beta_0 + J_{dn} - W_{hn})}{\rho + \alpha_{ds}\theta_d},$$

$$\eta_1 = \frac{\alpha_{ds}\theta_d\beta_1}{\rho + \delta_2 + \alpha_{ds}\theta_d}.$$

Thus, we can rewrite  $\beta_0$  and  $\beta_1$  as:

$$\beta_0 = \frac{\rho\omega_0 - z_l\delta_0 + c_s + \gamma_s\theta_s(W_{hn} + \omega_0) + \gamma_{sd}(1 - \theta_d)\left(\frac{k + \rho J_{dn} + \theta_d \alpha_{ds} W_{hn}}{\rho + \theta_d \alpha_{ds}} + \omega_0\right)}{\gamma_s\theta_s + \gamma_{sd}(1 - \theta_d)\frac{\alpha_{ds}\theta_d}{\rho + \alpha_{ds}\theta_d}}, \quad (33)$$

$$\beta_1 = \frac{\rho\omega_1 - z_l\delta_1 + \gamma_s\theta_s\omega_1 + \gamma_{sd}(1 - \theta_d)\omega_1 + \omega_1\delta_2}{\gamma_s\theta_s + \gamma_{sd}(1 - \theta_d)\frac{\alpha_{ds}\theta_d}{\rho + \delta_2 + \alpha_{ds}\theta_d}}. \quad (34)$$

Equating (31) with (33), and (32) with (34), we can solve for  $\omega_0$  and  $\omega_1$ :

$$\omega_0 = \frac{\frac{c_s - \delta_0 z_h}{\lambda + \rho} + \frac{c_s - \delta_0 z_l + \theta_s \gamma_s W_{hn} + \gamma_{sd}(1 - \theta_d)\frac{k + \rho J_{dn} + \theta_d \alpha_{ds} W_{hn}}{\rho + \theta_d \alpha_{ds}}}{\frac{\lambda}{\lambda + \rho} - \frac{\gamma_s \theta_s + \gamma_{sd}(1 - \theta_d)\frac{\alpha_{ds}\theta_d}{\rho + \alpha_{ds}\theta_d}}{\gamma_s \theta_s + \gamma_{sd}(1 - \theta_d)\frac{\alpha_{ds}\theta_d}{\rho + \alpha_{ds}\theta_d}}},$$

$$\omega_1 = \delta_1 \frac{\left(\gamma_s \theta_s + \gamma_{sd}(1 - \theta_d)\frac{\alpha_{ds}\theta_d}{\rho + \delta_2 + \alpha_{ds}\theta_d}\right) z_h + (\delta_2 + \rho + \lambda) z_l}{(\delta_2 + \rho + \lambda)(\rho + \gamma_s \theta_s + \gamma_{sd}(1 - \theta_d) + \delta_2) - \left(\gamma_s \theta_s + \gamma_{sd}(1 - \theta_d)\frac{\alpha_{ds}\theta_d}{\rho + \delta_2 + \alpha_{ds}\theta_d}\right) \lambda}.$$

Substituting  $\omega_0$  and  $\omega_1$  into equations (31) and (32), we obtain that  $\beta_0$  and  $\beta_1$  are equal to:

$$\beta_0 = \frac{c_s - z_l \delta_0 + \gamma_s \theta_s W_{hn} + \gamma_{sd} (1 - \theta_d) \frac{k + \rho J_{dn} + \theta_d \alpha_{ds} W_{hn}}{\rho + \theta_d \alpha_{ds}}}{\frac{\lambda}{\lambda + \rho} \left( \gamma_s \theta_s + \gamma_{sd} (1 - \theta_d) \frac{\alpha_{ds} \theta_d}{\rho + \alpha_{ds} \theta_d} \right) - \gamma_s \theta_s - \gamma_{sd} (1 - \theta_d)} \left( \frac{\lambda}{\lambda + \rho} + \frac{\rho}{\gamma_s \theta_s + \gamma_{sd} (1 - \theta_d) \frac{\alpha_{ds} \theta_d}{\rho + \alpha_{ds} \theta_d}} \right)$$

$$+ \frac{\frac{c_s - \delta_0 z_h}{\lambda + \rho} (\rho + \gamma_s \theta_s + \gamma_{sd} (1 - \theta_d))}{\gamma_s \theta_s + \gamma_{sd} (1 - \theta_d) \frac{\alpha_{ds} \theta_d}{\rho + \alpha_{ds} \theta_d} \frac{\lambda}{\lambda + \rho} - \frac{\gamma_s \theta_s + \gamma_{sd} (1 - \theta_d)}{\gamma_s \theta_s + \gamma_{sd} (1 - \theta_d) \frac{\alpha_{ds} \theta_d}{\rho + \alpha_{ds} \theta_d}}},$$

$$\beta_1 = \delta_1 \frac{(\rho + \gamma_s \theta_s + \gamma_{sd} (1 - \theta_d) + \delta_2) z_h + \lambda z_l}{(\delta_2 + \rho + \lambda) (\rho + \gamma_s \theta_s + \gamma_{sd} (1 - \theta_d) + \delta_2) - \left( \gamma_s \theta_s + \gamma_{sd} (1 - \theta_d) \frac{\alpha_{ds} \theta_d}{\rho + \delta_2 + \alpha_{ds} \theta_d} \right) \lambda}.$$

Moreover, substituting equations (28), (29), (30) and prices into  $W_{hn}$  and  $V_{dn}$ , we can rewrite them as:

$$W_{hn} = \frac{-c_s + \gamma_b (1 - \theta_s) \int (\beta_0 - \omega_0 + (\beta_1 - \omega_1) e^{-\delta_2 a}) dF(a)}{\rho + \lambda + \gamma_b (1 - \theta_s) + \gamma_{bd} (1 - \theta_d)} + \frac{\gamma_{bd} (1 - \theta_d) \int (\beta_0 - \eta_0 + J_{dn} + (\beta_1 - \eta_1) e^{-\delta_2 a}) dF(a)}{\rho + \lambda + \gamma_b (1 - \theta_s) + \gamma_{bd} (1 - \theta_d)},$$

$$J_{dn} = \frac{-k + \alpha_{db} \theta_d \int (\eta_0 - \omega_0 + (\eta_1 - \omega_1) e^{-\delta_2 a}) dF(a)}{\rho + \alpha_{db} \theta_d}.$$

## B Robustness of Estimates

In this Appendix, I evaluate the robustness of the estimates reported in Figure 2 and in Table 2 by performing the estimation on two separate sample periods: 1990-1999 and 2000-2008. The left and the right panels of Figure 5 display the set of values of the bargaining parameters consistent with the constraints imposed by the model in each sample period. Both panels confirm that sellers and dealers capture most surplus of transactions, although the set of bargaining parameters consistent with the model is larger in the earlier sample period than in the later one. The main reason for this difference in the estimated bargaining parameters is that the depreciation parameter  $\delta_2$  decreased over time. This decrease lowers the user cost of capital and, for a given level of asset prices, implies an increase in valuations. However, because of trading frictions, an increase in lower-value users' valuations decreases their gains from trade  $W_{lo}(a) - V_{lo}(a)$ , in particular for older vintages and up to a point that they become negative—i.e., lower-value users are better off not selling their assets. Thus,

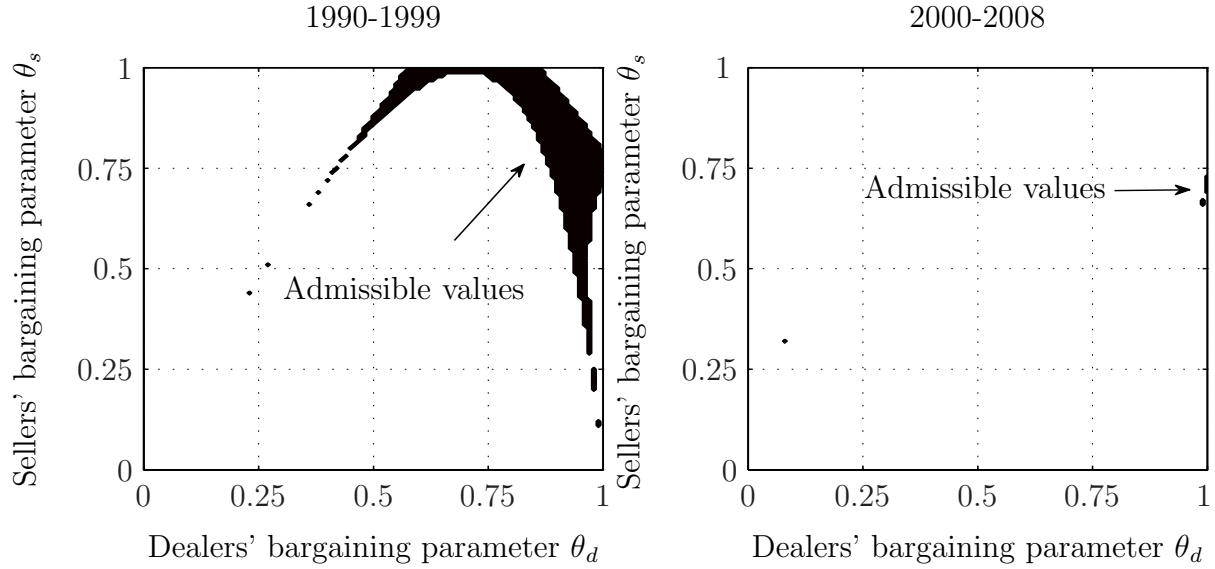


FIG. 5: Admissible values of the bargaining parameters. The left panel refers to the sample period 1990-1999, the right panel refers to the sample period 2000-2008.

the set of bargaining parameters consistent with the equilibrium of the model shrinks as depreciation decreases.

Overall, Table 6 indicates that most estimates of the parameters are stable over time, and very similar to those previously reported in Table 2. Moreover, the estimates on the different sample periods indicates that search costs have decreased over time—as observed in other consumer markets (Brown and Goolsbee, 2002). Similarly, dealers have become more efficient at offloading their inventories over time, perhaps due to higher investments (i.e., higher fixed cost  $k$ ). Furthermore, the table confirms that valuations have become more volatile over time.

TABLE 6: Homogeneous Aircraft: GMM Estimates 1990-1999 and 2000-2008

	1990 – 1999	1990 – 1999	2000 – 2008	2000 – 2008
	$\theta_s = 1$	$\theta_s = .5$	$\theta_s = .66$	$\theta_s = .73$
	$\theta_d = .7$	$\theta_d = .96$	$\theta_d = .99$	$\theta_d = 1$
$\lambda$	0.0523 [0.0512; 0.0532]	0.0523 [0.0512; 0.0532]	0.0556 [0.0546; 0.0566]	0.0556 [0.0546; 0.0566]
$\gamma_s$	0.2701 [0.2611; 0.2784]	0.2701 [0.2611; 0.2784]	0.1939 [0.1882; 0.1992]	0.1939 [0.1882; 0.1992]
$\gamma_{sd}$	0.4731 [0.4568; 0.4889]	0.4731 [0.4568; 0.4889]	0.2733 [0.2653; 0.2816]	0.2733 [0.2653; 0.2816]
$\alpha_{ds}$	0.6345 [0.6168; 0.6526]	0.6345 [0.6168; 0.6526]	0.9715 [0.9459; 0.9972]	0.9715 [0.9459; 0.9972]
$\gamma$	0.0005 [0.0005; 0.0006]	0.0005 [0.0005; 0.0006]	0.00015 [0.00014; 0.00016]	0.00015 [0.00014; 0.00016]
$\gamma'$	0.0013 [0.0012; 0.0013]	0.0013 [0.0012; 0.0013]	0.00075 [0.00073; 0.00078]	0.00075 [0.00073; 0.00078]
$A$	7, 969 [7, 808; 8, 158]	7, 969 [7, 808; 8, 158]	12, 948 [12, 712; 13, 206]	12, 948 [12, 712; 13, 206]
$\mu$	397.2 [387.1; 408.8]	397.2 [387.1; 408.8]	695.6 [680.1; 710.8]	695.6 [680.1; 710.8]
$\mu_d$	747.6 [747.6; 747.6]	747.6 [747.6; 747.6]	737.1 [737.1; 737.1]	737.1 [737.1; 737.1]
$\delta_2$	0.0285 [0.0273; 0.0295]	0.0285 [0.0273; 0.0295]	0.0192 [0.0188; 0.0196]	0.0192 [0.0188; 0.0196]
$\delta_0$	0.0896 [0.0842; 0.1300]	0.1680 [0.1584; 0.1806]	0.0665 [0.0623; 0.0708]	0.0451 [0.0420; 0.0483]
$z_h$	724, 437 [691, 430; 757, 730]	887, 419 [847, 930; 927, 470]	772, 042 [752, 980; 793, 010]	744, 555 [726, 110; 764, 880]
$z_l$	3, 937 [−27, 101; 35, 332]	43, 844 [20, 938; 66, 442]	122, 839 [110, 540; 137, 770]	134, 582 [122, 320; 149, 340]
$c_s$	19, 427 [11, 330; 196, 933]	25, 222 [18, 782; 31, 646]	381.3 [−1, 551; 2, 426]	99.7 [−1583.2; 1929.0]
$k$	38, 287 [33, 973; 41, 358]	202, 733 [194, 450; 212, 670]	337, 298 [325, 430; 348, 740]	299, 314 [288, 760; 309, 600]

Notes—This table reports GMM estimates of the parameters obtained from two separate sample periods: 1990-1999 and 2000-2008 . 95-percent confidence intervals in brackets are obtained bootstrapping the data using 1,000 replications.

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